SOME RESULTS REGARDING THE COMPARISON
OF THE EARTH'S ATMOSPHERIC MODELS

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SUMMARY: In this paper we examine air densities derived from our realization of aeronomic atmosphere models based on accelerometer measurements from satellites in a low Earth's orbit (LEO). Using the adapted algorithms we derive comparison parameters. The first results concerning the adjustment of the aeronomic models to the total-density model are given.

Key words. Earth

1. SATELLITE DRAG AND
DENSITY VARIABILITY

In this paper we examine densities derived from aeronomic atmosphere models based on accelerometer measurements in a low Earth's orbit (LEO) and compare the measured densities with those predicted by different empirical models.

As in the previous paper (Šegan 1988, Šegan and Pejović 1992, 1993) we start from the equation of hydrostatic equilibrium

$$\frac{dp}{dr} = -\frac{\mu}{r^2}MN = -g(r)\rho, \qquad (1)$$

where the Earth's parameter $\mu = 398601.3\ km^3/s^2$, M is the mean molecular mass of the air, N is the number of molecules, ρ is the air density and r is the geocentric distance, assuming $\Delta r << r$ and $g(r) = const.$ (Δr - height of the atmospheric layer; g(r) - gravitational acceleration). Then, for ideal gas

$$P = NkT = \rho RT, \qquad (2)$$

where $R = k/M$ is the Earth's atmosphere constant, from (1) we have

$$\frac{dp}{p} = -\frac{\mu M}{kT}\frac{dr}{r^2} = -\frac{gM}{k/T}dh = -\frac{dh}{H}, \qquad (3)$$

where h is the altitude and H scale height. Via the barometric equation

$$p(h) = p(h_0)e^{-(h-h_0)/H},$$

we can calculate the total number of particles (molecules, atoms) by

$$N(r) = \int_r^\infty N(r)dr = \int_0^{p(r)} \frac{r^2}{\mu/M}dp = N(r)H. \qquad (4)$$

If the atmosphere is oblate following the Earth's spheroid (ε – flattening factor), for an arbitrary concentric stratum we have expressions

$$r_{p_0} = R_E^{p_0}(1 - \varepsilon \sin^2 \varphi_{p_0})$$

$$R = r_{p_0}(1 + \varepsilon \sin^2 \varphi_{p_0})(1 - \varepsilon \sin^2 \varphi),$$

where R_E is the equatorial radius of a given spheroid, R is the geocentric distance for an arbitrary point of

the spheroid, so that at a geocentric distance r and latitude φ the density is

$$\rho = \rho_{p_0} e^{-(r-R)/H}. \qquad (5)$$

For many years a number of satellites have been supplied with accelerometers to measure the thermospheric density, as well as with neutral and ion mass spectrometers to measure the number densities of the neutral and ionized components in the upper atmosphere. In addition, the thermospheric densities have also been derived from the tracking of satellites and a careful analysis of orbital-parameter changes with time. All of these data sets have led to the development of composite thermospheric-density and temperature models which were used in orbital prediction studies, as well as for a number of scientific purposes. The "Mass Spectrometer Incoherent Scatter Extension XXXX" (CIRAyy, MSISEyy), where for "XXXX" can be substituted "1972", "1986", "1990" is a designation for successive models. One of the most recent models is MSISE90.

The various components of the Solar-Terrestrial System (Sun, Solar Wind, Magnetosphere, Thermosphere, Ionosphere, Middle Atmosphere) and the interactions among them are examined to provide a solid understanding of the reentry and orbital environments within which space vehicles operate.

