A Model for the Exchange of Water and Salt Between the Baltic and the Skagerrak

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ABSTRACT

A model for the exchange of salt and water between the Baltic and the Sea (the Skagerrak) is presented. Because of strong inter-basin interactions in the Baltic entrance area, the model must include the Kattegat and the Belt Sea. These are modeled by horizontally homogeneous two-layer sub-models. The most prominent dynamical properties of the sub-models are wind-driven entrainment flows and rotational-baroclinic, hydraulic controls. The model is driven by a meteorologically forced barotropic transport Q_b [calculated from the freshwater supply to the Baltic (Q_f) and the sea level fluctuations in the Kattegat], and turbulent entrainment flows coupled to the wind speed W and, in the Belt Sea, also to the barotropic transport. The most important bathymetric features of the basins are included.

The model equations are integrated numerically for a test period of 1½ years. The stratification in the Kattegat, as well as in the Belt Sea, is quite well predicted. It is found that approximately one-half of the salt transport into the Baltic is carried out by the dispersive mode associated with the barotropic fluctuations.

The effects of (short-term) changes in the external parameters Q_b, Q_f, W and S_{2K} (the salinity of the Skagerrak water) upon the stratification in the Belt Sea and the Kattegat are also investigated. Finally, the effects of long-term changes in the external parameters upon the surface salinity of the Baltic are investigated.

1. Introduction

The Baltic can be regarded as a large estuary. The long-term mean freshwater supply is $\sim$15 000 m^3 s^{-1}. The freshwater supply varies both on annual and interannual time scales. The Baltic proper has a vertically homohaline layer (salinity $S \approx$ 7–8‰), $\sim$50 m in thickness. Below this there is an $\sim$10 m thick halocline overlying the deep water ($S \approx$ 11–13‰) [see Kullenberg (1981) for a description of the hydrography of the Baltic]. The low salinities encountered in the Baltic are generally accepted as due to the freshwater surplus in combination with the long, shallow and partly narrow connection to the sea.

The mean depths of the Belt Sea and the Kattegat (see Fig. 1) are 13 and 26 m, respectively (Svansson, 1975). The narrows in the Belt Sea typically have vertical cross-sectional areas of 300 000 m^2. The sill depth between the Belt Sea and the Baltic is 18 m (the Darss Sill). There is one direct connection between the Baltic and the Kattegat through the Öresund. However, the sill depth here is only 8 m and the vertical cross-sectional area is less than 100 000 m^2. Approximately 25% of the instantaneous flows into/out of the Baltic is believed to pass this way [see Jacobsen (1980) who gives a thorough description of the Belt-Sea and the Öresund].

As a long-term mean the upper layer in the Belt Sea is $\sim$10 m thick and has a salinity of the order of 15‰, while the corresponding figures for the Kattegat upper layer are 15 m and 25‰. Below the pycnoclines, the typical Belt Sea and Kattegat salinities are 18 and 33‰, respectively. However, the stratification in these two seas is very variable. Fig. 2 shows a quasi-synoptic vertical section from the northern Kattegat to the Bornholm Basin. Several fronts in the Belt Sea (the Belt fronts) can be seen. Note that both position and strength of the fronts vary from time to time. This is also the case with the northern Kattegat front (separating Kattegat surface water from Skagerrak water) which cannot, however, be seen in Fig. 2 as, on this occasion, it occupies its most preferred position farther north in the Kattegat.

The exchange of water and suspended and dissolved material, as, for example, salt, between the Baltic and the Sea is a very much studied subject [see Jacobsen (1980), for a review]. The first estimate of the water exchange between the Baltic and the Belt Sea was made by Knudsen [around 1900 (see Jacobsen, 1980)] who assumed a steady baroclinic flow over the Darss Sill (no flow through the Öresund). From measurements, he estimated that the outflowing water has the mean salinity 8.7‰ while the inflowing water has the mean salinity of 17.4‰. Using his, now famous, hydrographical theorems and a reasonable estimate of the freshwater supply to the Baltic, he obtained an approximate figure for the baroclinic flow over the Darss Sill.

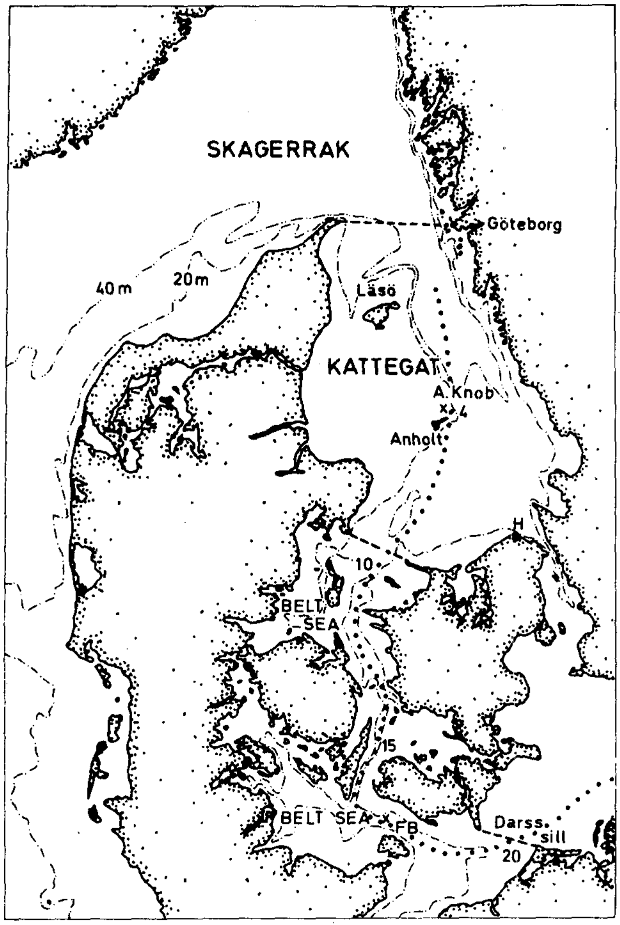

FIG. 1. Map of the actual area. The dotted line from Läsö to the southern Baltic shows the track of the vertical cross section in Fig. 2. H = Hornbaek; FB = Fehmarnbelt.

It has been known for a long time that the instantaneous transports in the Danish Sounds, into and out of the Baltic, may exceed those caused by the mean freshwater surplus by one order of magnitude. These large transports are caused by longitudinal sea-level slopes in the sounds. In the past, several attempts have been made to model the barotropic flow (Jacobsen, 1980). The present author obtained a good description of these barotropic flows from a frictional model driven by sea-level fluctuations in the Kattegat and the freshwater supply to the Baltic (Stigebrandt, 1980); see Fig. 3.

The simple description of a steady, baroclinic water exchange holds only if there is no correlation between the fluctuating components of the transport and the salinity. Then there is no dispersive salt flux. Several studies have shown that the dispersive component of the salt transport across the Darss Sill should be large (Jacobsen, 1980). The present study indicates that approximately one-half of the salt transport into the Baltic is actually carried out by the dispersive mode.

In order to determine the exchange of salt and water between the Baltic and the Sea, one evidently has to know the stratification and the current field on the boundary between the Baltic and the Belt Sea/Öresund (see the map in Fig. 1). In the model presented in this paper the Öresund is closed. Thus we are concerned only with the boundary on the Darss Sill. Upon constructing a model for the Belt Sea, one finds that the vertical stratification in the Kattegat is crucial. As this stratification is strongly coupled by forced interbasin interactions to the stratification in the Belt Sea, it cannot be prescribed. Thus it is necessary to extend the model also to include the Kattegat. Fortunately, it seems possible to stop here as the feedback between the Skagerrak and the Kattegat surface stratification appears weak.

The model for the exchange of salt (buoyancy) and water presented here is probably the simplest possible, physically meaningful model for this complex process. The model consists of three sub-models: 1) the frictional model for the barotropic exchange mentioned above, 2) a model for the stratification in the Kattegat, and 3) a model for the stratification in the Belt Sea. For simplicity, but retaining the most prominent baroclinic properties, the Kattegat and the Belt Sea are both simulated by two horizontally homogeneous layers. Regarding the exchange over the

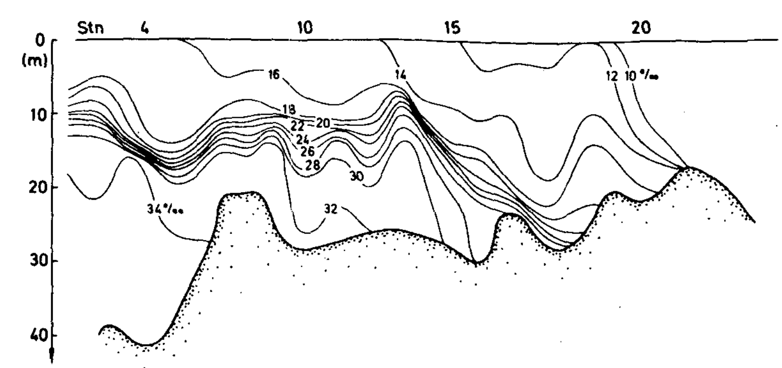

FIG. 2. Vertical cross-section on 3–5 April 1974 showing the salinity distribution from the Kattegat through the Belt Sea and into the Baltic (see Fig. 1).

