Determination of the Pressure Along a Closed Hydrographic Section. Part I: The Ideal Case[1]

PIERRE WELANDER

School of Oceanography, University of Washington, Seattle, 98195

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ABSTRACT

The pressure along a closed hydrographic section can be correctly calculated from density data, in the ideal case of perfectly steady, geostrophic, density-conserving flow; and from dense, error-free data, excluding certain degenerate cases. A corresponding practical method, aimed at an estimate of the pressure from real hydrographic data, has been designed.

The calculation is made by a minimization of the volume enclosed by the surface $B = F(\rho, P)$ in the P-ρ-B space, where ρ is the density, $P = f\rho_z$ the potential vorticity, and $B = B^* + p_0$ the Bernoulli function, split in a known baroclinic part B^* and an unknown pressure p_0, defined at a chosen depth z_0. The minimization is made under free variation of $p_0(s)$, as a function of the tangential coordinate s; the minimum volume is zero under the ideal conditions. Practically, one minimizes a moment rather than the volume, with identical results in the ideal case.

The minimization requires an identification of "corresponding points" (endpoints of the same streamline) from the P-conservation; this may become impractical in the presence of strong noise. In such cases an alternative method based on an integral equation expressing the detailed flux balance of P and B is proposed.

1. Introduction

The problem of determining the pressure, or equivalently the normal geostrophic current, at a closed hydrographic section has recently been revived by Wunsch (1977, 1978), who suggests a calculation based on density flux conservation for a number (M) of layers bounded by isopycnals. If the unknowns are the normal geostrophic velocities b_i between a number (N) of hydrographic stations at a chosen level $z = z_0$, the problem is generally undetermined, since practically $M < N$. Wunsch makes the problem determined by applying a method of inverse calculation, which becomes equivalent to a statement that $\Sigma\, b_i^2$ is minimum. Fiaderio and Veronis (1982) vary z_0 to obtain an absolute minimum for $\Sigma\, b_i^2$, removing an arbitrariness in the choice of initial reference level.

The inverse method can be extended in different ways. Fiadeiro and Veronis have suggested the use of potential vorticity layers rather than isopycnal layers. More generally, one can combine conservations of density, potential vorticity $P = f\rho_z$ (which can be estimated from the density data along the section), and possibly other variables (salinity, chemical tracers, etc.) to improve on the calculation, in cases where one believes that such conservation laws are applicable. However, as long as one only deals with net integrated fluxes for a few layers, the problem is still an undetermined one. An added principle, such as the minimization mentioned, is required to make the problem determined.

One may ask whether the problem, in principle, would be a determined one when using *all* physical information, applying for steady, geostrophic and density-conserving flow combined with complete, error-free density data along a closed section. As will be shown later, the entire physics is expressed by the *detailed conservation* of the density ρ, the potential vorticity P and Bernoulli function B, expressed either as conservation along streamlines or as flux-conservation along streamtubes, and it is thus sufficient to explore the consequences of such conservations in the present problem. If a unique solution exists and can be calculated, it should also be possible to design a practical method for calculating the pressure from real hydrographic data which has the advantage of "ideal correctness": as the assumed model equations get better satisfied, and as the boundary data get denser and more accurate, the calculated pressure will converge toward the correct solution. Such "ideal correctness" holds for the locally applied "β-spiral method" (Stommel and Schott, 1977) and also for the inverse method when the region is made small (Willebrand, private communication, 1982), but it does not hold for the inverse method as applied by Wunsch and by Fiadeiro and Veronis (*loc. cit.*) for

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non-local problems: in the latter case one generally obtains solutions which do not conserve all the three variables ρ, P and B along streamlines.

In the present paper, a geometric discussion of the surface $B = F(\rho, P)$ in the ρ-P-B space, expressing an existing functional relation between ρ, P and B in the ideal limit (Welander, 1971), is used to show the existence of a solution to the ideal problem, except in certain degenerate cases (for example, no unique solution is found for purely zonal flows, or for flows with P uniform on isopycnal surfaces). The calculation of the solution can be made by a minimization of a certain volume or moment in the ρ-P-B space, and it is suggested that such a minimization is a practically useful way for estimating the pressure along closed hydrographic sections in the real oceans.