2. OVERVIEW OF ATMOSPHERE MODELS

Early models of the thermosphere emerged about 1965 (e.g. Harris-Priester and Jacchia-65). These, as well as their descendants Jacchia-71 (Jacchia 1971), CIRA-72, and Jacchia-77 (Jacchia 1977), were based on a numerical quadrature of the species-wise diffusion equations. In these models the altitude profiles of the number densities n_i are largely determined by the magnitude of the exospheric temperature T_∞. This quantity is used to accommodate all activity concerning the diurnal effects, while semi-annual variations are introduced via empirical correction functions. In the Jacchia-77 model species-wise corrections are also introduced for the diurnal, seasonal/latitudinal, and geomagnetic effects. The numerical integration can be very CPU demanding in orbit predictions. In order to improve the time dependence for calls to such routines, Mueller (1982) implemented the Jacchia-Lineberry algorithm for the Jacchia-71 model, and Lafontaine and Hughes (1983) developed an equally efficient method to approximate the Jacchia-77 model. The MET-87 model (Marshall Engineering Thermosphere Model, Hickey 1988) is also based on the early Jacchia-71 atmosphere, but it extends the range of output quantities, including the pressure, the pressure scale height and the ratio of specific heats.

Another line of atmosphere models directly applies analytical solutions of the simplified diffusion equations to derive concentration profiles. The most prominent class of these models is called MSIS (Mass Spectrometer and Incoherent Scatter, Hedin 1987, 1991, 1991).

The MSIS models were continuously improved in 1977, 1983, and 1986 on the basis of new measurement data so that new results became available. MSIS-86 also became the reference atmosphere model referred to as CIRA-86 for thermospheric altitudes. Recently, MSIS-86 was upgraded to MSISe-90 by a continuation to ground level with smooth density and temperature profiles. The DTM-77 model (Density and Temperature Model) by Barlier et al. (1977) has a similar structure as MSIS-77 but it limits itself to the constituents N2, O2, O, and He (Barlier et al. 1977, Koehnlein 1979). Hydrogen, which becomes dominant at high altitudes especially for low activity levels, is not taken into account. The C model by Proelss et al. (1985) also has a MSIS-77 structure with modified correction functions. The advantage of MSIS, DTM, and C lies in their model flexibility to account for observed changes, and in their comprehensive range of output results (including number densities).

In these models, for simplicity we have assumed

$$T(z) = T_\infty - (T_\infty - T_{120})e^{-\sigma\zeta},$$

where T_∞ is the temperature in the thermopause, T_{120} is the temperature at the 120 km boundary and

$$\zeta = (z - 120)(R + 120)/(R + z), \quad R = 6356.77 \text{ km},$$

is the geopotential height, whereas σ is connected with the temperature-gradient parameter s ($s \approx 0.02$) via

$$\sigma = s + (R + 120)^{-1}.$$

The different constituent concentration will be expressed through the spherical functions (Hedin 1986, Barlier 1978). Explicitly, the number densities are

$$n_i(z) = A_i e^{(G_i(L)-1)f_i(z)}$$

where (Walker 1965, Bates 1959)

$$f_i(z) = ((1-a)/(1-ae^{\sigma\zeta})^{1+\alpha_i+\gamma_i} e^{-\sigma\gamma_i\zeta}$$

and

$$a = (T_\infty - T_{120})/T_\infty, \quad \gamma_i = (m_i g_{120})/(\sigma k T_\infty).$$

Here m_i is the molecular mass, k is Boltzmann's constant, g_{120} gravitational acceleration at 120 km height; the functions $G_i(L)$ are dependent of the physical parameters and their expression via the spherical functions is

$$G_i(L) = 1 + f_i(F, \bar{F}, K_p) + \beta[\sum_{p=1}^{\infty} f_2(p, \Omega, d) +$$

$$+ \sum_{n=1}^{\infty} \sum_{m=1}^{n} C_n^m f_3(n, m, \omega, t)]. \qquad (6)$$

A third class of thermosphere models only aims at total densities as an output result. The underlying data of the Russian GOST-84 model are solely derived from the satellite drag analysis. The total density is computed from a reference altitude profile which is adjusted with four factors accounting for (1) diurnal, (2) seasonal/latitudinal, (3) solar activity, and (4) geomagnetic activity effects. An updated set of model coefficients was published in 1990. The TD-88 model (Sehnal et al. 1988) is more flexible in its formulation since TD-88 does not assume a rigorous separation of perturbing effects (factorization of corrections) as done in GOST-84. The TD-88 model, however, should be applied only at altitudes from 150 km to 750 km.

