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A Model of Turbulent-Laminar Gas-Liquid Stratified Flow

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Abstract The time-dependent liquid film thickness and pressure drop are measured by using parallel-wire conductance probes and capacitance differential-pressure transducer. A mathematical model with iterative procedure to calculate holdup and pressure drop in horizontal and inclined gas-liquid stratified flow is developed. The predictions agree well with over a hundred experimental data in 0.024 and 0.04 m diameter pipelines.

Keywords two-phase flow, stratified flow, holdup, pressure drop

1 INTRODUCTION

Stratified two-phase flow in pipes may occur in various chemical and nuclear industrial processes. Examples include the flows of oil and natural gas in pipelines, and flows of steam and water in pipe networks during certain postulated loss-of-coolant accidents. An accurate estimate of the holdup and pressure drop is essential to the design of two-phase flow systems. However, a solution for stratified flow is quite complex due to the two dimensionality of the flow field and the uncertainty of the interfacial shear. As a result, the approaches taken in the past were considerably simplified by using generally either empirical correlations or an one-dimensional approach. The first empirical correlation for pressure drop and holdup in horizontal pipe was proposed by Lockhart and Martinelli[1]. It was based on experimental data spanning various flow patterns, such as slug and annular flows. This is the main reason for errors over 100% reported by the application of the correlations in stratified flow. Russell *et al.*[2] calculated pressure drop by using a geometrical model for the flow cross-section. Taitel and Dukler[3] proposed a model to predict flow regime transition. Their model for stratified flow is an one-dimensional model, since both phases are treated as bulk flows. This approach neglected the detailed velocity profile and the shear stresses were calculated *via* empirical correlations based on the average velocity. Li *et al.*[4] presented experimental data of interfacial shear stress and gave an appropriate correlation of interfacial shear stress in stratified flows. The aim of this study is to develop an analysis and design procedure for turbulent/laminar gas-liquid stratified flow, and to compare predicted pressure drop and holdup results with new experimental data.

2 EXPERIMENTAL

Figure 1 shows a schematic of experimental facility used in this study. It consists of two parallel circuits; one is for air and the other for water. The two streams were brought together in a T pipe and then passed through the test section. A compressor circulated the air, a centrifugal pump circulated the water, and bypass lines allowed the gas and water flows to be adjusted by manual valves. The test section was a lexan pipe, 8 m long with 24 mm or 40 mm(ID). The test section was upwardly inclined with angle 0°, 2°, 4°, −4°. To generate holdup and pressure drop data, the loop was brought to the desired operating condition. The volumetric rate of water was adjusted to a preselected value and the flow rate of gas increased in steps. The liquid level was measured by a parallel wire conductance probe. The probes were inserted at a position about 130-diameter length from the inlet of the pipe. This measuring technique relies on the fact that the conductance between two parallel wires is uniquely related to the liquid level between them. The parallel-wire probes require an electronic analyzer circuit[4], which measures the conductance of the liquid film between the wires and produces a DC output corresponding to the film height. A 100 kHz AC sine signal was applied to the parallel-wire probes while the signal of the wire electrode was fed into the electronic analyzer circuit. The circuits consist of four parts: (1) a current-to-voltage converter circuit, (2) a full-wave rectifier circuit, (3) a second-order low-pass filter, and (4) an amplifying circuit. The second-order low-pass filter was set at 5 kHz. Carrier frequency will not introducing significant distortion of any signal component lowered to 1 kHz. In most experiments, dissolved gasses and variations of temperature can cause slow changes in conductivity. To solve this problem, the conductivity was measured by a reference probe located in the downstream liquid line. The reference probe used the same circuit as the thickness probe, but the circuit was connected to a constant geometry

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test cell. This cell is constructed so that the geometry of the liquid between the wire is independent of flow rate or other conditions of the film. The output of the reference probe therefore varies only with the conductivity of the liquid. The output of the thickness probe is non-dimensionalized by dividing the output of the reference probe. Once a thickness probe is calibrated to relate the dimensionless output to film thickness, the change of the liquid conductivity is eliminated as a variable. The accuracy of the measurements throughout the covered range is around 5%, and the details on the probe accuracy and the calibration procedure were deceribed elewhere[6]. The volumetric flow rates of water and gas were measured by orifice flowmeters, with the error within 2%. Pressure drop measurements were obtained by using a capacity differential pressure transmitter, with a resolution 1%. All experiments were performed at atmospheric pressure. The superficial gas Reynolds number ranged from 400 to 30000, and the superficial liquid Reynolds number ranged from 400 to 2500.