The method requires that "corresponding points", that are the two endpoints of a streamline, can be paired by use of P-conservation. Such pairing may be unique or non-unique; in the latter, each possible case must be explored separately. If the data are very noisy, the number of possible cases become large, and the proposed method may become impractical. In such a case one may go to another method which is based on detailed flux balances. These balances are applied for streamtubes defined by given ranges ΔP and ΔB of the potential vorticity and Bernoulli function, and do not require pairing of corresponding points. Mathematically, the problem is described by a nonlinear integral equation. Practical tests of the methods mentioned to real hydrographic sections in the oceans are presently being carried out. The results of these tests will be presented in Part II of the paper.

2. Basic equations, conservation equations and the first integral

We consider an idealized situation in which the oceanic motion is steady, incompressible (Boussinesq-type), geostrophic-hydrostatic and non-diffusive. A top boundary is placed at $z = 0$ (rigid lid approximation), and a bottom at $z = -H(\lambda, \phi)$, where λ, ϕ, z are longitude, latitude and height, respectively. We further assume the existence of a free, lateral boundary L, at which the density is prescribed as a function of z and a tangential coordinate s. The basic equations are

$$\bar{\rho} f \mathbf{z} \times \mathbf{v} = -\nabla p - \mathbf{z}\rho g, \qquad (1\text{a, b, c})$$

$$\nabla \cdot \mathbf{v} = 0, \qquad (2)$$

$$\mathbf{v} \cdot \nabla \rho = 0, \qquad (3)$$

where $\mathbf{v} = (u, v, w)$ is the velocity, p the pressure, ρ the density, $\bar{\rho}$ a mean density, g the gravity acceleration, $f = 2\Omega \sin\phi$ the Coriolis parameter (Ω earth's angular speed), and $\mathbf{z}$ a vertical unit vector. When applying this model to the real ocean, the model density ρ cannot be identified with the *in situ* density. The model density corresponds better to such oceanographic variables as σ_t or σ_θ, but the correspondence is not a perfect one, and in calculations involving P and B the errors could be critical, in certain situations. The problem will be further discussed in Part II.

Boundary conditions are zero normal velocity at the top and bottom[2]

$$w = 0 \quad \text{at} \quad z = 0, \qquad (4)$$

$$v_n = 0 \quad \text{at} \quad z = -H(\lambda, \phi). \qquad (5)$$

At the free, lateral boundary L the model density is assumed known, i.e.,

$$\rho = \rho(s, z) = 0 \quad \text{on } L, \qquad (6)$$

where s is a tangential coordinate along the boundary; positive direction counterclockwise (see Fig. 1).

It can be shown that the potential vorticity $P = f\rho_z$ and the Bernoulli function $B = p + g\rho z$ are conserved along streamlines:

$$\mathbf{v} \cdot \nabla B = 0, \qquad (7)$$

$$\mathbf{v} \cdot \nabla P = 0. \qquad (8)$$

If we denote $\mathbf{v} \cdot \nabla$ by D/Dt and denote horizontal components by subscript h, we have

$$\frac{D}{Dt}(f\rho_z) = \frac{Df}{Dt}\rho_z + f\frac{D}{Dt}\rho_z = \frac{Df}{Dt}\rho_z + f\left\{\left(\frac{D\rho}{Dt}\right)_z - (\mathbf{v}_h)_z \cdot \nabla_h \rho - w_z \rho_z\right\} = 0,$$

since $D\rho/Dt = 0$, $(\mathbf{v}_h)_z \cdot \nabla_h \rho = 0$ (thermal wind equation), and $Df/Dt = fw_z$ (vorticity equation in the geostrophic approximation). For the Bernoulli function

$$\frac{D}{Dt}(p + g\rho z) = \frac{Dp}{Dt} + g\frac{D\rho}{Dt}z + g\rho\frac{Dz}{Dt} = 0,$$

since $D\rho/Dt = 0$, $p_t = 0$ (steady state), $\mathbf{v}_h \cdot \nabla_h p = 0$ (geostrophic balance), and $p_z = -g\rho$ (hydrostatic balance).