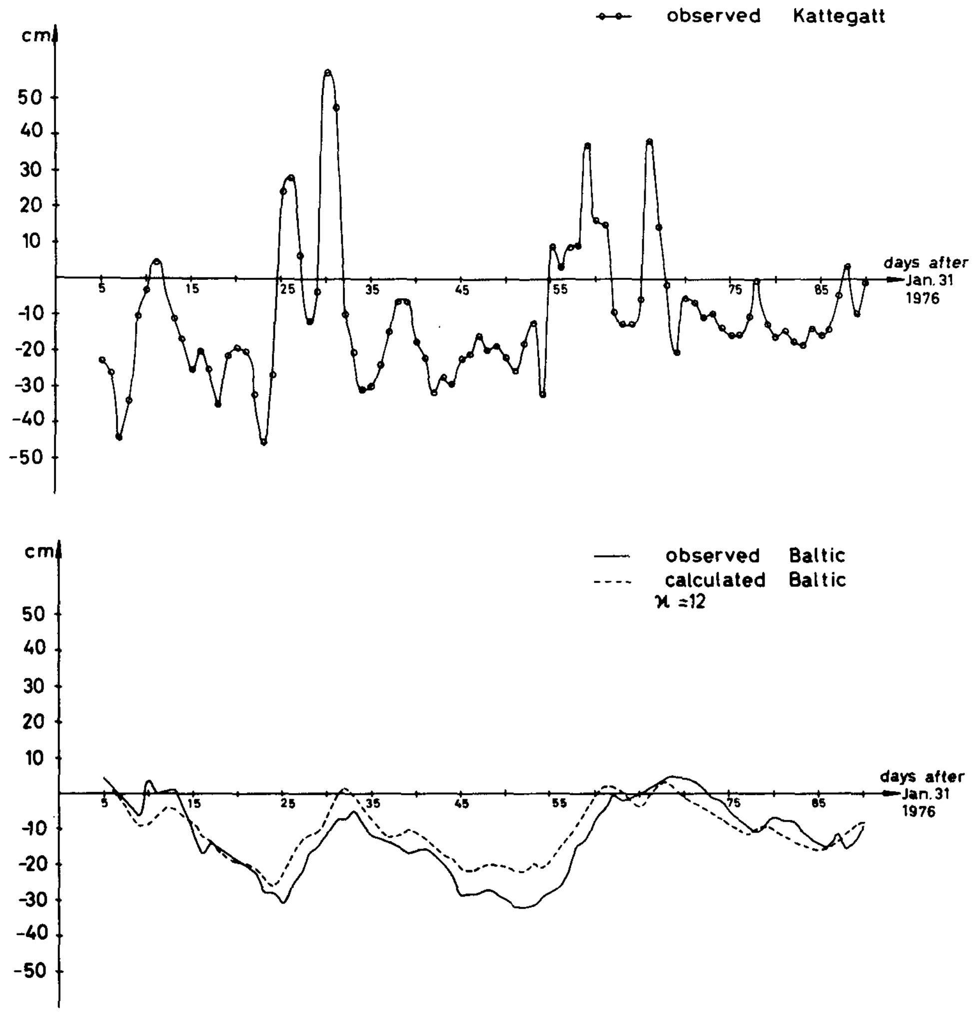

FIG. 3. The mean sea level in (a) the Kattegat (daily means) for a period of three months, and (b) the Baltic. Observed (solid line) and computed by the barotropic model (dashed line). From Stigebrandt, 1980.

Darss Sill there is no need for a model of the stratification in the Baltic Sea as long as the pycnocline there is situated below the sill level (Welander, 1974).

A self-contained model can, in principle, be constructed only if there is no feedback across the downstream section (boundary). Such a case occurs if there is a hydraulic control at this section. This means that the flow through the control section is completely determined by upstream parameters. Thus, ideally, there is no downstream influence upstream of the control.

Our model has two sections with assumed hydraulically controlled flow. Both controls are rotational–baroclinic (two-layer) in character. One is situated in the upper layer in the northern Kattegat, controlling the outflow of Kattegat surface water. The other is situated on the Darss Sill, controlling the flow into the Baltic of relatively dense water from the lower layer in the Belt Sea.

There are two further boundaries where conditions have to be considered. One is the boundary between the Skagerrak and the lower layer in the Kattegat. In the model this layer is treated as a deep, passive reservoir of Skagerrak water, which eliminates the need for boundary conditions here. The other boundary is that between the Baltic surface layer and the upper layer in the Belt Sea. This boundary is situated on the Darss Sill. Here we apply the condition that the transport must be equal to the barotropic transport minus the baroclinic transport in the lower layer. Thus, the total transport across the Darss Sill is the one calculated from the barotropic model. On this boundary we also apply a condition upon the salinity of the flow. When the current switches from one direction to the other, one expects the salinity to approach only gradually the salinity of the upper layers in the Baltic or the Belt Sea. This is accounted for in the model by the introduction of a buffer volume in which the salinity varies from the value in the Baltic to the value in the Belt Sea.

The upper layer of the Kattegat model has some important dynamical properties. On the boundary between the Kattegat and the Skagerrak, a front, separating the Kattegat surface water from the Skagerrak water, is assumed to exist. The front is assumed to be stationary and essentially oriented in the east–west direction. However, near the Swedish coast, the front becomes parallel to the coast and between the coast and the front a coastal current runs northward, draining the surface layer of the Kattegat. The transport effected by this current is determined from an assumption of geostrophy. In the model the wind over the Kattegat creates entrainment flows from the lower to the upper layer, thus raising the salinity of the Kattegat surface layer. Possible downward entrainment flows, caused by bottom currents below the pycnocline, are neglected.

During periods with barotropic inflow to the Baltic, water from the southern Kattegat enters the Belt Sea. The salt content in this water is, of course, heavily dependent upon the vertical stratification in the Kattegat and the topography of the narrows in the northern Belt Sea.

The third sub-model is of the Belt Sea. Also, the Belt Sea is assumed to develop a two-layer structure. In the model, the lower layer consists of essentially surface water from the Kattegat, and water entrained from the upper layer in the Belt Sea (this entrainment in the model is driven by the bottom turbulence generated by barotropic flows). When the barotropic current switches from one direction to the other, the salinity of the water crossing the boundary between the Kattegat and the Belt Sea only gradually attains the value expected from the vertical stratification in the Kattegat or the Belt Sea. This feature is included in the model where a buffer volume is inserted between the Kattegat and the Belt Sea. The Darss Sill divides the deeper parts of the Belt Sea from the Baltic. Water from the dense, lower layer in the Belt Sea flows over the Darss Sill. This flow is assumed to be in geostrophic balance. Of course, the wind may, by entrainment of water from the lower layer in the Belt Sea, create a relatively salty upper layer in the Belt Sea. Intermittently this is swept into the Baltic by the barotropic current.

The two sub-models (for the Kattegat and the Belt Sea) are coupled and the resulting model is run for a period of a year and a half in length. It is driven by the observed wind (from Anholt) and calculated barotropic flows (from the barotropic sub-model).

The stratification in the Kattegat and the Belt Sea is astonishingly well predicted with regard to the complexity of the system. As a result, the exchange of water and salt between the Baltic and the Belt Sea is also quantitatively realistically predicted. The quantitative effects on the stratification in the Kattegat and the Belt Sea as well as on the inflow functions caused by medium-term variations (1–10 years) in the wind speed, the barotropic flow, the freshwater supply and the salinity of the Skagerrak water are also explored. Furthermore, steady-state values of the surface salinity in the Baltic for various values of the external parameters mentioned above are computed.

2. A model for the salinity and thickness of the upper layer in the Kattegat

The Kattegat has a wide and partly deep connection with the Skagerrak as shown in Fig. 1. This implies that the Skagerrak water, of near-oceanic salinity, has an easy access to the Kattegat. Water from the Belt Sea intermittently flows into the Kattegat maintaining a brackish upper layer there. Water of high salinity is added to the brackish upper layer. Wind-excited entrainment is probably the dominating process responsible for this. The brackish layer is drained by leakage into the Skagerrak along the Swedish coast in the so-called Baltic current and, intermittently, into the Belt Sea because of the barotropic current field. The Baltic outflow has a dominant influence upon the stratification in the Kattegat and it also plays a significant role along the Scandinavian coast in the Skagerrak (see Svansson, 1975).

a. The two-layer model

We will investigate the properties of the simple two-layer model verbally sketched in the Introduction (see also Fig. 4). The depth of the pycnocline and the salinities in the two layers are assumed not to vary horizontally. This is a rather coarse simplification. Conservation of volume in the brackish layer gives

$$\frac{dV_{1K}}{dt} = Q_0 - Q_S + Q_{eK}, \tag{1}$$

where V_{1K} is the volume of the brackish layer, Q_S the loss of brackish water to the Skagerrak, Q_{eK} the water entrained from below (by wind-mixing) and Q_0 the exchange of water between the Kattegat surface layer and the Belt Sea. [The relationship between Q_0 and the barotropic flow is given in Eq. (7).] Volume changes of the brackish layer are accompanied by depth changes of the pycnocline. Depth changes could also possibly occur through horizontal compression of the upper layer, caused by advective effects of wind and current upon the open boundary in the northern Kattegat. However, such effects are not considered here.

The topography of the Kattegat is modeled by a rectangular channel of length L_K, with a triangular cross section. Hence, the width $B_K(z)$ of the Kattegat is assumed to decrease linearly with depth; z is the

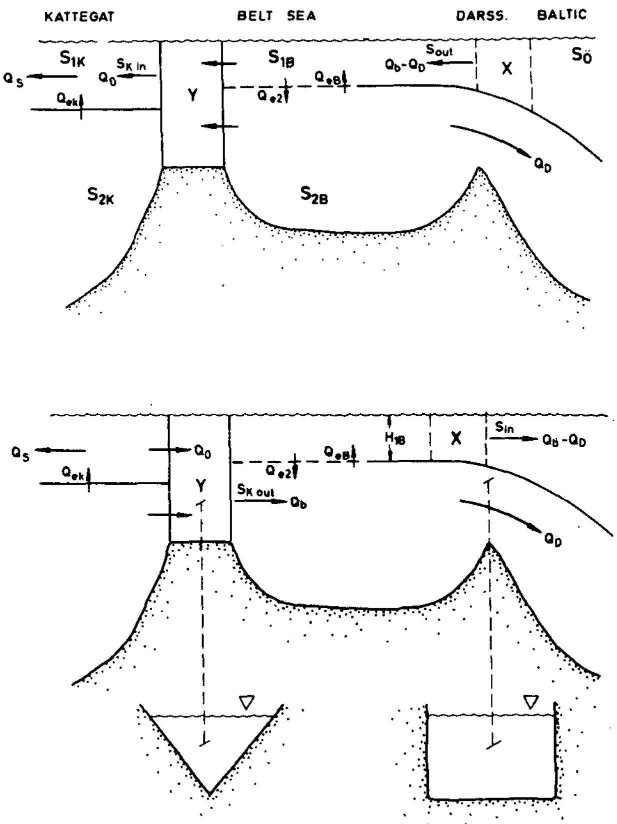

FIG. 4. Sketch showing some of the important features of the model. The buffer volumes X and Y are indicated.

