On the Iso-Taxicab Trigonometry

İsmail Kocayusufoğlu and Tuba Ada

Abstract. The Iso-Taxicab Geometry is defined in 1989. The Iso-Taxicab trigonometric functions $\cos_I(\theta)$, $\sin_I(\theta), \tan_I(\theta)$, and $\cot_I(\theta)$ are defined in [1]. The aim of this paper is to give the reduction, addition-substraction and half-angle formulas of Iso- taxicab trigonometry. Finally, we will give the *"Table of Iso-Taxicab Trigonometry"*.

M.S.C. 2000: 51F05, 51F99, 51K05.
Key words: Iso-Taxicab Geometry, Non-Euclidean Geometry.

1 Introduction

Trigonometric functions are the basic ingredients of elementary mathematics. Although they are elementary concepts, we use them almost in everywhere, all the branches of mathematics, engineer, physics, ...

So, defining the trigonometric functions on any geometry has an important place.

Iso-Taxicab Geometry is a new geometry. It is defined in 1989 by K. O. Sowell. It is a non-Euclidean geometry. As it is mentioned in [5] that in iso-taxicab geometry three axes occur at the origin: the x-axis, the y-axis and the $y^{'}$-axis. This latter axis forms an angle of 60° with the x-axis and with the y-axis. But, the points will still be named by ordered pairs of real numbers with respect to the x-axis and the y-axis. These three axes separate the plane into six regions, called hextants. These hextants will be numbered from I to VI in a counterclockwise direction beginning with the hextant where the coordinates of the points are both positive (Figure 1).

At any point in the plane three lines may be drawn parallel to the axes which separate the plane into six regions. Two points, then, may have $I-IV$ or $II-V$ or $III-VI$ orientation to one another. With these orientations, three distance functions arises:

$$d_I(A,B) = \begin{cases} (i) & |x_1 - x_2| + |y_1 - y_2| \quad , \quad I-IV \text{ orientation} \\ (ii) & |y_1 - y_2| \quad , \quad II-V \text{ orientation} \\ (iii) & |x_1 - x_2| \quad , \quad III-VI \text{ orientation} \end{cases}$$

If the points lie on a line parallel to x-axis, the formula (iii) holds; parallel to y or y'-axes, the formula (ii) holds [5].

Applied Sciences, Vol.8, 2006, pp. 101-111.

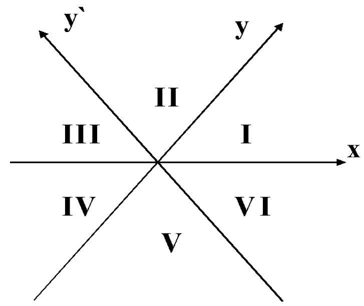

Figure 1

Definitions of iso-taxicab trigonometric functions $\cos_I(\theta)$, $\sin_I(\theta)$, $\tan_I(\theta)$, and $\cot_I(\theta)$ are given in [1]. We, first, recall them here and then we will define the reduction, addition, substraction and half-angle formulas of Iso- taxicab trigonometry. Finally, we will give the "*Table of Iso-Taxicab Trigonometry*".

2 Iso-Taxicab Trigonometric Functions

We first start with defining the iso-taxicab unit circle and then will define iso-taxicab trigonometric functions $\cos_I(\theta)$, $\sin_I(\theta)$, $\tan_I(\theta)$, and $\cot_I(\theta)$.

Definition 1. *The set of points such that the iso-taxicab distance from the origin is* 1 *defines the* iso-taxicab unit circle.

The equation of Iso-taxicab unit circle can written as

$$C_I = \begin{cases} (x,y)| \quad |x|+|y|=1 & , \quad I-IV \text{ orientation} \\ (x,y)| \quad |y|=1 & , \quad II-V \text{ orientation} \\ (x,y)| \quad |x|=1 & , \quad III-VI \text{ orientation} \end{cases}$$

Definition 2. *Let* $f:[0,2\pi_I)\to C_I$ *be a function*

such that

$$f(\theta)=P \Leftrightarrow \begin{cases} d_I(P,A) & =\theta \quad , \quad \theta\in I & = \quad [0,\frac{\pi_I}{3}) \\ 1+d_I(P,B) & =\theta \quad , \quad \theta\in II & = \quad [\frac{\pi_I}{3},\frac{2\pi_I}{3}) \\ 2+d_I(P,C) & =\theta \quad , \quad \theta\in III & = \quad [\frac{2\pi_I}{3},\pi_I) \\ 3+d_I(P,D) & =\theta \quad , \quad \theta\in IV & = \quad [\pi_I,\frac{4\pi_I}{3}) \\ 4+d_I(P,E) & =\theta \quad , \quad \theta\in V & = \quad [\frac{4\pi_I}{3},\frac{5\pi_I}{3}) \\ 5+d_I(P,F) & =\theta \quad , \quad \theta\in VI & = \quad [\frac{5\pi_I}{3},2\pi_I) \end{cases}$$

f is called the *iso-taxicab trigonometric function*.