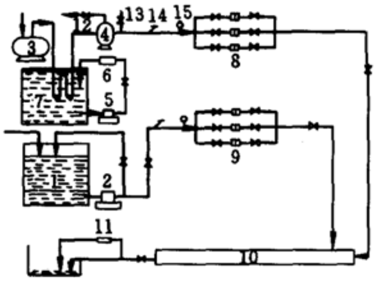

Figure 1 Diagram of experimental facility
1—water tank; 2—water pump; 3—air compressor; 4—air tank; 5—circuit pump; 6—cooling tower; 7—cooling water tank; 8—air orifice plate; 9—water orifice plate; 10—test pipe; 11—reference probe; 12—safe valve; 13—air adjusting valves; 14—temperature transducer; 15—pressure transducer

3 ANALYSIS AND MODEL DEVELOPMENT

From the Baker flow pattern chart[5], stratified flow exists when the air is in turbulent and water is either·laminar or turbulent. We take liquid Reynolds number 2100 as the criterion, which decides whether the stratified flow can be treated as laminar/turbulent motion or turbulent/turbulent gas-liquid stratified flow.

3.1 Model of predicting holdup and pressure drop in laminar/turbulent gas-liquid stratified flow

For gas-liquid stratified flow, we can get the motion equations of gas and liquid phases according to separate-phase model. These equations describe the physical situation shown in Fig. 2

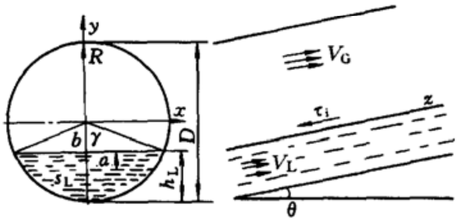

Figure 2 Gas-liquid stratified flow

$$\mu_G\frac{\partial^2 V_G}{\partial x^2}+\mu_G\frac{\partial^2 V_G}{\partial y^2}=\frac{\partial p_G}{\partial z}+\rho_G g\sin\theta \tag{1}$$

$$\mu_L\frac{\partial^2 V_L}{\partial x^2}+\mu_L\frac{\partial^2 V_L}{\partial y^2}=\frac{\partial P_L}{\partial z}+\rho_L g\sin\theta \tag{2}$$

The velocities at the wall are zero

$$V_G=0,\quad V_L=0 \tag{3}$$

The gas pressure drop equals the liquid pressure drop, for $\rho_G \ll \rho_L$

$$\frac{\partial p_G}{\partial z}=\frac{\partial p_L}{\partial z}+\rho_L g\sin\theta \tag{4}$$

The stress and velocity of the two phases at the interface are continuous

$$\mu_G\frac{\partial V_G}{\partial y}=\mu_L\frac{\partial V_L}{\partial y} \tag{5}$$

$$V_G=V_L \tag{6}$$

From Eqs. (1) and (2), we learn that the solution to this coupled set of equations yields velocity profiles V_G and V_L as functions of x and y. Volumetric flow rates can be obtained by integration of each velocity profile. The holdup and pressure drops are easy to be calculated once the interfacial position is known. We give the following definitions

$$Re_G=\frac{V_G h_G \rho_G}{\mu_G} \tag{7}$$

$$Re_L=\frac{V_L h_L \rho_L}{\mu_L} \tag{8}$$

$$V_G=\frac{Q_G}{R_G A},\quad V_L=\frac{Q_L}{R_L A} \tag{9}$$

$$R_G=\frac{A_G}{\pi R^2},\quad R_L=\frac{A_L}{\pi R^2} \tag{10}$$

$$A_G+A_L=\pi R^2,\quad \alpha=\frac{a}{R},\quad \beta=\frac{b}{R} \tag{11}$$

$$H_G=\frac{4A_G}{S_G},\quad H_L=\frac{4A_L}{S_L} \tag{12}$$

$$S_{\mathrm{G}}, S_{\mathrm{L}} = \pi D - R\theta + 2R\sin\gamma \tag{13}$$

where A is the area of the pipe.