vertical coordinate, positive downward. The volume of the brackish layer is then

$$V_{1K} = L_K \frac{B_K(0) + B_K(H_{1K})}{2} H_{1K}, \qquad (2)$$

where H_{1K} is the vertical distance between the sea surface and the pycnocline. If we specifically put $B_K(z) = B_K(0) \times (1 - z/a)$, where a is a length, we obtain

$$V_{1K} = \tfrac{1}{2} L_K B_K \left(2 - \frac{H_{1K}}{a}\right) H_{1K}, \qquad (2')$$

where $B_K = B_K(0)$. Differentiation of Eq. (2′) yields

$$\frac{dV_{1K}}{dt} = L_K B_K \left(1 - \frac{H_{1K}}{a}\right) \frac{dH_{1K}}{dt}, \qquad (3)$$

and Eqs. (1) and (3) together give

$$\frac{dH_{1K}}{dt} = [Q_0 - Q_S + Q_{eK}] \frac{1}{L_K B_K \left(1 - \frac{H_{1K}}{a}\right)}. \qquad (4)$$

The conservation equation for salt takes two different shapes, depending upon the sign of Q_0; thus

$$\left.\begin{aligned} \frac{d}{dt}(V_{1K} S_{1K}) &= -Q_S S_{1K} + Q_0 S_{K\,in} + Q_{eK} S_{2K}, \\ & \qquad Q_0 > 0 \\ \frac{d}{dt}(V_{1K} S_{1K}) &= -Q_S S_{1K} + Q_0 S_{1K} + Q_{eK} S_{2K}, \\ & \qquad Q_0 < 0 \end{aligned}\right\}, \qquad (5)$$

where $S_{K\,in}$ is the salinity of the water flowing from the Belt Sea into the Kattegat, S_{1K} is the salinity of the upper layer in the Kattegat and S_{2K} is the salinity of the lower layer (Skagerrak water). Utilizing Eq. (3) one obtains

$$\frac{dS_{1K}}{dt} = \frac{1}{B_K L_K H_{1K}} [\epsilon Q_0 (S_{K\,in} - S_{1K}) + Q_{eK}/(S_{2K} - S_{1K})] \frac{1}{1 - H_{1K}/2a}, \qquad (6)$$

where

$$\epsilon = \begin{cases} 1, & \text{if} \quad Q_0 > 0 \\ 0, & \text{if} \quad Q_0 < 0. \end{cases}$$

Hence the salinity of the upper layer in the Kattegat changes only when there are inflows of water from below and/or from the Belts. $S_{K\,in}$ is given by Eq. (18) in Section 3a.

b. *The parameterization of the Q's*

The volume exchange between the Baltic and the Kattegat is mainly driven by water-level fluctuations in the Kattegat. Stigebrandt (1980) presented a model that apparently quite accurately predicted the barotropic exchange Q_b on time scales of one day and longer, as illustrated in Fig. 3. That model takes into account the net freshwater supply to the Baltic. The volume flow of Belt Sea water into the Kattegat is directly taken from that model. The flow of Kattegat surface water into the Belt Sea is taken as the barotropic flow multiplied by the fraction of the vertical surface area of the upper layer in the narrows of the northern Belt Sea. Thus, we disregard possible baroclinic effects, like the process of selective withdrawal, which might modify the vertical distribution of the flow. The narrows in the northern Belt Sea are modeled by a channel of triangular vertical cross-sectional area. The area of the narrows occupied by the upper layer is denoted by A_K. The relation between Q_b and Q_0 is then

$$Q_0 = \begin{cases} -Q_b, & Q_b < 0 \\ -A_K Q_b, & Q_b > 0, \end{cases} \qquad (7)$$

where

$$A_K = \begin{cases} (2 - H_{1K}/b) H_{1K} b^{-1}, & H_{1K} < b \\ 1, & H_{1K} \geq b, \end{cases}$$

Here Q_b is taken from the barotropic model. The flow toward the Baltic is positive in that model which explains the minus sign.

The outflow Q_S from the Kattegat to the Skagerrak is assumed to create a front at the free surface in the Skagerrak. The velocity of the lower layer in the outflow region in the northeastern Kattegat is assumed to be negligible. This seems to be a reasonable assumption as the Kattegat has a large vertical cross-sectional area in this region. Thus

$$Q_S = \frac{g\beta(S_{2K} - S_{1K})}{2f} H_{1K}^2, \tag{8}$$

where f is the Coriolis parameter and β is defined by the simplified equation of state for the brackish water,

$$\rho = \rho_f(1 + \beta S),$$

where ρ_f is the density of fresh water. The assumption of a geostrophically balanced outflow from a two-layer system was used earlier by the present author in a model for the Arctic polar surface water (Stigebrandt, 1981b).

Finally, the entrainment flow, caused by the local wind over the Kattegat, is modeled by the Kato–Phillips entrainment law which seems to work well in estuaries of the fjord-type (strongly stratified) (see Stigebrandt, 1981a):

$$Q_{eK} = B_K L_K \left(1 - \frac{H_{1K}}{a}\right) \frac{2m_0 u_*^3}{g\beta(S_{2K} - S_{1K})H_{1K}}, \tag{9}$$

where m_0 is a constant and u_* the friction velocity in the upper layer. As can be seen the variation of the surface area of the pycnocline with H_{1K} is included in Eq. (9); u_* is related to the wind speed W by the well-known relation

$$u_*^2 = c_d \frac{\rho_a}{\rho_{1K}} W^2, \tag{10}$$

where c_d is the drag coefficient for air flow over a sea surface and ρ_a is the density of air. The numerical value of c_d depends among other things, on the height at which the wind measurements are conducted and the stability of the air (e.g., Kraus 1972).

3. A model for the salinities and the halocline depth in the Belt Sea

In this section we develop a two-layer model for the Belt Sea (see Fig. 4). The oceanographic and topographic conditions in the Belt Sea are described by Jacobsen (1980). The Belt Sea is modeled by a box having the surface area $L_B B_B$ at sea level. The bottom is inclined, and we assume the surface area to decrease linearly with depth; thus

$$L_B B_B(z) = L_B B_B(1 - z/c),$$

where c is a constant. The volume of the Belt Sea below the pycnocline, at a depth H_{1B}, is then

$$V_{2B} = \tfrac{1}{2} B_B L_B (c - H_{1B})^2 c^{-1}. \tag{11}$$

The volume above the pycnocline is

$$V_{1B} = \tfrac{1}{2} B_B L_B H_{1B}(2c - H_{1B}) c^{-1}. \tag{12}$$

a. Equations for the lower layer

Conservation of volume demands

$$\frac{dV_{2B}}{dt} = \begin{cases} Q_b - Q_D - Q_{eB} + Q_{e2}, & Q_b > 0, \\ (1 - A_B)Q_b - Q_D - Q_{eB} + Q_{e2}, & Q_b < 0, \end{cases} \tag{13}$$

where A_B is the fraction of the vertical cross-sectional area of the upper (Belt Sea) layer in the narrows of the northern Belt Sea during outflow to the Kattegat. For this flow direction, too, the baroclinic effects are disregarded in the narrows. Hence

$$A_B = \begin{cases} \left(2 - \dfrac{H_{1B}}{b}\right) \dfrac{H_{1B}}{b}, & H_{1B} < b \\ 1, & H_{1B} \geq b. \end{cases}$$

When the flow is directed from the Kattegat to the Belt Sea, Eq. (13) shows that all water emanating from the Kattegat is assumed to go into the lower layer in the Belt Sea. Q_{eB} is the entrainment flow from the lower to the upper layer caused by wind action; Q_D is the flow from the lower layer in the Belt Sea over the Darss Sill into the Baltic.

It is assumed that there is a rotational hydraulic control at the Darss Sill (analogous to the one at the northern boundary of the Kattegat surface layer), as the magnitude of the (internal) Rossby radius of deformation is considerably smaller than the width of the sill. We assume that the net effect of the alternating currents in the upper layer on Q_D is small. Thus Q_D is determined by

$$Q_D = \frac{g\beta(S_{2B} - S_{1B})}{2f} (H_D - H_{1B})^2, \tag{14}$$

where H_D is the depth of the Darss Sill and f is the Coriolis parameter. Here we have assumed, for simplicity, that the Darss Sill has a rectangular cross-sectional area.

The water Q_{e2} is entrained from the upper to the lower layer. This entrainment is driven by bottom turbulence in the lower layer caused by the barotropic flow. As Q_{e2} may be expected to be proportional to $|Q_b{}^3|A^{-1}$, where A is the vertical cross-sectional area of a section perpendicular to the flow, most of the downward entrainment will occur in the narrows of the Belt Sea. We parameterize Q_{e2} as described below.

A typical velocity in the narrows of the Belt Sea is $U = Q_b/A$. The narrows occupy the horizontal surface area LB where L is the total length and B the typical width of the narrows. If we assume that the hypsographic curve (the distribution of depths) is the same for the narrows as for the Belt Sea (we can then use the same length c), we get, using the Kato–Phillips formula,

$$Q_{e2} = LB\left(1 - \frac{H_{1B}}{c}\right) \times \frac{2m_0 u_{*b}^3}{g\beta(S_{2B} - S_{1B})^{1/2}(c - H_{1B})}. \qquad (15')$$

Here we have used the same efficiency factor (m_0) as for wind-driven entrainment. There are some indications that tidal mixing efficiency might be larger (e.g., Simpson *et al.*, 1978).

The horizontal surface area of the narrows may be obtained from the barotropic model of Stigebrandt (1980). He showed that the best fit is obtained if

$$LB = \frac{6A}{c_{db}}, \qquad (15a)$$

where c_{db} is the drag coefficient for the barotropic flow over the bottom. Eqs. (15′) and (15a) thus give

$$Q_{e2} = 24\frac{A}{c_{db}} \cdot \frac{m_0 u_{*b}^3}{g\beta(S_{2B} - S_{1B}) \cdot c}. \qquad (15)$$

The relation between the friction velocity u_{*b} and U is defined by

$$u_{*b} = \sqrt{c_{db}}|U| = \sqrt{c_{db}}|Q_b|/A. \qquad (16)$$

Thus the frictional resistance against the barotropic flow and the entrainment flow into the lower layer are both caused by the same bottom-generated turbulence field.