It is easy to show that f is one-to-one and surjective function. Using the definition

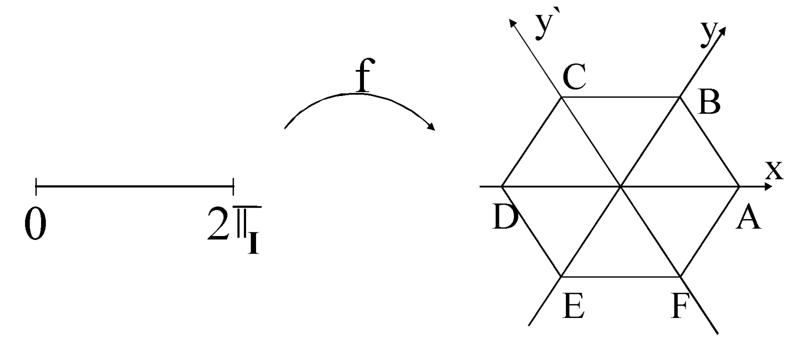

Figure 2

of C_I, we can write

$$f(\theta) = (x,y) \Leftrightarrow \begin{cases} y &= \theta \quad , \quad \theta \in I \\ 1-x &= \theta \quad , \quad \theta \in II \\ 3-y &= \theta \quad , \quad \theta \in III \\ 3-y &= \theta \quad , \quad \theta \in IV \\ 4+x &= \theta \quad , \quad \theta \in V \\ 6+y &= \theta \quad , \quad \theta \in VI \end{cases}$$

Thus, using the iso-taxicab trigonometric function, each angle θ can be represented by a real number in the interval $[0,6)$.

2.1 Iso-Taxicab Cosine and Sine Functions

Definition 3. *Let θ be an angle and Q be a point as in Figure 3. Let $P = (x,y)$ be the intersection point of the line $\overline{OQ}$ and the unit circle C_I. Then, the x coordinate of P defines the Cosine function and the y coordinate of P defines the Sine function. Namely, $x = \cos_I(\theta), y = \sin_I(\theta)$.*

So, by using Definition 1 and Definition 2, we have the following:

$$\begin{array}{llll} \theta \in I & \Rightarrow & \cos_I(\theta) = 1-\theta & , \ \sin_I(\theta) = \theta \\ \theta \in II & \Rightarrow & \cos_I(\theta) = 1-\theta & , \ \sin_I(\theta) = 1 \\ \theta \in III & \Rightarrow & \cos_I(\theta) = -1 & , \ \sin_I(\theta) = 3-\theta \\ \theta \in IV & \Rightarrow & \cos_I(\theta) = \theta-4 & , \ \sin_I(\theta) = 3-\theta \\ \theta \in V & \Rightarrow & \cos_I(\theta) = \theta-4 & , \ \sin_I(\theta) = -1 \\ \theta \in VI & \Rightarrow & \cos_I(\theta) = 1 & , \ \sin_I(\theta) = \theta-6 \end{array}$$

As a result, we have

θ	0	$\frac{\pi_I}{6}$	$\frac{\pi_I}{4}$	$\frac{\pi_I}{3}$	$\frac{\pi_I}{2}$	$\frac{2\pi_I}{3}$	π_I	$\frac{4\pi_I}{3}$	$\frac{3\pi_I}{2}$	$\frac{5\pi_I}{3}$	$2\pi_I$
$\cos_I(\theta)$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-1	-1	0	$\frac{1}{2}$	1	1
$\sin_I(\theta)$	0	$\frac{1}{2}$	$\frac{3}{4}$	1	1	1	0	-1	-1	-1	0