The conservation of ρ, P, B along streamlines implies a functional relation, say, $F(\rho, P, B) = 0$, which could be multivalued. Since $\rho = -g^{-1}p_z$, $P = -fg^{-1}p_{zz}$, $B = p - zp_z$, by the hydrostatic equation, this relation represents a second-order differential equation for p after z, with λ and ϕ occurring parametrically. From this first integral certain classes of exact solutions can

[2] These boundary conditions will only be relevant for isopycnal surfaces that intersect the top and bottom. Excluding these surfaces, and dealing only with isopycnals that cross the lateral boundary L between the top and bottom everywhere, the method of calculation described in the next section applies for arbitrary top and bottom boundary conditions; for example, we may have an Ekman vertical velocity applied at the top.

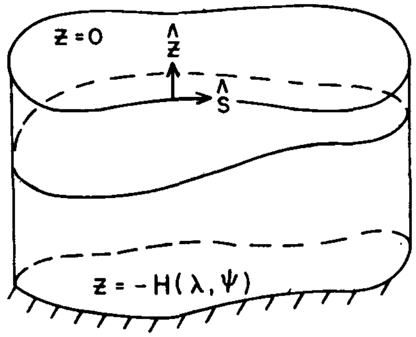


FIG. 1. Schematic picture of an oceanic region bounded by a top surface at $z = 0$ (rigid lid approximation), a bottom at $z = -H(\lambda, \phi)$, and a lateral boundary L. The density $\rho(s, z)$ on L is assumed known. Here λ, ϕ and z are longitude, latitude and height, and s a tangential distance along the boundary, measured counterclockwise from an arbitrary origin. The intermediate curve indicates an isopycnal line along the boundary.

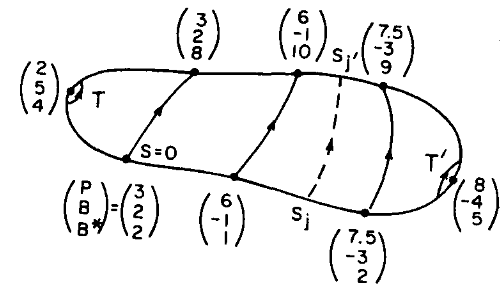


FIG. 2. Schematic picture of an isopycnal surface with contained streamlines, in a horizontal projection. It is assumed that P and B vary monotonically across the streamlines. Minimum and maximum values of P and B appear simultaneously at the points T, T' where the streamlines are tangent to the boundary. Except at these points, each point s_j on the boundary with values P_j, B_j has a different "corresponding point" $s_{j'}$, with the same values P_j, B_j; these represent the entrance and exit points of the same streamline. Hypothetical values for P, B and the (non-conserved) baroclinic Bernoulli function B^* are given at selected points.

be generated (Welander, 1971). Any solution for p generated from this differential equation will produce associated density and velocity fields that satisfy all the original equations, when we solve for ρ from the hydrostatic equation, $\mathbf{v}_h$ from the geostrophic equation, and w from the density conservation equation (3).

3. Determination of the pressure from collapse of the B^* (ρ, P) function

We consider the picture of an isopycnal surface, in a horizontal projection, shown in Fig. 2. A simple streamline pattern is assumed, and arbitrary values of P and B are assigned to the streamlines; the values are shown at the two "corresponding points" on the boundary L. If we plot P and B as a function of a tangential coordinate s we generate the curves shown in Fig. 3. If we plot B as a function of P we get the curve shown in Fig. 4a—a line segment covered twice as we go around the section. If we only have density data we do not know B, but can estimate the *baroclinic Bernoulli function*

$$B^* = -g \int_{z_0}^{z} \rho dz + g\rho(z - z_0). \qquad (9)$$

The relation between B and B^* is

$$B = B^*(s, z) + p_0(s), \qquad (10)$$

where $p_0(s)$ is the pressure at $z = z_0$. With the B^* values assigned to the boundary points in Fig. 2, which are now unequal at corresponding points, we generate the curve $B^*(s)$ shown in Fig. 3, and the curve $B^*(P)$ shown in Fig. 4b. The curve $B^*(P)$ obviously "opens", enclosing a certain area. To come from this curve to the curve $B(P)$ shown in Fig. 4a, we must add a function $p_0(s)$ to B^*, making the curve collapse to a line segment. This can obviously be done in an infinity of ways. However, if such a collapse is required at many isopycnals, only one function $p_0(s)$ will work, in the ideal case. Actually, two isopycnals will generally be sufficient to determine the solution. We assume that we have N "stations", and correspondingly N unknown values, denoted p_j. The collapse of the B-values at two corresponding points (assumed for simplicity to fall at the stations, as in Fig. 2) is then expressed by the equation $B^{*\prime} + p_j' = B^* + p_j$, determining the difference between two p_j values. For the $N/2$ pairs of points (N assumed even) we get $N/2$ equations, and for two isopycnals we generate N equations. There is a degeneracy in the problem, as only pressure differences can be determined, but this is physically acceptable, as we are only interested in the horizontal variations of pressure in the geostrophic calculation.