Superimposed on the slowly varying barotropic flow, calculated from the barotropic model, there are high-frequency, fluctuating flows. These are mainly caused by the semi-diurnal tidal components in the southern Kattegat. Together these give rise to a semi-diurnal tidal current in the narrows of the Belt Sea which has the typical amplitude $u' = 0.25$ m s^{-1} (T. S. Jacobsen, personal communication). If these fluctuations are to be included in the entrainment formula we must replace $|Q_b|$ in Eq. (16) by the time average of $|Q_b + Q_b'|$, where $Q_b' = u'A$. Thus we obtain

$$u_{*b} = \sqrt{c_{db}}\frac{|Q_b|}{A}\left(1 + 3\frac{Q_b'^2}{Q_b^2}\right)^{1/3}. \qquad (16')$$

We can express dV_{2B}/dt in dH_1B/dt. From Eq. (11) we obtain

$$\frac{dV_{2B}}{dt} = -B_B L_B \frac{1}{c}(c - H_{1B})\frac{dH_{1B}}{dt}.$$

Thus we have

$$\frac{dH_{1B}}{dt} = \begin{cases} -\dfrac{c}{B_B L_B(c - H_{1B})}(Q_b - Q_D - Q_{eB} + Q_{e2}), & Q_b > 0, \\ -\dfrac{c}{B_B L_B(c - H_{1B})}[(1 - A_B)Q_b - Q_D - Q_{eB} + Q_{e2}], & Q_b < 0. \end{cases} \qquad (17)$$

When the barotropic current switches from one direction to the other, we expect the salinity of the water crossing the boundary between the Kattegat and the Belt Sea to approach slowly a limiting value. This is accomplished by inserting a buffer volume Y, which then has to be flushed before the limiting value is reached. The salinity of the water flowing through the buffer volume into the upper layer in the Kattegat (when $Q_b < 0$) is denoted by $S_{K\,\mathrm{in}}$ and the salinity of the water flowing into the lower Belt Sea layer (when $Q_b > 0$) is denoted by $S_{K\,\mathrm{out}}$. If we define

$$S_a = A_B S_{1B} + (1 - A_B)S_{2B},$$

$$S_b = A_K S_{1K} + (1 - A_K)S_{2K},$$

we then have

$$S_{K\,\mathrm{out}} = \begin{cases} S_a + (S_b - S_a)U(I)/Y, & U(I)/Y < 1 \\ S_b, & U(I)/Y \geq 1, \end{cases}$$

$$S_{K\,\mathrm{in}} = \begin{cases} S_b + (S_b - S_a)U(I)/Y, & |U(I)|/Y < 1 \\ S_a, & |U(I)|/Y \geq 1. \end{cases} \qquad (18)$$

Here $U(I)$ is the cumulative flow function defined by

$$U(I) = \begin{cases} U(I-1) + Q_b(I)dt, & Q_b(I)/Q_b(I-1) > 0, \\ Q_b(I)dt, & Q_b(I)/Q_b(I-1) < 0. \end{cases}$$

Here I is the (discrete) time variable and dt is the time difference between the moments I and $I - 1$. Eq. (18) is used in Eq. (6) in the Kattegat model.

The salinity of the lower layer may change by the addition of water of different salinities from the Kattegat and by entrainment from above. Thus

$$\frac{d}{dt}(V_{2B}S_{2B}) = \begin{cases} Q_b S_{K\,\mathrm{out}} - Q_D S_{2B} - Q_{eB}S_{2B} + Q_{e2}S_{1B}, \\ \qquad Q_b > 0, \\ S_{2B}\{(1 - A_K)Q_b - Q_D - Q_{eB}\} + Q_{e2}S_{1B}, \\ \qquad Q_b < 0, \end{cases} \qquad (19)$$

Eqs. (13) and (19) together give

$$\frac{dS_{2B}}{dt} = \begin{cases} \dfrac{Q_b}{V_{2B}}(S_{K\ out} - S_{2B}) - \dfrac{Q_{e2}}{V_{2B}}(S_{2B} - S_{1B}), & \\ & Q_b > 0, \quad (20) \\ -\dfrac{Q_{e2}}{V_{2B}}(S_{2B} - S_{1B}), & Q_b < 0. \end{cases}$$

Thus, if appropriate initial values are given, the depth of the upper layer is determined by Eq. (17) and the salinity of the lower layer by Eq. (20).

b. Equations for the upper layer

The salinity of the upper layer must be determined. Conservation of volume for the upper layer gives

$$\frac{dV_{1B}}{dt} = \begin{cases} Q_{eB} - (Q_b - Q_D) - Q_{e2}, & Q_b > 0, \\ A_B Q_b + Q_{eB} - (Q_b - Q_D) - Q_{e2}, & Q_b < 0, \end{cases} \quad (21)$$

In this model we have neglected the (small) contribution to volume changes in the upper layer that is introduced by sea-level changes.

Conservation of salt gives

$$\frac{d}{dt}(V_{1B}S_{1B}) = \begin{cases} Q_{eB}S_{2B} - (Q_b - Q_D)S_{in} - Q_{e2}S_{1B}, & Q_b > 0, \\ Q_{eB}S_{2B} - A_B Q_b S_{1B} - (Q_b - Q_D)S_{out} - Q_{e2}S_{1B}, & Q_b < 0. \end{cases} \quad (22)$$

Here S_{in} and S_{out} are the salinities of the barotropically transported water over the Darss Sill. Eqs. (21) and (22) give

$$\frac{dS_{1B}}{dt} = \begin{cases} \dfrac{1}{V_{1B}}\{Q_{eB}(S_{2B} - S_{1B}) + (Q_b - Q_D)(S_{1B} - S_{in})\}, & Q_b > 0, \\ \dfrac{1}{V_{1B}}\{Q_{eB}(S_{2B} - S_{1B}) + (Q_b - Q_D)(S_{1B} - S_{out})\}, & Q_b < 0. \end{cases} \quad (23)$$

When the current switches from one direction to the other, here too one expects the salinity gradually to approach S_{1B} or $S_{Ö}$. $S_{Ö}$ is the salinity of the Baltic surface layer. Thus a certain volume X must pass before the limiting value of the salinity is reached. This is expressed in the following way:

$$S_{in} = \begin{cases} S_{Ö} + (S_{1B} - S_{Ö})T(I)/X, & \text{if} \quad T(I)/X < 1, \\ S_{1B}, & \text{if} \quad T(I)/X \geq 1. \end{cases}$$

$$S_{out} = \begin{cases} S_{1B} + (S_{1B} - S_{Ö})T(I)/X, & \text{if} \quad |T(I)|/X < 1, \\ S_{Ö}, & \text{if} \quad |T(I)|/X \geq 1. \end{cases}$$

Here T is a cumulative flow function similar to $U(I)$ and it is defined by

$$T(I) = T(I-1) + [Q_b(I) - Q_D(I)]dt, \quad \text{if} \quad Q_b(I)/Q_b(I-1) > 0,$$

$$T(I) = [Q_b(I) - Q_D(I)]dt, \quad \text{if} \quad Q_b(I)/Q_b(I-1) < 0.$$

The wind-driven entrainment velocity is also described by the Kato–Phillips algorithm. For the Belt Sea, we then obtain

$$Q_{eB} = B_B L_B\left(1 - \frac{H_{1B}}{c}\right)\frac{2m_0 u_*^3}{g\beta(S_{2B} - S_{1B})H_{1B}}. \quad (24)$$

During periods with strong inflow and weak winds, the upper layer in the Belt Sea may possibly be completely eroded from below and the water becomes homogeneous from top to bottom. This feature can also be handled in the model.

4. Some remarks on the exchange of salt between the Baltic and the Belt Sea

By numerical integration of the equations presented in the earlier sections the stratification in the Belt Sea (S_{1B}, S_{2B}, H_{1B}) and the barotropic and baroclinic transport (Q_b, Q_D) are obtained for N equally spaced moments. Thus, we have at our disposal the fields of current and salinity necessary to determine the exchange of salt and water between the Baltic and the Belt Sea. The mean transport of salt out of the Baltic is

$$Z_{out} = \frac{1}{N}\sum_{n=1}^{N} \epsilon(Q_{bn} - Q_{Dn})S_{out}, \quad (25)$$

while the mean transport of salt into the Baltic is

$$Z_{in} = \frac{1}{N}\sum_{n=1}^{N} \{(1-\epsilon)(Q_{bn} - Q_{Dn})S_{in} + Q_{Dn}S_{2Bn}\}. \quad (26)$$

The parameter ϵ is defined by

$$\epsilon = \begin{cases} 1, & \text{if} \quad Q_{bn} - Q_{Dn} < 0 \\ 0, & \text{if} \quad Q_{bn} - Q_{Dn} > 0. \end{cases}$$

The salt balance ($Z_{out} = Z_{in}$) of the Baltic, as it appears for the narrow outlet on the Darss Sill, is maintained by both barotropic and baroclinic modes. As can be seen from Eqs. (25) and (26) it is even more complicated as the baroclinic mode also influences the barotropic mode.