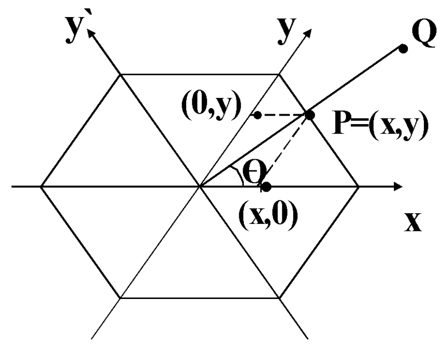

Figure 3

2.2 Iso-Taxicab Tangent and Cotangent Functions

Definition 4. *Let P be a point on the iso-taxicab unit circle, but not on the y-axis. Let $T = (1, t)$ be the intersection point of the line $x = 1$ and the line $\overline{OP}$. The ordinate of the point T is called* the iso-taxicab tangent *of the angle θ and is denoted by* $\tan_I(\theta)$.

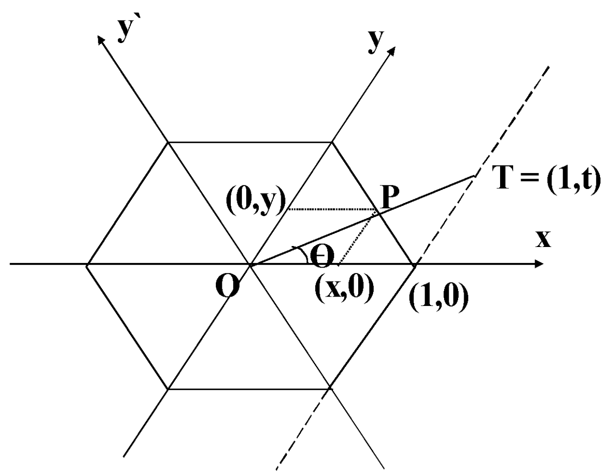

Figure 4

It is clear that the equation of the line $\overline{OP}$ is $y = mx$. Since $T = (1, t) \in \overline{OP}$, we have $t = m \cdot 1 = m$. Thus,

$$\tan_I(\theta) = t = m = \frac{y}{x} = \frac{\sin_I(\theta)}{\cos_I(\theta)}.$$

Definition 5. *Let P be a point on the iso-taxicab unit circle, but not on the x-axis. Let $C=(c,1)$ be the intersection point of the line $y=1$ and the line $\overline{OP}$. The abcyssa of the point C is called* the iso-taxicab cotangent of the angle θ *and is denoted by* $\cot_I(\theta)$.

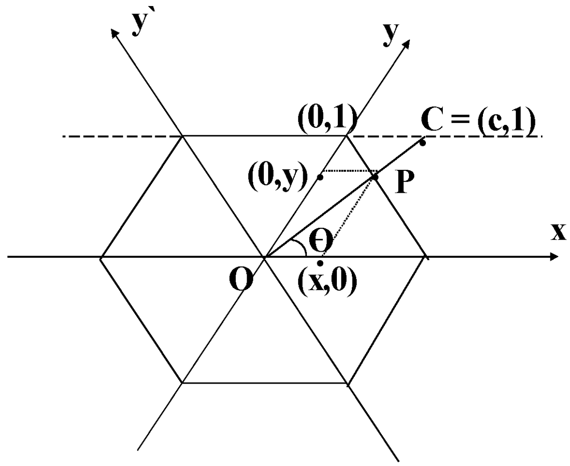

Figure 5

In this case, we have $C=(c,1)\in\overline{OP}$. So,

$$\cot_I(\theta)=c=\frac{1}{m}=\frac{x}{y}=\frac{\cos_I(\theta)}{\sin_I(\theta)}.$$

Therefore, from Definition 4 and Definition 5, we have

$$\tan_I(\theta)=\begin{cases}\frac{\theta}{1-\theta} & , \quad \theta\in I\\ \frac{1}{1-\theta} & , \quad \theta\in II\\ \theta-3 & , \quad \theta\in III\\ \frac{3-\theta}{\theta-4} & , \quad \theta\in IV\\ \frac{1}{4-\theta} & , \quad \theta\in V\\ \theta-6 & , \quad \theta\in VI\end{cases}$$

and $\cot_I(\theta)=1/\tan_I(\theta)$. Thus,

θ	0	$\frac{\pi_I}{6}$	$\frac{\pi_I}{3}$	$\frac{2\pi_I}{3}$	π_I	$\frac{5\pi_I}{3}$	$2\pi_I$
$\tan_I(\theta)$	0	1	$+\infty-\infty$	-1	0	-1	0
$\cot_I(\theta)$	$-\infty+\infty$	1	0	-1	$-\infty+\infty$	-1	$-\infty+\infty$

3 Reduction Formulas

Let $\theta \in I$. It is not difficult to see that the reduction formulas in Iso-taxicab geometry are as folllows.