For the case of gas in turbulent and liquid in laminar motion, the boundary conditions for Eq. (2) are as follows:

At the wall

$$V_{\mathrm{L}} = 0 \tag{14}$$

At gas-liquid interface

$$\mu_{\mathrm{L}}\frac{\partial V_{\mathrm{L}}}{\partial y} = \tau_i \tag{15}$$

For laminar liquid flow, we treat the interface as smooth. The interface shear stress τ_i is related to $\Delta p_{\mathrm{G}}/L$ as follows

$$\tau_i = \frac{\Delta p_{\mathrm{G}}}{L}\cdot\frac{H_{\mathrm{G}}}{4} \tag{16}$$

For laminar liquid flow, assuming that[2]

$$\frac{\partial^2 V_{\mathrm{L}}}{\partial x^2} = C\frac{\partial^2 V_{\mathrm{L}}}{\partial y^2} \tag{17}$$

then Eq. (2) can be written as

$$\frac{\partial^2 V_{\mathrm{L}}}{\partial y^2} = C_{\mathrm{L}}k \tag{18}$$

where

$$k = \frac{1}{\mu_{\mathrm{L}}}\left(\frac{\Delta p_{\mathrm{L}}}{L} + \rho_{\mathrm{L}}g\sin\theta\right) \tag{19}$$

$$C_{\mathrm{L}} = \frac{1}{1+C}$$

The liquid velocity profiles is obtained by integration of Eq. (18)

$$\frac{\partial V_{\mathrm{L}}}{\partial y} = \int C_{\mathrm{L}}k\mathrm{d}y = C_{\mathrm{L}}ky + C_1(x)$$

$$\begin{aligned} V_{\mathrm{L}} &= \int [C_{\mathrm{L}}ky + C_1(x)]\mathrm{d}\,y \\ &= \frac{1}{2}C_{\mathrm{L}}ky^2 + C_1(x)y + C_2(x) \end{aligned}$$

The boundary conditions are

$$\frac{\partial V_{\mathrm{L}}}{\partial y} = \frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}} \quad \text{when} \quad y = b$$

$$V_{\mathrm{L}} = 0 \quad \text{when} \quad y = \sqrt{R^2 - x^2}$$

from the boundary conditions

$$C_{\mathrm{L}}(x) = \frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}} - C_{\mathrm{L}}kb$$

and

$$\begin{aligned} C_2(x) = &-\frac{1}{2}C_{\mathrm{L}}k(R^2 - x^2) - \\ &\left(\frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}} - C_{\mathrm{L}}kb\right)(R^2 - x^2)^{\frac{1}{2}} \end{aligned}$$

The liquid velocity profile is

$$\begin{aligned} V_{\mathrm{L}} = &\frac{1}{2}C_{\mathrm{L}}ky^2 + \left(\frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}} - C_{\mathrm{L}}kb\right)y - \\ &\frac{1}{2}C_{\mathrm{L}}k(R^2 - x^2) - \left(\frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}} - C_{\mathrm{L}}kb\right)(R^2 - x^2)^{\frac{1}{2}} \end{aligned} \tag{20}$$

The liquid volumetric flow rate is

$$\begin{aligned} Q_{\mathrm{L}} =&2\int_0^a \mathrm{d}x\int_0^{\sqrt{R^2-x^2}}\Bigg[\frac{1}{2}C_{\mathrm{L}}ky^2 + \left(\frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}} - C_{\mathrm{L}}kb\right)y - \\ &\frac{1}{2}C_{\mathrm{L}}k(R^2 - x^2) - \left(\frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}} - C_{\mathrm{L}}kb\right)(R^2 - x^2)^{\frac{1}{2}}\Bigg]\mathrm{d}\,y \\ =&\frac{15}{12}C_{\mathrm{L}}kR^2ab - \frac{1}{6}C_{\mathrm{L}}ka^3b - \Bigg(\frac{C_{\mathrm{L}}k}{4}R^4 + \\ &C_{\mathrm{L}}kb^2\alpha R^2\Bigg)\arcsin\alpha - \frac{\tau_{\mathrm{i}}R^2a}{\mu_{\mathrm{L}}} + \\ &\frac{\tau_{\mathrm{i}}a^3}{3\mu_{\mathrm{L}}} + \frac{\tau_{\mathrm{i}}bR^2}{\mu_{\mathrm{L}}}\arcsin\alpha \end{aligned} \tag{21}$$