If we let ρ vary continuously, the curve $B^*(p)$ will generate a surface $B^*(\rho, P)$ in the ρ-P-B^* space which encloses a certain volume. An example of such a surface calculated from an analytical example is given in Fig. 6 (see next section for details). The solution is obtained by collapsing this surface to one that encloses no volume, by adding a function $p_0(s)$ to B^*.

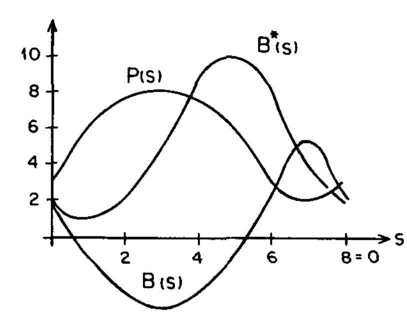


FIG. 3. Plot of interpolated functions $P(s)$, $B(s)$ and $B^*(s)$ corresponding to the values given in Fig. 2.

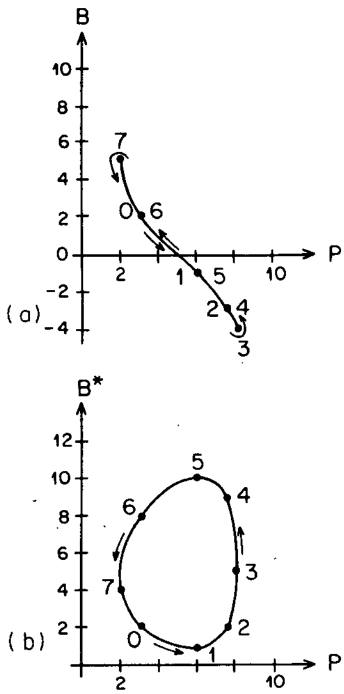


FIG. 4. The functions $B(P)$, case a, and $B^*(P)$, case b, obtained from the curves shown in Fig. 2.

Geometrically, we achieve the collapse by translating individually the isopleths s = constant (dashed lines in Fig. 6) along the B^* axis, until they all make contact. In the example shown in Fig. 6 the s-isopleths are actually plane curves, and the collapsed surface a single plane. Note that the surface always can be displaced uniformly along the B^* axis, corresponding to the addition of a constant to all pressure values.

There is an obvious extremum principle that can be applied to obtain the solution: if we define the positive volume $V = \iint |B' - B| dPd\rho$, the solution is obtained from minimizing V. The minimum value is zero in an ideal case.

If we work with real data, the set of linear equations for the p_j's are inconsistent, if we use more than two isopycnals. We will thus arrive at a non-zero minimum V. The solution obtained by this minimization will be one that, in a certain sense, lies as close as possible to an ideal one. For practical reasons the minimization of the volume is replaced by a minimization of a quadratic moment:

TABLE 1. Parameter values for the analytical example (cgs units).

$\lambda_0 = -15°$, $\lambda_1 = 15°$,	$\phi_0 = 10°$, $\phi_1 = 20°$,	$g = 981$
$a = 4 \times 10^{-6}$	$k = -1.47 \times 10^{-5}$	$q = -3.31070 \times 10^{7}$
$b = -2.039 \times 10^{-13}$	$l = 2.56 \times 10^{-5}$	$r = 1.38809 \times 10^{7}$
$c = 0$	$m = 1.0261$	$\rho_m = 1.028$

$$M = \iint (B' - B)^2 dPd\rho. \quad (11)$$

The moment vanishes together with the volume, and presumably is as good a measure as the volume when calculating approximate solutions.

The calculation of the pressure by the present method does not necessarily require a closed hydrographic section. Points on an open section can be used as long as they are connected by streamlines. This contrasts with total budget methods, which balance net fluxes of water, potential vorticity, etc., across the boundary: in the latter case a closed section (or a section closed partly by a solid boundary) is obviously required.