A useful framework for the description of stratified flows has been developed by Walin (1977). The volume flow is divided into discrete salt intervals, i.e., $q(S)$ means the volume flow having salinities in the interval $(S, S + dS)$. As $S_{\rm in}$, $S_{\rm out}$ and S_{2B} vary with time, the inflows and outflows calculated by the model presented in this paper occur in several salt intervals. The net mean flow, during the period under consideration, across the Darss Sill is, in a specific salt interval,

$$q(S) = q(S_{\rm \ddot{o}} + mdS) = \frac{1}{N}\sum_{n=1}^{N} \{(1 - \epsilon)(Q_{bn} - Q_{Dn})\delta_1 + Q_{Dn}\delta_2 + \epsilon(Q_{bn} - Q_{Dn})\delta_3\}, \quad (27)$$

where m is an integer, dS is the chosen width of the salt interval and $S_{\rm \ddot{o}}$ is the salinity of the Baltic surface layer (the lowest salinity in the system considered here). The functions δ_1, δ_2 and δ_3 are defined by

$$\delta_1 = \begin{cases} 1, & \text{if } S_{\rm \ddot{o}} + mdS \le S_{\rm in} < S_{\rm \ddot{o}} + (m+1)dS \\ 0, & \text{otherwise} \end{cases},$$

$$\delta_2 = \begin{cases} 1, & \text{if } S_{\rm \ddot{o}} + mdS \le S_{2B} < S_{\rm \ddot{o}} + (m+1)dS \\ 0, & \text{otherwise} \end{cases},$$

$$\delta_3 = \begin{cases} 1, & \text{if } S_{\rm \ddot{o}} + mdS \le S_{\rm out} < S_{\rm \ddot{o}} + (m+1)dS \\ 0, & \text{otherwise} \end{cases}.$$

The cumulative distribution function $M(S)$ is

$$M(S) = \int_S^\infty q(S')dS'.$$

In discrete form it takes the shape

$$M(S) = M(S_{\rm \ddot{o}} + mdS) = \sum_{m'=m}^{\infty} q(S_{\rm \ddot{o}} + m'dS). \quad (28)$$

The distributions $M(S)$ are calculated for the test period for various values of the external parameters and presented in Section 6.

5. Model results

a. *Field data for forcing and comparison*

In the numerical solution of the coupled model, the salinities $S_{1K}(t)$, $S_{1B}(t)$ and $S_{2B}(t)$ and the halocline depths $H_{1K}(t)$ and $H_{1B}(t)$ are calculated for each day of the test period. This runs from 750 701 to 761 231. The barotropic flow Q_b is first calculated separately by the barotropic model. This model is driven by the observed sea-level fluctuations in the southern part of the Kattegat (daily means from Hornbaek) and the net freshwater supply to the Baltic (daily values were interpolated from the monthly means given by Jacobsen, 1980).

In order to calculate the wind-driven entrainment flows the wind speed is needed. From observations of the wind speed at Anholt, daily means of the "mixing wind," defined by

$$W_n = [1/6 \sum_{k=1}^{6} |W_{nk}|^3]^{1/3},$$

where W_{nk} is the wind speed at the kth observational occasion during the nth day, were calculated. In the present model the wind at Anholt is also allowed to represent the wind over the Belt Sea.

The salinities S_{K0} at the depths 0, 5, 10, 15, 20 and 30 m are measured once every day at the Danish lightship Anholt Knob. The depth H_{1K0} of the upper layer in the Kattegat is here defined as that depth where the salinity is equal to the surface salinity plus 60% of the difference in salinity between 0 and 30 m. The mean salinity of the upper layer defined in this way is calculated. As the horizontal surface area of the Kattegat decreases with the depth (as $1 - z/a$, see Section 2) we have to take this into account when calculating the mean salinity of the upper layer. Thus we define S_{1K0} as

$$S_{1K0} = \frac{1}{H_{1K0}\left(1 - \dfrac{H_{1K0}}{2a}\right)} \int_0^{H_{1K0}} S_{K0}(z)\left(1 - \frac{z}{a}\right)dz. \quad (29)$$

The observed salinity S_{1K0} and depth H_{1K0} of the upper layer in the Kattegat will be compared to the calculated ones (S_{1K} and H_{1K}, respectively).

In the Belt Sea we use salinity observations from the German lightship *Fehmarnbelt*. These are obtained once every day at depths of 0, 5, 10, 15, 20, 25 and 28 m. We define the depth H_{1B0} of the upper layer as that depth where the salinity is equal to the surface salinity plus 60% of the difference in salinity between 0 and 15 m. Here we chose 15 m as there may be a secondary pycnocline at the sill depth. The salinities of the upper and lower layers are calculated from the expressions

$$S_{1B0} = \frac{1}{H_{1B0}\left(1 - \dfrac{H_{1B0}}{2c}\right)} \int_0^{H_{1B0}} S_{B0}(z)(1 - z/c)dz, \quad (30)$$

$$S_{2B0} = \frac{2c}{(c - H_{1B0})^2} \int_{H_{1B0}}^{c} S_{B0}(z)(1 - z/c)dz, \quad (31)$$

where c is defined in section 3.

All numerical constants and initial values used in the computations are given in Table 1. Some results obtained from the model are presented below.

b. *The thickness and salinity of the upper layer in the Kattegat*

The layer of brackish water of height H_{1K} and salinity S_{1K} can be thought of as being a mixture of a layer of freshwater of height $H_f = (S_{2K} - S_{1K})H_{1K}/S_{2K}$ and a layer of Skagerrak water (salinity S_{2K}) of height $H_{1K} - H_f$. H_f may be termed the freshwater height. In the real Kattegat, H_f normally decreases

TABLE 1. Parameter values used in the model.

$B_K L_K = 2.2 \times 10^{10}$ (m^2)	$c_{db} = 3 \times 10^{-3}$
$B_B L_B = 2 \times 10^{10}$ (m^2)	$\left(c_d \dfrac{\rho_a}{\rho_{1K}}\right)^{1/2} = 1.25 \times 10^{-3}$
$a = 46$ (m)	
$b = 15$ (m)	
$c = 26$ (m)	$S_{2K} = 33$ (‰)
$A = 3 \times 10^5$ (m^2)	$S_Ö = 8.5$ (‰)
$H_D = 15$ (m)	
$m_0 = 1$	Initial values at $t = 0$:
$\beta = 8 \times 10^{-4}$ (‰)$^{-1}$	
$g = 10$ (m s^{-2})	$S_{1K} = 20$ (‰)
$f = 1.4 \times 10^{-4}$ (s^{-1})	$S_{1B} = 12$ (‰)
$P = 1.5$	$S_{2B} = 20$ (‰)
$X = 2.9 \times 10^{10}$ (m^3)	$H_{1K} = 15$ (m)
$Y = 3 \times 10^{10}$ (m^3)	$H_{1B} = 10$ (m)

toward the Skagerrak. However, in the model Kattegat the brackish layer is chosen to be horizontally homogeneous and the model H_f is thus constant. The outflow from the Kattegat to the Skagerrak is assumed to be geostrophically balanced [see Eq. (8)], and thus proportional to *the local value* of H_f. The H_f calculated by the model is meant to be the horizontal mean for the Kattegat.

Thus the model H_f may be expected to be not quite representative for the outflow region. Hence, in order to take care of the effect of the horizontal gradient of the real H_f in the model outflow to the Skagerrak, we may multiply the right-hand side in Eq. (8) by a factor P^{-1} ($P > 1$). From the material presented by Svansson (1975), an approximate estimate gives $P \approx 1.5$. In future models for the Kattegat, the internal dynamics will also be considered and factors like P will be superfluous.

As the observed thickness of the upper layer in the Kattegat exhibits a great deal of high-frequency variability, which is not shown by the predicted one, the observed thickness is smoothed (6-day running average). This variability is believed to be caused mainly by baroclinic waves of different kinds that are known to exist in the Kattegat. As long ago as the early 1930's Pettersson and Kullenberg (1933) and Kullenberg (1935) recorded and described internal waves in the Kattegat.

The computed thickness and salinity of the upper layer in the Kattegat are drawn in Figs. 5a and 5b, together with the observed thickness and salinity [from Eq. (29)] of the upper layer at the lightship *Anholt Knob*. The development with time of the thickness of the upper layer is shown in the upper diagram. The dashed line shows the observed thickness at *Anholt Knob* while the solid line shows the thickness predicted by the model. As can be seen from the diagram, the predicted thickness is of the right order of magnitude. However, there seems to be at least one event where a new shallow pycnocline is established. Such events should be expected when windy periods with inflow to the Baltic are succeeded by long periods with strong outflow and calm weather. One event of this kind appears to start about day 210.

The next diagram shows the predicted salinity of the brackish upper layer (solid line) and the salinity observed at the lightship (not smoothed). The general trends are very well predicted and the correlation between the curves is quite high for frequencies lower than the residence time of water parcels in this layer. For some situations with strong winds, the salinity at the *Anholt Knob* increases faster and, for short periods, attains higher values than predicted by the model. This is probably due to the fact that *Anholt Knob* is situated in the deep part of the Kattegat where entrainment occurs, while the model has a large amount of inertia as it calculates an integrated salinity value and thus also includes the shallow areas that lack a pycnocline.

There is much less high-frequency noise in the observed salinity than in the thickness (not smoothed) of the upper layer. This strengthens our previous conclusion that high-frequency variations in the thickness of the upper layer are associated with baroclinic waves and not with advective phenomena such as passing of fronts. Some single examples of the passing of low-salinity water parcels can be found, e.g., on day 210. The third and fourth diagrams in Fig. 5 show the mixing wind W and the inflow/outflow Q_b to/from the Baltic. Q_b is positive when the transport is directed toward the Baltic.

c. The salinities and halocline depth in the Belt Sea

Figs. 6a, 6b show the result of the calculations for the Belt Sea. The upper diagram gives the smoothed observed thickness at *Fehmarnbelt* (dashed line) and the computed thickness from the model (solid line). (Also in the Belt Sea there appears to be a great internal-wave activity.) The upper layer may occasionally disappear; both the observed and computed thicknesses vanish for certain short periods.

The next diagram shows the salinity of the upper layer, calculated from the salinity observations at *Fehmarnbelt* (dashed line) and the salinity computed by the model (solid line). As can be seen, the agreement is good and major trends are well captured. For periods when the upper layer is absent, the salinity computed for the lower layer is plotted.

The third diagram from the top shows the observed and computed salinities of the lower layer. Here, too, the model is quite realistic as it correctly gives the level and also some of the major changes.