3. THE COMPARISON OF THE MODELS

In applications which require a smooth, continuous density profile with altitude the 'deviation from the diffusive equilibrium' option in MSISe-90 (MSISe-00) should be switched off (corresponding to switch setting $SW(15) = 0.0$).

The MSISe-90 Model will be compared with others with special attention given to the TD-88 model.

We have exploited a partially changed original utility (Hedin 1991) which is designed to give the MSIS total mass density evaluated at a given local time and latitude location over a course of day and annual season. For a Sun-synchronous orbit the local time is fixed, but the longitude changes as the Earth rotates. The input parameters necessary to obtain the MSISe-90(00) density values are the following:

Input Year and Day of Year	*YD*
Time step in hours	*LTh*
Altitude in km	*ATh*
Latitude in degrees	φ
Local Time in hours	*LT*
F10.7 cm (2800MHz) Solar Flux Values	*F*
Three-Month Average of F10.7	$\bar{F}$
Daily Kp Indices	*Kp*

We have prepared a series of plots to make comparisons.

For each of the following comparisons, we have commented on the behavior of the measured and modelled atmospheric densities. For the purpose of comparing the two most interesting models - TD-88 and MSISe-00 - we present Figs. 1-8.
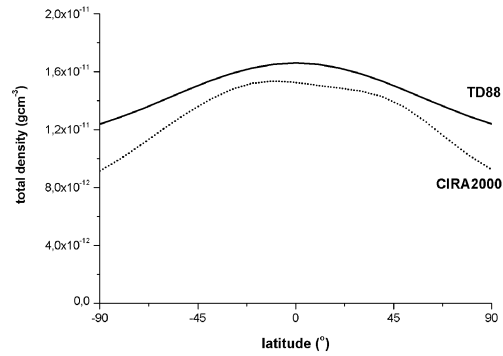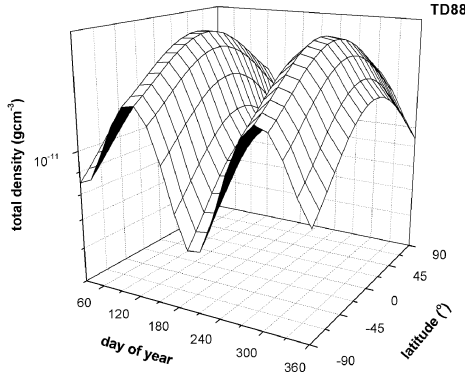

Fig. 1. *Dependence of the total density on the latitude obtained by using TD-88 and MSISe-00; YD=80, LTh=24, ATh=350, φ=(-90,90), LT=14, F=150, $\bar{F}$=150, Kp=3.*
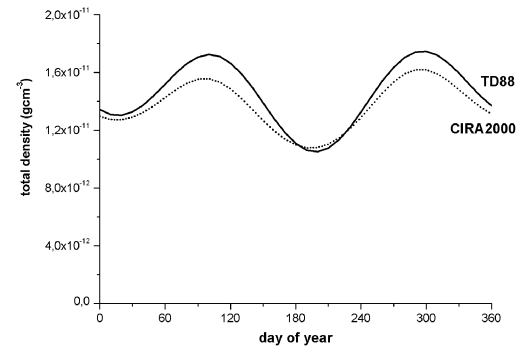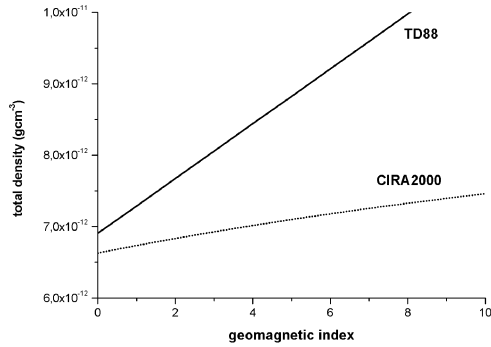