$\cos_I(\frac{\pi_I}{3}-\theta)$	$=$	$\sin_I(\theta)$	$\sin_I(\frac{\pi_I}{3}-\theta)$	$=$	$\cos_I(\theta)$
$\cos_I(\frac{\pi_I}{3}+\theta)$	$=$	$-\sin_I(\theta)$	$\sin_I(\frac{\pi_I}{3}+\theta)$	$=$	1
$\cos_I(\frac{2\pi_I}{3}-\theta)$	$=$	$-1+\sin_I(\theta)$	$\sin_I(\frac{2\pi_I}{3}-\theta)$	$=$	1
$\cos_I(\frac{2\pi_I}{3}+\theta)$	$=$	-1	$\sin_I(\frac{2\pi_I}{3}+\theta)$	$=$	$1-\sin_I(\theta)$
$\cos_I(\pi_I-\theta)$	$=$	-1	$\sin_I(\pi_I-\theta)$	$=$	$\sin_I(\theta)$
$\cos_I(\pi_I+\theta)$	$=$	$-1+\sin_I(\theta)$	$\sin_I(\pi_I+\theta)$	$=$	$-\sin_I(\theta)$
$\cos_I(\frac{4\pi_I}{3}-\theta)$	$=$	$-\sin_I(\theta)$	$\sin_I(\frac{4\pi_I}{3}-\theta)$	$=$	$-1+\sin_I(\theta)$
$\cos_I(\frac{4\pi_I}{3}+\theta)$	$=$	$\sin_I(\theta)$	$\sin_I(\frac{4\pi_I}{3}+\theta)$	$=$	-1
$\cos_I(\frac{5\pi_I}{3}-\theta)$	$=$	$1-\sin_I(\theta)$	$\sin_I(\frac{5\pi_I}{3}-\theta)$	$=$	-1
$\cos_I(\frac{5\pi_I}{3}+\theta)$	$=$	1	$\sin_I(\frac{5\pi_I}{3}+\theta)$	$=$	$-1+\sin_I(\theta)$
$\cos_I(2\pi_I-\theta)$	$=$	1	$\sin_I(2\pi_I-\theta)$	$=$	$-\sin_I(\theta)$
$\cos_I(2\pi_I+\theta)$	$=$	$\cos_I(\theta)$	$\sin_I(2\pi_I+\theta)$	$=$	$\sin_I(\theta)$

We can conclude that

$$\begin{aligned}\cos_I(2k\pi_I+\theta) &= \cos_I(\theta)\\ \sin_I(2k\pi_I+\theta) &= \sin_I(\theta)\end{aligned}$$

for any natural number k. Therefore, the period of the Cosine and Sine functions is $2\pi_I = 6$.

4 Addition Formulas

There are 42 cases. For each case, we have to apply to Definition 1 and Definition 2 for the vectors θ_1, θ_2, and $\theta_3 = \theta_1 + \theta_2$. We give an example, first. Then we will give the table.

Example 6. *Suppose $\theta_1 \in I$, $\theta_2 \in I$ and $\theta_3 = \theta_1 + \theta_2 \in I$. Let*

$$\begin{aligned}P &= (x,y) &&= (\cos_I(\theta_1), \sin_I(\theta_1))\\ P' &= (x',y') &&= (\cos_I(\theta_2), \sin_I(\theta_2))\\ P'' &= (x'',y'') &&= (\cos_I(\theta_3), \sin_I(\theta_3))\end{aligned}$$

Thus, we have

$$\begin{aligned}&\textit{For } \theta_1 : y = \theta_1 \ , \ x+y = 1\\ &\textit{For } \theta_2 : y' = \theta_2 \ , \ x'+y' = 1\\ &\textit{For } \theta_3 : y'' = \theta_3 \ , \ x''+y'' = 1\end{aligned}$$

Solving these equations for x'' and y'', we get

$$\begin{aligned}x'' &= -1+x+x'\\ y'' &= y+y'\end{aligned}$$

which implies that

$$\begin{aligned} \cos_I(\theta_1+\theta_2) &= -1+\cos_I(\theta_1)+\cos_I(\theta_2) \\ \sin_I(\theta_1+\theta_2) &= \sin_I(\theta_1)+\sin_I(\theta_2)\ . \end{aligned}$$