For comparison, the fourth diagram shows the barotropic transport calculated from the barotropic model.

d. The predicted inflow of salt to the Baltic

The calculated mean distribution of inflow/outflow to/from the Baltic [$q(S)$] defined by Eq. (27) is plotted

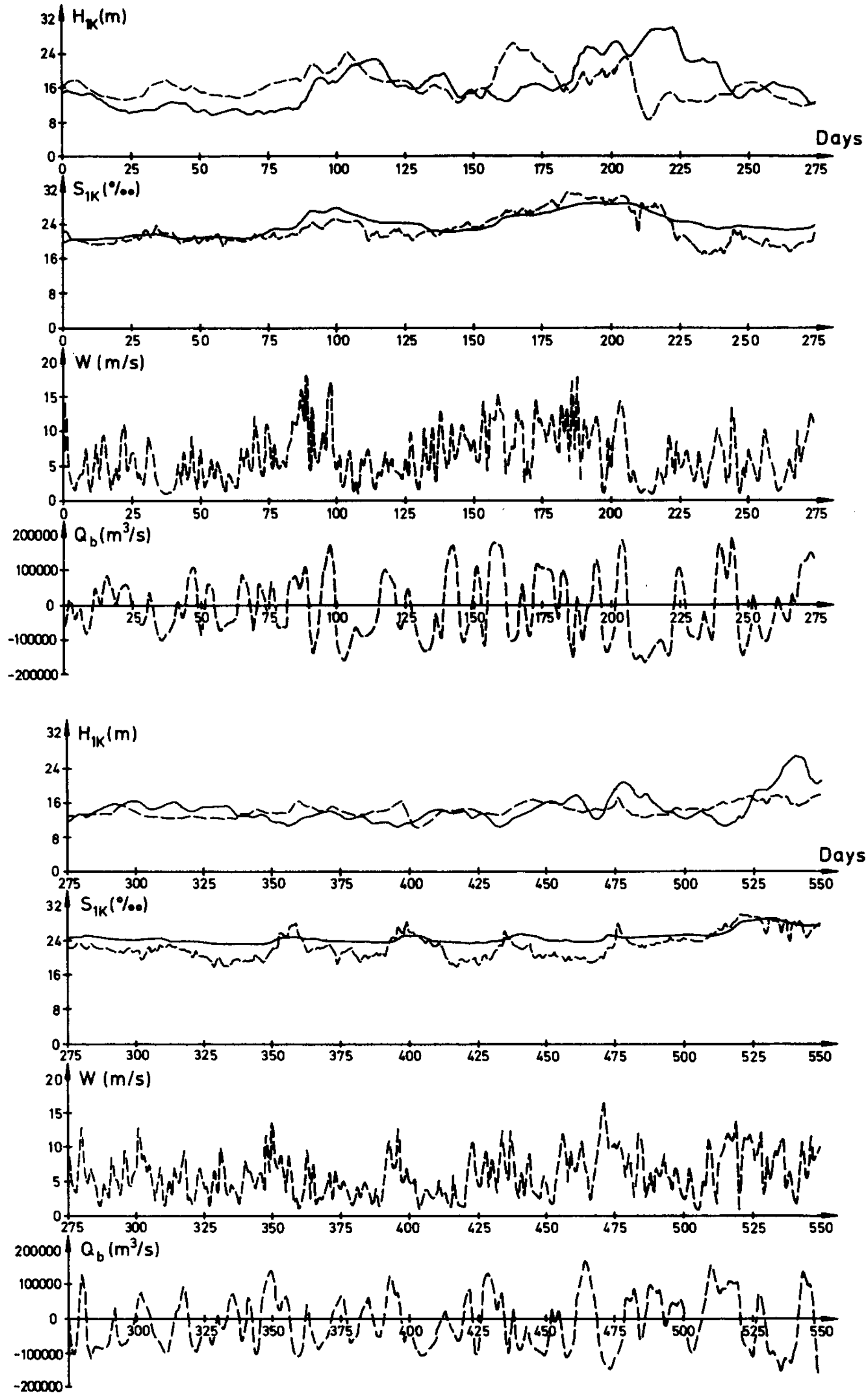

FIG. 5. Diagrams showing, from top to bottom, H_{1K}, S_{1K}, W, Q_b for days 1–275 (a) and 275–550 (b). Dashed lines show measured, and solid lines computed, quantities. Day 1 = 750 701.

in Fig. 7 for the "normal" case (see Section 6.1); ($Q_f/Q_{fN} = 1.5$, $Q_b/Q_{bN} = 1$, $W/W_N = 1$, $S_{2K} = 33‰$ and $S_{Ö} = 8.5‰$). The inflow has a peak centered at $\sim 18‰$ (incidentally, this conforms very well with the mean value obtained by Knudsen) while the outflow is largest for the lowest salinities. The magnitude of the mean salt transport into/out of the Baltic for this (salt-balanced) normal case is 258 tons s^{-1}.

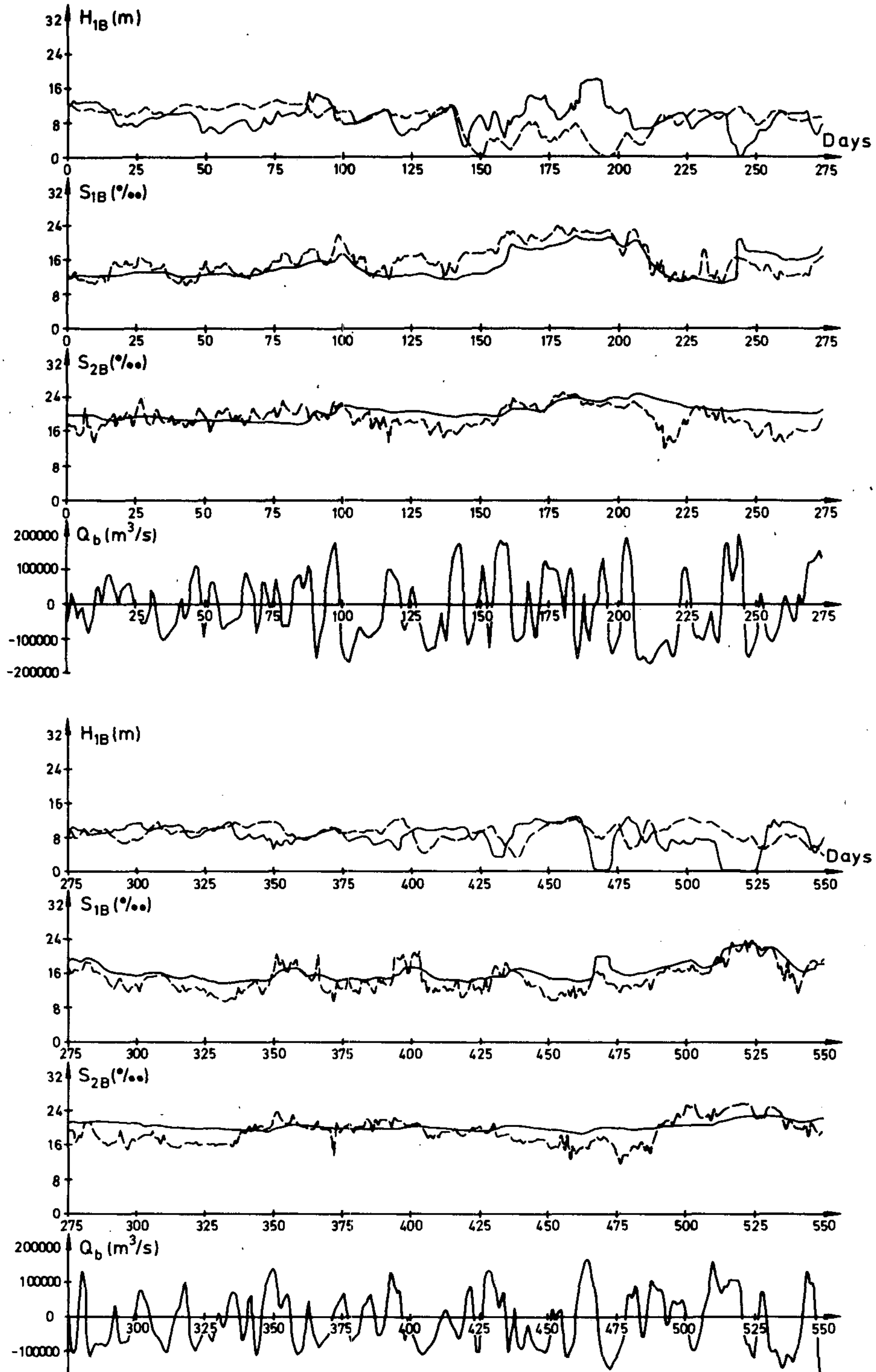

FIG. 6. As in Fig. 5, except for H_{1B}, S_{1B}, S_{2B} and Q_b.

e. Some comments upon the model

Waters with salinities in the range between S_{δ} and S_{2K} must obviously be created in the Belt Sea–Kattegat as there is no source for such waters at the borders. A simple estimate shows that the wind generally should be the dominating energy source for mixing. In particular, the small part of the basic potential

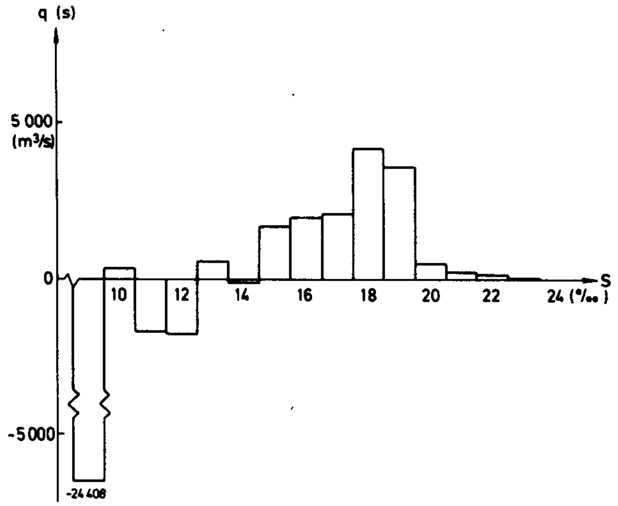

FIG. 7. The computed distribution of inflows (positive) and outflows (negative) to/from the Baltic at the Darss Sill for the "normal" case. $q(S)$ is the volume flow in the salinity (‰) range S–$(S + 1)$.

energy in the system, which might be available for mixing, is of minor importance. It is noteworthy that the model predicts the right level of salinities and pycnocline depths in the system. This could not have been achieved in a model where the parameterization of the mixing was very wrong.