4. An analytical example

We consider an analytical example in the class of error-integral type solutions (Welander, 1971). This solution is generated from a linear form of the first integral

$$P = a\rho + bB + c. \quad (12)$$

Inserting $P = f\rho_z$, $B = p + g\rho z$, and taking a z derivative, gives the ρ equation $f\rho_{zz} = (a + bgz)\rho_z$ which has a solution of the form

$$\rho = \rho_0(\lambda, \phi) + C(\lambda, \phi) \int_0^z \exp[a\zeta + \tfrac{1}{2} bg\zeta^2) f^{-1}] d\zeta. \quad (13)$$

The pressure solution is found from the integrated hydrostatic equation

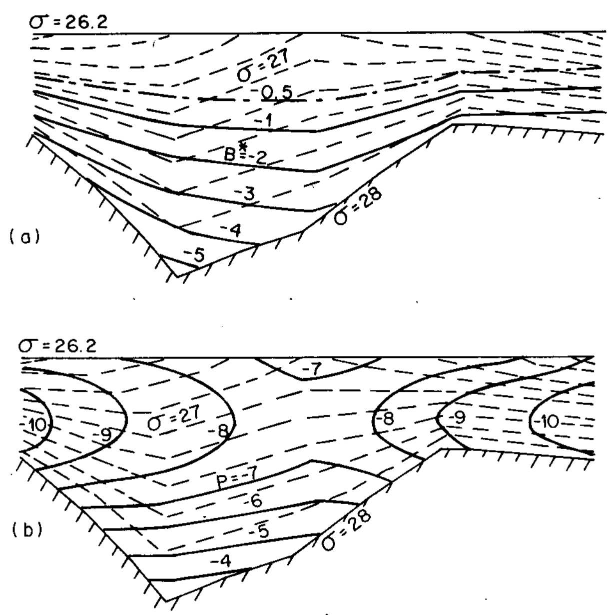


FIG. 5. Isopleths for B^* and ρ (dashed), case a, and P and ρ, case b, along the boundary in the analytical example.

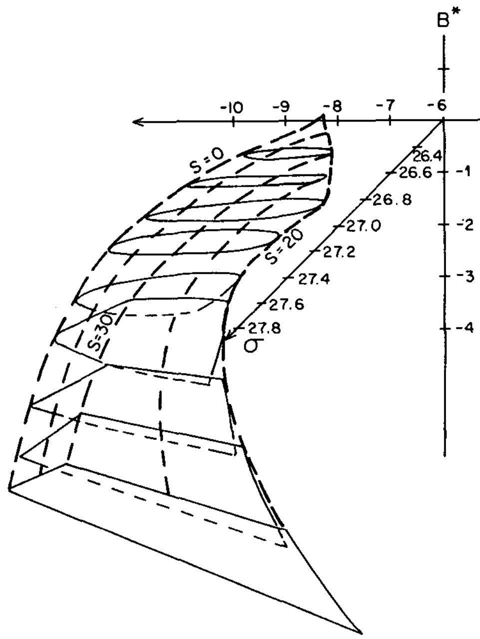


FIG. 6. The surface $B^*(\rho, P)$ calculated for the analytical example, with a few s-isopleths. These isopleths are all contained in parallel planes.

$$p = p_0(\lambda, \phi) - g\rho_0(\lambda, \phi)z - gC(\lambda, \phi) \times \int_0^z dz' \int_0^{z'} \exp[a\zeta + \tfrac{1}{2}bg\zeta^2)f^{-1}]d\zeta. \quad (14)$$

We have one relation between p_0, ρ_0 and C from Eq. (12), i.e.,

$$fC = a\rho_0 + bp_0 + c \quad (15)$$

(an extra arbitrary function was introduced by the previous z differentiation); further we have the two boundary conditions (4) and (5) for a total of three conditions. We assume a surface density of the form $\rho_0 = k\lambda + l\phi + m$, and a surface pressure of the form $p = q\rho_0 + r$, which makes the top boundary condition automatically satisfied; further we let the bottom be represented by an isopycnal surface $\rho = \rho_m$, which makes the bottom boundary condition automatically satisfied.