Let's, now, give the table of formulas for 42 cases:

	θ_1	θ_2	θ_3	$\cos(\theta_1+\theta_2)$	$\sin(\theta_1+\theta_2)$
1	I	I	I	$-1+\cos_I(\theta_1)+\cos_I(\theta_2)$	$\sin_I(\theta_1)+\sin_I(\theta_2)$
2	I	I	II	$1-\sin_I(\theta_1)-\sin_I(\theta_2)$	1
3	I	II	II	$-\sin_I(\theta_1)+\cos_I(\theta_2)$	1
4	I	II	III	1	$1+\cos_I(\theta_1)+\cos_I(\theta_2)$
5	I	III	III	-1	$-\sin_I(\theta_1)+\sin_I(\theta_2)$
6	I	III	IV	$-1+\sin_I(\theta_1)-\sin_I(\theta_2)$	$-\sin_I(\theta_1)+\sin_I(\theta_2)$
7	I	IV	IV	$-1+\sin_I(\theta_1)-\sin_I(\theta_2)$	$-\sin_I(\theta_1)+\sin_I(\theta_2)$
8	I	IV	V	$-1+\sin_I(\theta_1)-\sin_I(\theta_2)$	-1
9	I	V	V	$\sin_I(\theta_1)+\cos_I(\theta_2)$	-1
10	I	V	VI	1	$-2+\sin_I(\theta_1)+\cos_I(\theta_2)$
11	I	VI	VI	1	$\sin_I(\theta_1)+\sin_I(\theta_2)$
12	I	VI	I	$1-\sin_I(\theta_1)-\sin_I(\theta_2)$	$\sin_I(\theta_1)+\sin_I(\theta_2)$
13	II	II	III	-1	$1+\cos_I(\theta_1)+\cos_I(\theta_2)$
14	II	II	IV	$-\cos_I(\theta_1)-\cos_I(\theta_2)$	$1+\cos_I(\theta_1)+\cos_I(\theta_2)$
15	II	III	IV	$-\cos_I(\theta_1)-\sin_I(\theta_2)$	$-1+\cos_I(\theta_1)+\sin_I(\theta_2)$
16	II	III	V	$-\cos_I(\theta_1)-\sin_I(\theta_2)$	-1
17	II	IV	V	$-\cos_I(\theta_1)-\sin_I(\theta_2)$	-1
18	II	IV	VI	1	$-2-\cos_I(\theta_1)-\sin_I(\theta_2)$
19	II	V	VI	1	$-1-\cos_I(\theta_1)+\cos_I(\theta_2)$
20	II	V	I	$2+\cos_I(\theta_1)-\cos_I(\theta_2)$	$-1-\cos_I(\theta_1)+\cos_I(\theta_2)$