Fig. 2. *Dependence of the total density on the day of the year obtained by using TD-88 and MSISe-00; YD=(0,360), LTh=24, ATh=350, φ=0, LT=14, F=150, $\bar{F}$=150, Kp=3.*
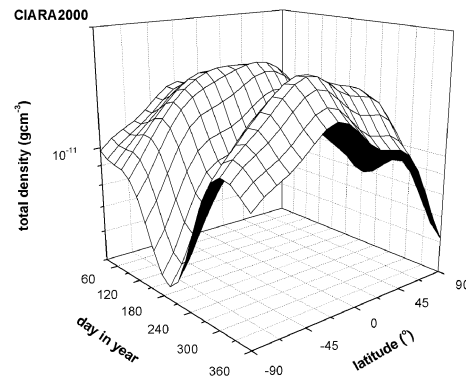

Fig. 3. *Dependence of the total density on the day of the year and on the latitude obtained by using MSISe-00; YD=(0,360), LTh=24, ATh=350, φ=(-90,90), LT=14, F=150, $\bar{F}$=150, Kp=3.*

Fig. 4. *Dependence of the total density on the day of year and on the latitude obtained by using TD-88; YD=(0,360), LTh=24, ATh=350, φ=(-90,90), LT=14, F=150, F̄=150, Kp=3.*


Fig. 5. *Dependence of the total density on geomagnetic index obtained by using TD-88 and MSISe-00; YD=80, LTh=24, ATh=350, φ=0, LT=14, F=150, F̄=150, Kp=(0,10).*
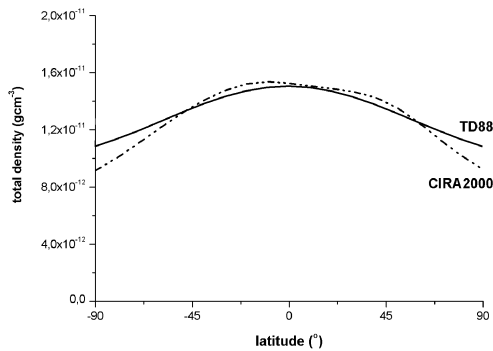
Fig. 6. *Corrected total-density value varying with latitude, obtained by using TD-88; the corresponding values for MSISe-00 are not corrected; YD=80, LTh=24, ATh=350, φ=(-90,90), LT=14, F=150, F̄=150, Kp=3.*
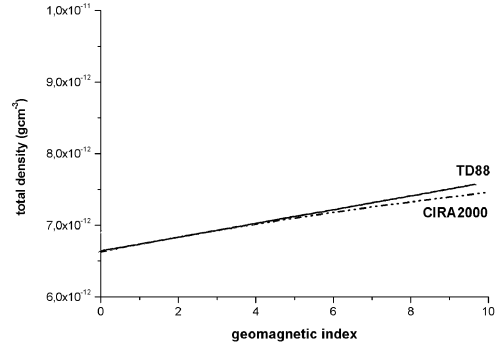
Fig. 7. *Corrected total-density value varying with geomagnetic index, obtained by using TD-88; the corresponding values for MSISe-00 are not corrected; YD=80, LTh=24, ATh=350 φ=0, LT=14, F=150, F̄=150, Kp=(0,10).*
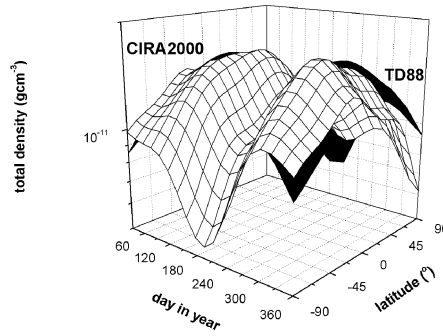
Fig. 8. *Corrected total-density value varying with the day of year and latitude, obtained by using TD-88; the corresponding values for MSISe-00 are not corrected; YD=(0,360), LTh=24, ATh=350, φ=(-90,90), LT=14, F=150, F̄=150, Kp=3.*

As we have stated above, the main result is that the TD-88 model can be improved so to become as close to MSISe-00 as possible (Fig. 6, Fig. 7, Fig. 8). The agreement of the two models is achieved by fitting their differences by using least squares.