One might think that the Kato–Phillips formula is unnecessarily sophisticated for the present description. However, theoretical arguments and vast experience in the oceanographic community from two-layer modeling of thermoclines, shelf fronts, estuarine circulation in fjords, etc., support this parameterization and there is, at this stage, no reason to abandon it.

In fact, the model was run for different values of the efficiency factor m_0 (0.75, 1, 1.25) [see Eq. (9)]. The distinctly best fit was obtained for $m_0 = 1$. Actually it seems that the model area is particularly suited for tests of mixing parameterizations as mixing appears to be a dominating process in the system.

The output from a horizontally integrated model should, of course, be compared to horizontally averaged field data. As there normally are horizontal gradients in the system even between the more persistent fronts, some of the discrepancy between the model results and field data can be said to depend upon incomplete sets of field data. Thus, the model might be even better than the results in Figs. 5 and 6 appear to show.

6. The sensitivity of the system to changes in the external parameters

a. Effects upon the Kattegat and the Belt Sea

In order to look into the sensitivity of the model to changes in the freshwater supply, the barotropic current fluctuations, the "mixing wind" and the salinity of the Skagerrak water, the model was run with different sets of Q_f, Q_b, W and S_{2K} (but retaining the salinity in the Baltic surface layer at its contemporary value). Three of the four parameters were held constant at their "normal" values (denoted by index N, the value used during the test period), while the fourth was changed by multiplying the amplitude by a constant factor.

There are reasons to believe that the mixing wind W during the period really may represent a long-term mean; see Kullenberg (1977) who calculated the annual mean of the wind cubed for this century. If the barotropic water level fluctuations in the Kattegat are mainly caused by the wind field, Q_b during the period may also represent a long-term mean. However, to the present author's knowledge there is no analysis in the literature showing this. The net freshwater supply to the Baltic was extremely low during the period (<10 000 m^3 s^{-1}, while the normal value is ~15 000 m^3 s^{-1}). Thus $Q_f/Q_{fN} \approx 1.5$ should represent the case of normal freshwater supply. In fact, the magnitudes (X and Y) of the buffer-volumes were determined by balancing the inflows and outflows of salt for the test period using $Q_f/Q_{fN} = 1.5$, $Q_b/Q_{bN} = 1$, $W/W_N = 1$, $S_{Ö} = 8.5$‰ and $S_{2K} = 33$‰.

1) EFFECTS OF A CHANGED FRESHWATER SUPPLY

From Table 2 it can be seen that an increased freshwater supply has a rather large effect on the surface salinity in the Kattegat and in the Belt Sea. The lower layer in the Belt Sea is more sensitive than the upper layer. The depth of the halocline decreases both in the Kattegat and in the Belt Sea when the freshwater discharge increases. Note that these effects of changes in the freshwater supply are caused by the (instantaneous) accompanying changes in the mean transports out of the Baltic. Thus they have nothing to do with changes in $S_{Ö}$ (which will occur for low-frequency or permanent changes in Q_f, see Section 6d).

The effect of a changed freshwater supply to the Baltic upon the surface salinity in the Kattegat, given in Table 2, seems to agree with the empirical results obtained by Svansson (1975).

2) EFFECTS OF CHANGED "MIXING WIND" CONDITIONS

Some results are given in Table 3. It can be seen that the halocline depths both in the Belt Sea and in

TABLE 2. Mean properties for various values of Q_f.

Q_f/Q_{fN}	H_{1K} (m)	S_{1K} (‰)	H_{1B} (m)	S_{1B} (‰)	S_{2B} (‰)
1	15.61	24.16	6.89	14.99	19.50
1.5	15.17	23.02	6.72	14.01	17.78
2.25	14.79	21.38	6.07	12.51	15.85

TABLE 3. Mean properties for various values of W/W_N.

W/W_N	H_{1K} (m)	S_{1K} (‰)	H_{1B} (m)	S_{1B} (‰)	S_{2B} (‰)
0.9	13.45	21.87	6.58	13.35	17.33
1	15.17	23.02	6.72	14.01	17.78
1.1	17.29	24.28	7.61	15.24	19.25

TABLE 5. Mean properties for various values of S_{2K}.

S_{2K} (‰)	H_{1K} (m)	S_{1K} (‰)	H_{1B} (m)	S_{1B} (‰)	S_{2B} (‰)
32	15.57	22.54	6.68	13.83	17.48
33	15.17	23.02	6.72	14.01	17.78
34	14.87	23.62	6.96	14.36	18.34

the Kattegat increase markedly with increasing wind speed. Also, the surface salinities in the Kattegat and the Belt Sea, as well as the salinity of the lower layer in the Belt Sea, increase with the wind speed.

3) EFFECTS OF CHANGED BAROTROPIC FLOWS

The amplitudes of the barotropic flows were changed by the factors 0.8 and 1.2, respectively (but retaining Q_f/Q_{fN} at 1.5). The computed effects (see Table 4) of changing the barotropic flow are not so easily understood. The reason for this is, of course, that the parameter Q_b enters the model in so many places and in different roles. For instance, Q_b completely determines the rate of exchange between the Kattegat and the Belt Sea and partly the rate of exchange between the Belt Sea and the Baltic, while Q_b^3 is the main parameter governing the downward entrainment flow in the Belt Sea.

4) EFFECTS OF A CHANGED SALINITY IN THE SKAGERRAK

The salinity of the lower layer in the Kattegat was allowed to attain the values 32, 33 and 34‰. The results of the computations are shown in Table 5. The effects on the salinities are largest in the Kattegat upper layer and in the lower layer in the Belt Sea. The amplitudes of the changes are typically one-half that of the forced change in amplitude. The depth of the pycnocline decreases slightly in the Kattegat, while it increases still less in the Belt Sea for an increase in S_{2K}.

b. The renewal of the lower layer in the Kattegat

The renewal of the water in the lower layer in the Kattegat may be of interest. In the model, this water is renewed by inflow from the Skagerrak. The rate of renewal is determined by the wind-driven entrainment into the upper layer and by the flow directly into the lower layer in the Belt Sea. In our model the entrainment term completely dominates the renewal of the Kattegat lower layer. A 10% change in the mixing wind from its normal value changes the entrainment flow by as much as 20%. Also the freshwater supply to the Baltic is of importance.

TABLE 4. Mean properties for various values of Q_b/Q_{bN}.

Q_b/Q_{bN}	H_{1K} (m)	S_{1K} (‰)	H_{1B} (m)	S_{1B} (‰)	S_{2B} (‰)
0.8	15.59	23.53	7.98	13.74	17.84
1	15.17	23.02	6.72	14.01	17.78
1.2	15.39	23.09	6.03	14.50	18.82

If we take the mean depth of the upper layer to be 15 m, we can calculate the volume of the lower layer with the help of Eq. (2′). The typical residence time for the Kattegat deep water then turns out to be approximately two months (in the normal case the long-term mean of the entrainment flow is $\sim$30 000 m^3 s^{-1}). Svansson (1975) shows that maximum and minimum in temperatures at 30 m depth occur approximately two months earlier in the northern Kattegat than in the southern part, which thus supports the result of the model presented here.

c. The dependence of the inflow of salt over the Darss Sill on Q_f, Q_b, W and S_{2K} (short time scales)

When we run the model with different sets of Q_f, Q_b, W and S_{2K} [and retaining the surface salinity of the Baltic at its contemporary value ($S_♉ = 8.5$‰)], we study changes of the inflow on a time scale that is short compared to the one over which the Baltic responds by changing $S_♉$ (20–30 years). (Steady-state values of $S_♉$ for various values of the external parameters are investigated in Section 6d.) The cumulative distribution function $M(S)$ for the whole test period was calculated from Eq. (28). This experiment is interesting because it shows how sensitive the salt inflow to the Baltic is to changes in the external parameters.

Note that the "normal" case $Q_f/Q_{fN} = 1$ is highly abnormal as it represents the freshwater supply during the test period which was just about ⅔ of the long-term mean. Fig. 8 shows the effect of changing the freshwater supply. The general trend is that the total inward volume flow decreases and occurs for lower salinities for increasing freshwater supplies. The volume flow out of the Baltic, however, increases with an increasing freshwater supply. Thus, the results already presented in this figure suggest that a permanently higher freshwater supply to the Baltic (compared with the contemporary value) will decrease the salinity of the Baltic and vice versa.

Fig. 9 shows the effect of changing the magnitude of the "mixing wind." The inflow distribution changes somewhat. The total inflow of salt increases slightly with an increasing mixing wind.

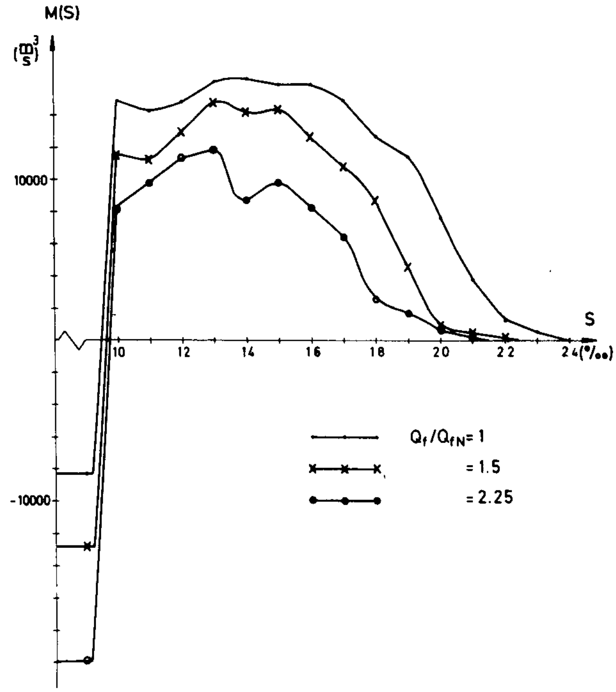

FIG. 8. The computed cumulative distribution function for inflows $M(S)$ for various values of Q_f.