	θ_1	θ_2	θ_3	$\cos_I(\theta_1+\theta_2)$	$\sin_I(\theta_1+\theta_2)$
21	II	VI	I	$-1+\cos_I(\theta_1)-\sin_I(\theta_2)$	$2-\cos_I(\theta_1)+\sin_I(\theta_2)$
22	II	VI	II	$\cos_I(\theta_1)-\sin_I(\theta_2)$	1
23	III	III	V	$2-\sin_I(\theta_1)-\sin_I(\theta_2)$	-1
24	III	III	VI	1	$-\sin_I(\theta_1)-\sin_I(\theta_2)$
25	III	IV	VI	1	$-\sin_I(\theta_1)-\sin_I(\theta_2)$
26	III	IV	I	$1+\sin_I(\theta_1)+\sin_I(\theta_2)$	$-\sin_I(\theta_1)-\sin_I(\theta_2)$
27	III	V	I	$\sin_I(\theta_1)-\cos_I(\theta_2)$	$1-\sin_I(\theta_1)+\cos_I(\theta_2)$
28	III	V	II	$\sin_I(\theta_1)-\cos_I(\theta_2)$	1
29	III	VI	I	$-2+\sin_I(\theta_1)-\cos_I(\theta_2)$	$3-\sin_I(\theta_1)+\cos_I(\theta_2)$
30	III	VI	II	$2-\sin_I(\theta_1)+\sin_I(\theta_2)$	1
31	IV	IV	I	$1+\sin_I(\theta_1)+\sin_I(\theta_2)$	$-\sin_I(\theta_1)-\sin_I(\theta_2)$
32	IV	IV	II	$1+\sin_I(\theta_1)+\sin_I(\theta_2)$	1
33	IV	V	II	$\sin_I(\theta_1)-\cos_I(\theta_2)$	1
34	IV	V	III	-1	$2+\sin_I(\theta_1)-\cos_I(\theta_2)$
35	IV	VI	III	-1	$\sin_I(\theta_1)-\sin_I(\theta_2)$
36	IV	VI	IV	$-1-\sin_I(\theta_1)+\sin_I(\theta_2)$	$\sin_I(\theta_1)-\sin_I(\theta_2)$
37	V	V	III	-1	$1-\cos_I(\theta_1)-\cos_I(\theta_2)$
38	V	V	IV	$\cos_I(\theta_1)+\cos_I(\theta_2)$	$1-\cos_I(\theta_1)-\cos_I(\theta_2)$
39	V	VI	IV	$2+\cos_I(\theta_1)+\sin_I(\theta_2)$	$-1-\cos_I(\theta_1)-\sin_I(\theta_2)$
40	V	VI	V	$\cos_I(\theta_1)+\sin_I(\theta_2)$	-1
41	VI	VI	V	$2+\sin_I(\theta_1)+\sin_I(\theta_2)$	-1
42	VI	VI	VI	1	$\sin_I(\theta_1)+\sin_I(\theta_2)$

5 Substraction Formulas

There are 36 cases. For each case, we have to apply to Definition 1 and Definition 2 for the vectors θ_1, θ_2, and $\theta_3=\theta_1-\theta_2$. Let's give an example, first. Then we will give the table.

Example 7. *Suppose $\theta_1 \in I$, $\theta_2 \in I$. Then $\theta_3=\theta_1-\theta_2 \in I$. If*

$$\begin{array}{lllll} P & = & (x,y) & = & (\cos_I(\theta_1),\sin_I(\theta_1)) \\ P' & = & (x',y') & = & (\cos_I(\theta_2),\sin_I(\theta_2)) \\ P'' & = & (x'',y'') & = & (\cos_I(\theta_3),\sin_I(\theta_3)) \end{array}$$

then, we have

$$\begin{array}{llllllll} \text{For } \theta_1 & : & y & = & \theta_1 & , & x+y & = & 1 \\ \text{For } \theta_2 & : & y' & = & \theta_2 & , & x'+y' & = & 1 \\ \text{For } \theta_3 & : & y'' & = & \theta_3 & , & x''+y'' & = & 1 \end{array}$$

Solving these equations for x'' and y'', we get

$$\begin{array}{lll} x'' & = & 1+x-x' \\ y'' & = & y-y' \end{array}$$

which implies that

$$\begin{aligned}\cos_I(\theta_1-\theta_2) &= 1+\cos_I(\theta_1)-\cos_I(\theta_2)\\ \sin_I(\theta_1-\theta_2) &= \sin_I(\theta_1)-\sin_I(\theta_2)\ .\end{aligned}$$

We, now, give the table of formulas for 36 cases:

	θ_1	θ_2	θ_3	$\cos_I(\theta_1-\theta_2)$	$\sin_I(\theta_1-\theta_2)$
1	I	I	I	$1+\cos_I(\theta_1)-\cos_I(\theta_2)$	$\sin_I(\theta_1)-\sin_I(\theta_2)$
2	II	II	I	$1+\cos_I(\theta_1)-\cos_I(\theta_2)$	$-\cos_I(\theta_1)+\cos_I(\theta_2)$
3	III	III	I	$1+\sin_I(\theta_1)-\sin_I(\theta_2)$	$-\sin_I(\theta_1)+\sin_I(\theta_2)$
4	IV	IV	I	$1-\cos_I(\theta_1)+\cos_I(\theta_2)$	$-\sin_I(\theta_1)+\sin_I(\theta_2)$
5	V	V	I	$1-\cos_I(\theta_1)+\cos_I(\theta_2)$	$\cos_I(\theta_1)-\cos_I(\theta_2)$
6	VI	VI	I	$1-\sin_I(\theta_1)+\sin_I(\theta_2)$	$\sin_I(\theta_1)-\sin_I(\theta_2)$
7	II	I	I	$1+\cos_I(\theta_1)-\cos_I(\theta_2)$	$-\cos_I(\theta_1)+\cos_I(\theta_2)$
8	II	I	II	$1+\cos_I(\theta_1)-\cos_I(\theta_2)$	1
9	III	I	II	$-1+\sin_I(\theta_1)-\cos_I(\theta_2)$	1
10	III	I	III	-1	$\sin_I(\theta_1)+\sin_I(\theta_2)$
11	III	II	I	$-1+\sin_I(\theta_1)-\cos_I(\theta_2)$	$2-\sin_I(\theta_1)+\cos_I(\theta_2)$
12	III	II	II	$-1+\sin_I(\theta_1)-\cos_I(\theta_2)$	1
13	IV	I	III	-1	$\sin_I(\theta_1)+\sin_I(\theta_2)$
14	IV	I	IV	$-1+\cos_I(\theta_1)+\cos_I(\theta_2)$	$\sin_I(\theta_1)+\sin_I(\theta_2)$
15	IV	II	II	$-1+\sin_I(\theta_1)-\cos_I(\theta_2)$	1
16	IV	II	III	-1	$-\cos_I(\theta_1)-\cos_I(\theta_2)$
17	IV	III	I	$1+\sin_I(\theta_1)-\sin_I(\theta_2)$	$-\sin_I(\theta_1)+\sin_I(\theta_2)$
18	IV	III	II	$1+\sin_I(\theta_1)-\sin_I(\theta_2)$	1
19	V	I	IV	$-1+\cos_I(\theta_1)+\cos_I(\theta_2)$	$-\cos_I(\theta_1)-\cos_I(\theta_2)$
20	V	I	V	$-1+\cos_I(\theta_1)+\cos_I(\theta_2)$	-1

	θ_1	θ_2	θ_3	$\cos_I(\theta_1-\theta_2)$	$\sin_I(\theta_1-\theta_2)$
21	V	II	III	-1	$-\cos_I(\theta_1)-\cos_I(\theta_2)$
22	V	II	IV	$-1-\cos_I(\theta_1)+\cos_I(\theta_2)$	$-\cos_I(\theta_1)-\cos_I(\theta_2)$
23	V	III	II	$-\cos_I(\theta_1)-\sin_I(\theta_2)$	1
24	V	III	III	-1	$2-\cos_I(\theta_1)-\sin_I(\theta_2)$
25	V	IV	I	$1-\cos_I(\theta_1)+\cos_I(\theta_2)$	$\cos_I(\theta_1)-\cos_I(\theta_2)$
26	V	IV	II	$1-\cos_I(\theta_1)+\cos_I(\theta_2)$	1
27	VI	I	V	$2+\sin_I(\theta_1)-\sin_I(\theta_2)$	-1
28	VI	I	VI	1	$\sin_I(\theta_1)-\sin_I(\theta_2)$
29	VI	II	IV	$1+\sin_I(\theta_1)+\cos_I(\theta_2)$	$-2-\sin_I(\theta_1)-\cos_I(\theta_2)$
30	VI	II	V	$1+\sin_I(\theta_1)+\cos_I(\theta_2)$	-1
31	VI	III	III	-1	$-\sin_I(\theta_1)-\sin_I(\theta_2)$
32	VI	III	IV	$-1+\sin_I(\theta_1)+\sin_I(\theta_2)$	$-\sin_I(\theta_1)-\sin_I(\theta_2)$
33	VI	IV	II	$-2-\sin_I(\theta_1)-\sin_I(\theta_2)$	1
34	VI	IV	III	-1	$-\sin_I(\theta_1)-\sin_I(\theta_2)$
35	VI	V	I	$-1-\sin_I(\theta_1)+\cos_I(\theta_2)$	$2+\sin_I(\theta_1)-\cos_I(\theta_2)$
36	VI	V	II	$-1-\sin_I(\theta_1)+\cos_I(\theta_2)$	1

Using the addition formulas, we can easily find the half-angle formulas in Iso-taxicab geometry.