The corrected values from Fig. 6 are given by

$$\Delta\rho = +1.44 \times 10^{-12} g/m^3. \tag{7}$$

An analogous situation is found for Fig. 7 where we have the equation

$$\Delta\rho = \Delta\rho_0 + K_p \tan\alpha \tag{8}$$

with $\alpha = 26^o$, $\Delta\rho_0 = 3.04 \times 10^{-13} g/m^3$.

Repeating procedure which gave (7) according to the least squares method in two dimensions we obtain the total densities for the corrected TD-88 model which is presented in Fig. 8.

4. CONCLUSIONS

Exploiting the simpler analytical formulation of the TD-88 model, we showed that it is possible to improve its results by fitting the TD-88 values to those obtained from the MSIS model.

This improvement can be achieved by using least squares for all the changes of the model parameters at once. For this purpose we are preparing a special procedure, based on the use of computer algebra.

REFERENCES

anon.: 1972, "CIRA-72 (COSPAR International Reference Atmosphere 1972)", Akademie Verlag, Berlin.

anon.: 1984, "Density Model for Satellite Orbit Predictions", *GOST 25645-84*, Standards Editing House, Moscow, 1984.

Barlier, F., Berger, C., Falin, J.L., Kocharts, G. and Thuiller, G.: 1978, *Ann. Geophys.*, **34**, 9.

Hedin, A.E.: 1987, *Journal of Geophysical Research*, **92**, 4649.

Hedin, A.E.: 1991, *Journal of Geophysical Research*, **96**, 1159.

Hedin, A.E.: 1991, *Journal of Geophysical Research*, **96**, 7657.

Hickey, M.P.: 1988, *NASA CR-179359*.

Jacchia, L.G.: 1971, *SAO Report*, **332**, May 5.

Jacchia, L.G.: 1977, *SAO Report*, **375**, March 15

Koehnlein, W. et al.: 1979, *Geophys. Res.*, **84**, 4355.

Lafontaine, J.; Hughes, P.: 1983, *Celestial Mechanics*, **29**, 3.

Mueller, A.: 1982, *NASA Report*, 82-FM-52/JSC-18507, Oct.

Proelss, G. W. and Blum, P. W.: 1985, *Proceedings of ESA workshop on Re-Entry of Space Debris, Darmstadt*, 24-25 September 1985 (ESA SP-245).

Sehnal, L. and Pospišilova, L. 1988, *Preprint No. 67*, Observatory Ondrejov.

Šegan, S.: 1988, *Celestial Mechanics*, **41**, 381.

Šegan, S. and Pejović, N.: 1992, *Bull. Astron. Belgrade*, **145**, 101.

Šegan, S. and Pejović, N.: 1993, *Bull. Astron. Belgrade*, **148**, 17.

НЕКИ РЕЗУЛТАТИ ПОРЕЂЕЊА МОДЕЛА ЗЕМЉИНЕ АТМОСФЕРЕ

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Претходно саопштење

Испитане су тоталне густине атмосфере Земље изведене из аерономских атмосферских модела, добијене на бази мерења са сателита на ниским орбитама. Адаптирани су алгоритми за MSISe-00 и TD-88 да би се извели параметри за компарацију. Дати су први резултати за модификацију постојећих модела на једноставни модел тоталне густине у циљу побољшања слагања експерименталних и теоријских вредности и повећања брзине рачуна.