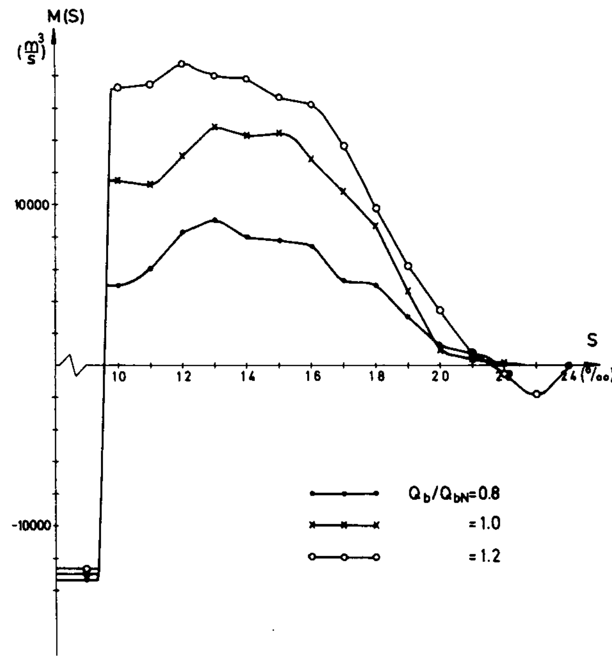

FIG. 10. As in Fig. 8 except for various values of Q_b.

Fig. 10 shows the effect of changing the amplitude of the barotropic current fluctuations. It had been mentioned in the Introduction that there is a general belief in the importance of the barotropic fluctuations for the salt balance of the Baltic. The correctness of this belief is strengthened by the computed results in this figure. Of special importance is the fact that a decrease in the amplitude of the barotropic fluctuations will lead to a large decrease in the inflow of salt to the Baltic (cf. also Section 6d).

Fig. 11 shows the effects of changing the salinity of the Skagerrak water. There is a clear tendency of increased inflow of salt to the Baltic for increasing values of S_{2K}. The main effect is to be found in the high-salinity part of the flow.

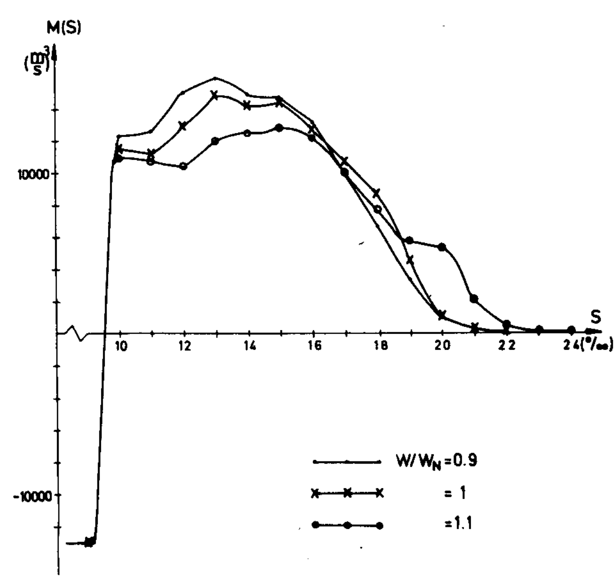

FIG. 9. As in Fig. 8 except for various values of W.

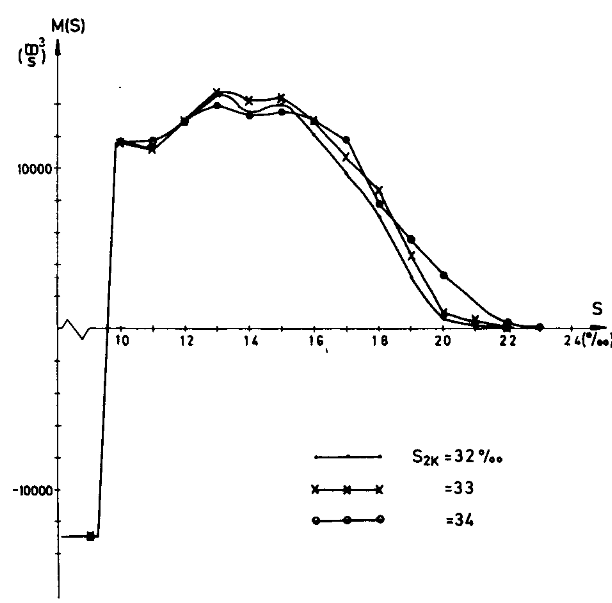

FIG. 11. As in Fig. 8 but for various values of S_{2K}.

d. The steady-state surface salinity in the Baltic for various values of Q_f, Q_b, W and S_{2K}

Earlier in this section we looked at short-period hydrographic changes in the Baltic entrance area (the Kattegat and the Belt Sea) caused by changes in the freshwater supply, the barotropic forcing, the wind and the salinity of the Skagerrak water. For that purpose, we kept $S_{Ö}$ equal to its contemporary value in the computations. Now we will investigate the effect of permanent or very-long-period changes in the same parameters upon the surface salinity in the Baltic. We assume that the halocline in the Baltic stays below the level of the Darss Sill. Thus for a given set of external parameters, we vary $S_{Ö}$ in the computations until we find that value which gives balance in transports of salt into/out of the Baltic at the Darss Sill.

The results of the computations are given in Figs. 12a, b. Fig. 12a shows that the surface salinity is very sensitive to changes in the freshwater supply. For example, a permanent change from 15 000 to 10 000 $m^3 s^{-1}$ would raise the surface salinity from 8.5 to 12.3‰ according to our model. The same figure also shows that an increase/decrease of the salinity in the Skagerrak by 1‰ would increase/decrease the surface salinity in the Baltic by ~0.3‰.

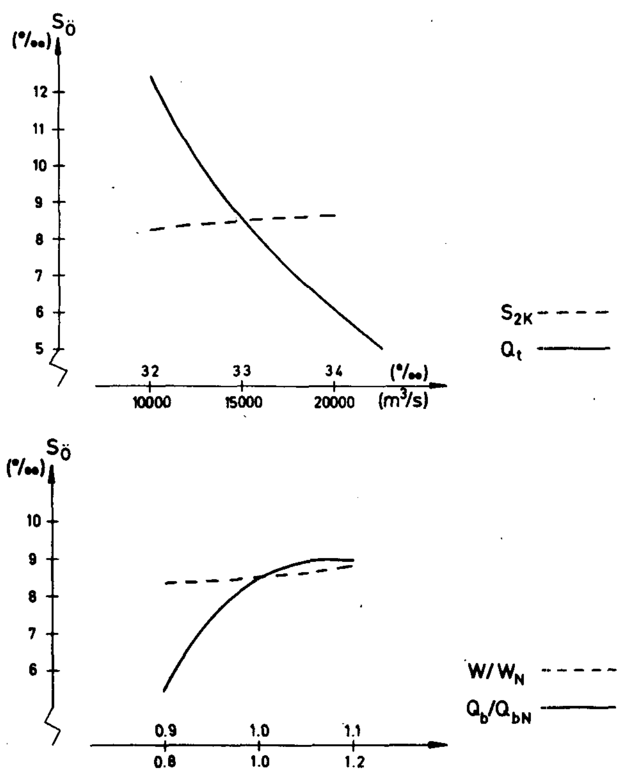

FIG. 12. Equilibrium salinity $S_{Ö}$ of the upper layer in the Baltic as a function of (a) the freshwater supply to the Baltic (Q_f) and the salinity (S_{2K}) of the Skagerrak water; (b) the normalized mixing wind (W/W_N) and the normalized barotropic flow (Q_b/Q_{bN}).

Fig. 12b shows that a change in the "mixing wind" over the Kattegat-Belt Sea area only has a slight effect upon the surface salinity in the Baltic. The intensity of the barotropic forcing, however, is an important factor for $S_{Ö}$. A decrease of this forcing from its contemporary level seems to have the largest effect.

7. Concluding remarks and an outlook on future work

The nature of the problem forces a realistic model for the exchange between the Baltic and the Skagerrak to be rather complex. Still, the model presented here is thought to be as simple as possible. The model predicts the stratification in both the Belt Sea and the Kattegat rather accurately. Actually, the salinities of the surface layer in the Kattegat and the two layers in the Belt Sea are well predicted, even at the end of the test period. Thus the model did not "lose track" even after 550 days of integration (representing a period which is 10–20 times longer than the residence time of water parcels in the layers). This strongly supports the conclusion that the model contains the basic physics of the system, and that the physical processes are adequately parameterized. This conclusion is further supported by the results of the sensitivity analysis as the response of the various values of the external parameters seems to be quite realistic.

Some improvements of the model have already been suggested in the text. One is to include the Öresund. Another is the possibility of creating a new halocline above the primary one in the Kattegat. Still another improvement concerns the barotropic model where the sea level in the Baltic close to the Belt Sea/Öresund is used. In the present version it is assumed that the sea level within the Baltic is flat. An obvious improvement is to add a separate model which calculates the discrepancy between the sea level near the entrance and the mean sea level in the Baltic.

One of the primary merits of the present two-layer model is that it, in a logical way, points out a number of processes that we do not know enough about. An increased understanding of these processes should be a goal for future research. Some of the questions remaining to be answered are:

1) What are the dynamical properties of the front in the northern Kattegat? How is the front advected in the north–south direction by wind and current?

2) Which are the processes that govern the horizontal gradients in the Kattegat and Belt Sea upper layers? What do the instantaneous gradients look like?

3) What kinds of free waves are generated in the Kattegat and the Belt Sea? Is it possible to model the wave-generation processes? [The characteristics of the topography and the barotropic current field seem to be favorable for topographical generation of baro-

clinic waves (cf. Stigebrandt, 1976) both in the Belt Sea and in the southern Kattegat.] Are the waves of importance for the mean circulation and stratification?

4) What are the dynamical properties of the buffer volumes X and Y?

The model presented in this paper will obviously be useful in providing boundary conditions for models of the (salt) stratification in the Baltic as well as for models of the coastal current in the Skagerrak (fed by Kattegat surface water).

Fjords with complicated entrance areas and strong barotropic forcing are common. Often the barotropic forcing is caused by the tides and the horizontal scales are usually smaller than in the present case (this may possibly eliminate effects of the rotation of the earth). In contrast to the present model, the pycnoclines of the fjords may be situated above the sill level. However, it should be possible to modify the present model to describe many estuaries of the kind depicted here.

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