6 Half-angle Formulas

The half-angle formulas in Iso-taxicab geometry are as follows:

	θ	2θ	$\cos_I(2\theta)$	$\sin_I(2\theta)$
1	I	I	$\cos_I(\theta)-\sin_I(\theta)$	$2\sin_I(\theta)$
2	I	II	$\cos_I(\theta)-\sin_I(\theta)$	1
3	II	III	-1	$1+2\cos_I(\theta)$
4	II	IV	$-2-2\cos_I(\theta)$	$1+2\cos_I(\theta)$
5	III	V	$2-2\sin_I(\theta)$	-1
6	III	VI	1	$-2\sin_I(\theta)$
7	IV	I	$1+2\sin_I(\theta)$	$-2\sin_I(\theta)$
8	IV	II	$1+2\sin_I(\theta)$	1
9	V	III	-1	$1-2\cos_I(\theta)$
10	V	IV	$-2+2\cos_I(\theta)$	$1-2\cos_I(\theta)$
11	VI	V	$2+2\sin_I(\theta)$	-1
12	VI	VI	1	$2\sin_I(\theta)$

Before giving the final concept of this paper, we note that these results may also be written in different way:

It is clear that

$$\begin{aligned} |\cos_I(\theta)|+|\sin_I(\theta)| &= 1 \quad , \quad \theta\in I-IV \\ |\sin_I(\theta)| &= 1 \quad , \quad \theta\in II-V \\ |\cos_I(\theta)| &= 1 \quad , \quad \theta\in III-VI \end{aligned}$$

So, in the case of 4, above, we also have

$$\sin_I(2\theta) = 1+2\cos_I(\theta) = -1+2\sin_I(\theta)$$

7 Table of Iso-Taxicab Trigonometry

Finally, let's give the table of values of Iso-taxicab trigonometric functions.

Degree	Cos_I	$\mathrm{Si}\,n_I$	Tan_I	Cot_I	
0	1	0	0	∞	60
1	0,983	0,016	0,016	59,036	59
2	0,966	0,033	0,034	29,090	58
3	0,950	0,050	0,052	19,000	57
4	0,933	0,066	0,071	14,090	56
5	0,916	0,083	0,090	11,036	55
6	0,900	0,100	0,111	9,000	54
7	0,883	0,116	0,132	7,612	53
8	0,866	0,133	0,153	6,511	52
9	0,850	0,150	0,176	5,666	51

10	0,833	0,166	0,200	5,018	50
11	0,816	0,183	0,224	4,459	49
12	0,800	0,200	0,250	4,000	48
13	0,783	0,216	0,276	3,625	47
14	0,766	0,233	0,304	3,287	46
15	0,750	0,250	0,333	3,000	45
16	0,733	0,266	0,363	2,755	44
17	0,716	0,283	0,395	2,529	43
18	0,700	0,300	0,428	2,333	42
19	0,683	0,316	0,463	2,158	41
20	0,666	0,333	0,500	2,000	40
21	0,650	0,350	0,538	1,857	39
22	0,633	0,366	0,579	1,729	38
23	0,616	0,383	0,621	1,609	37
24	0,600	0,400	0,666	1,500	36
25	0,583	0,416	0,713	1,402	35
26	0,566	0,433	0,765	1,307	34
27	0,550	0,450	0,818	1,222	33
28	0,533	0,466	0,874	1,143	32
29	0,516	0,483	0,936	1,069	31
30	0,5	0,5	1	1	30
	$\mathrm{Si}\,n_I$	Cos_I	Cot_I	Tan_I	Degree

References

[1] İ. Kocayusufoglu, *Trigonometry on Iso-Taxicab Geometry*, Mathematical and Computational Applications, Association for Scientific Research, 5, 3 (1999), 201-212.

[2] E.F. Krause, *Taxicab Geometry*, Addison-Wesley, Menlo Park, NJ, 1975.

[3] K. Menger, *You will like Geometry*, A Guide Book for the Ill. Inst. of Tech. Geometry Exhibition at the Museum of Science and Industry, Chicago, 1952.

[4] S.L. Salas, E. Hille, T.A. Anderson, *Calculus - One and Several Variables*, John Wiley & Sons, 1986.

[5] K.O. Sowell, *Taxicab Geometry -A new slant*, Mathematics Magazine 62 (1989), 238-248.

[6] K. Thompson, T. Dray, *Taxicab angles and trigonometry*, Pi Mu Epsilon 11 (2000), 87-96.

Authors' addresses:

İsmail Kocayusufoğlu
Eskişehir Osmangazi University, Dept. of Mathematics, Eskişehir -Turkey.
email: ikyusuf@ogu.edu.tr

Tuba Ada
Anadolu University, Education Faculty, Eskisehir - Turkey.
email: tyuzugul@anadolu.edu.tr