Rewriting Eq. (21) as

$$\begin{aligned} \frac{Q_{\mathrm{L}}}{kR^4} =&\frac{C_{\mathrm{L}}}{12}\Big[2\alpha^3\beta - 15\alpha\beta + (3 + 12\beta^2)\arcsin\alpha + \\ &\frac{\tau_{\mathrm{i}}}{\mu_{\mathrm{L}}kC_{\mathrm{L}}R}(12\alpha - 4\alpha^3 - 12\beta\arcsin\alpha)\Big] \end{aligned} \tag{22}$$

and defining a dimensionless term

$$Q^{\circ} = \frac{Q_{\mathrm{L}}}{8kR^4} \tag{23}$$

we have

$$\begin{aligned} Q^{\circ} =&\frac{C_{\mathrm{L}}}{96}\Big\{[2\alpha^3\beta - 15\alpha\beta + (3 + 12\beta^2)\arcsin\alpha] + \\ &\frac{H_{\mathrm{G}}}{C_{\mathrm{L}}D}[6\alpha - 2\alpha^3 - 6\beta\arcsin\alpha]\Big\} \end{aligned} \tag{24}$$

where

$$C_{\mathrm{L}} = 1.05\exp\left[-1.46\frac{h_{\mathrm{L}}}{D}\right] \tag{25}$$

3.2 Calculation procedure

Eq. (22) provides a means of computing pressure drop and holdup if fluid flow rate, pipe size and fluid properties are specified. The iterative calculation procedure required is given below:

(1) The value of h_{L}/D is assumed, α, β, V_{G}, V_{L}, Re_{G}, Re_{L}, R_{G}, R_{L}, H_{L}, H_{G}, S_{G}, S_{L} are easy to be computed.

(2) Q° is obtained from Eq. (24), since Q_{L} and μ_{L} are known, $\Delta p_{\mathrm{L}}/L + \rho_{\mathrm{L}}g\sin\theta$ can be obtained from Eqs. (19) and (23).

(3) The gas-phase pressure drop $\Delta p_{\rm G}/L$ must be equal to $\Delta p_{\rm L}/L + \rho_{\rm L} g \sin\theta$. $\Delta p_{\rm G}/L$ is computed by using single-phase flow procedures $\Delta p_{\rm G}/L = 2 f_{\rm G} V_{\rm G}^2 \rho_{\rm G}/H_{\rm G}$. The gas-phase friction factor can be obtained by using the Blasius equation $f_{\rm G} = 0.079 Re_{\rm G}^{-0.25}$.

(4) If $\Delta p_{\rm G}/L$ is not equal to $\Delta p_{\rm L}/L + \rho_{\rm L} g \sin\theta$, a new value $h_{\rm L}/D$ is selected and the procedure repeated until a satisfactory match is obtained.

Figure 3 shows the comparison of experimental data with prediction. $\mathrm{d}p_{\rm exp}$, $(h_{\rm L}/D)_{\rm exp}$ present experimental data, $\mathrm{d}p_{\rm pred}$, $(h_{\rm L}/L)_{\rm pred}$ present the prediction data of this model. The prediction agrees very well with over a hundred experimental data.

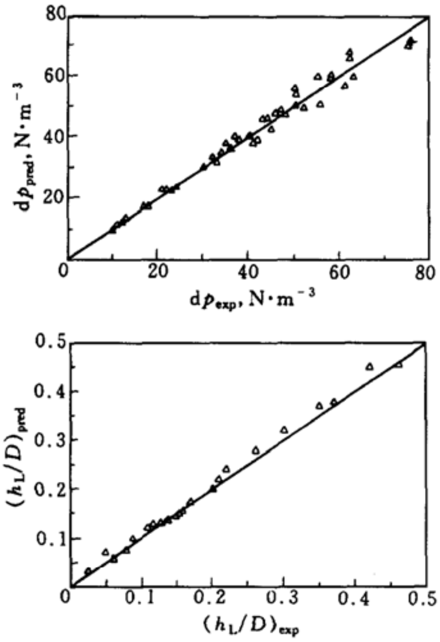

Figure 3 **Comparison experimental date of holdup and pressure drop with prediction**

4 CONCLUSIONS

In this study, the pressure drop and holdup are measured. A model of holdup and pressure drop in turbulent/laminar gas-liquid stratified flow is developed. A procedure for prediction of liquid holdup and pressure drop is presented for stratified flow in horizontal and inclined pipes. The prediction agrees well with experimental data.

NOMENCLATURE

A	area, m^2
a	half the interface length, m
b	distance from center of pipe to interface, m
C	constant
D	diameter, m
f	friction coefficient
h	liquid level, m
L	length, m
p	pressure, Pa
Q	volumetric flow rate, m^3·s^{-1}
R	radius, m
Re	Reynolds number
V	velocity, m·s^{-1}
x, y, z	coordinate
γ	angle, (°)
θ	angle of pipe inclination, (°)
μ	viscosity, kg·m^{-1}·s^{-1}
ρ	density, kg·m^{-3}
τ	shear stress, N·m^{-2}

Subscripts

G	gas
i	interface of gas and liquid
L	liquid
S	superficial

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