We consider a specific solution in a hypothetical oceanic region between free lateral boundaries $\lambda = \lambda_0, \lambda_1$; $\phi = \phi_0, \phi_1$. The parameter values chosen are given in Table 1.

The assumed known boundary variables are the density $\rho(s, z)$, the potential vorticity $P(s, z) = f\rho_z$, and the baroclinic Bernoulli function $B^*(s, z)$ relative to $z = 0$, $B^*(s, z) = -g \int_0^z \rho dz + g\rho z$. Isopleths for these three boundary variables along the boundary are shown in Fig. 5. The coordinate s is running counterclockwise, starting from $s = 0$ at the northeast corner. The surface $B^*(\rho, P)$ with a few s-isopleths inserted, is shown in Fig. 6.

In the present case we know that the collapsed surface must be a plane parallel to the one given by (12), and that the s-isopleths therefore must be plane curves. The collapse is obviously unique, except for uniform translations along the B^* axis.

We demonstrate how the method of minimizing the moment M works in this case. Eight "stations" were taken along the described hypothetical section, one in each corner and one at the midpoint of each side. The moment is evaluated as

$$M = \tfrac{1}{2} \sum_i \sum_j (B_{ij'} - B_{ij})^2 \Delta P_{ij} \Delta_i \rho. \quad (16)$$

The index i numbers the density surfaces, the index j the stations. The index j' indicates values at a "corresponding point", obtained by linear interpolation. If the potential vorticity at point j (index i omitted for the moment) is P_j, we look for two other consecutive stations k and $k + 1$ with P-values which straddle this value. Weight coefficients α and $\beta = 1 - \alpha$ are found from the relation $P_j = \alpha P_k + \beta P_{k+1}$, giving $P_{j'} = \alpha P_K + \beta P_{k+1} = P_j$ and $B_{j'} = \alpha B_k + \beta B_{k+1}$. The value of ΔP_{ij} is $\tfrac{1}{2}(P_{i,j+1} - P_{i,j-1})$, and $\Delta_i \rho$ is chosen constant. The factor ½ in relation (16) occurs because we count the moment twice when summing over all j's.

We have $B_{ij'} - B_{ij} = b^*_{ij} + p_{j'} - p_j$, where $b^*_{ij} = B^*_{ij'} - B^*_{ij}$ is a known quantity, and $p_{j'} - p_j = \alpha p_k + \beta p_{k+1} - p_j$ is a linear combination of three p_j-values. The equations $\partial M/\partial p_j = 0$ ($j = 1, 2, \cdots 8$), gives eight linear equations for the p_j's. The equations are unchanged if a constant is added to all p_j's. The degeneracy is removed by setting $p_1 = 0$ and dropping one linear equation.

Using three isopycnals $\sigma = 27.0, 27.5, 28.0$, the seven equations determining $p_2, p_3, \cdots p_8$ are

p_2	p_3	p_4	p_5	p_6	p_7	p_8		
3.825	0.287	−2.145	−0.868	0	0	−0.341		−0.599
0.287	2.594	−0.100	0	0	0	−1.382		−4.480
−2.145	−0.100	15.061	1.162	0	−0.106	−12.848		−20.134
−0.868	0	1.162	9.237	0.351	−2.508	−4.630	=	−10.275
0	0	0	0.351	5.923	−2.055	−4.051		−2.975
0	0	−0.106	−2.508	−2.055	2.923	1.745		4.620
−0.341	−1.382	−12.848	−4.630	−4.051	1.745	20.125		27.942

TABLE 2. Values for p_j calculated by minimization of M with three isopycnals (a), and two isopycnals (b). Exact values from the analytical solution are given by (c). In all cases $p_1 = 0$. Units are centibars.

	Value of j						
	2	3	4	5	6	7	8
(a)	−0.728	−1.460	−1.034	−0.612	0.119	0.848	0.425
(b)	−0.727	−1.459	−1.033	−0.612	0.120	0.848	0.427
(c)	−0.730	−1.460	−1.036	−0.612	0.118	0.848	0.424

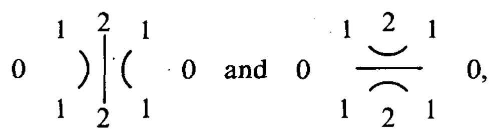
and direct solution of this gives the p_j values listed in the first line of Table 2. The second line gives the corresponding solution when the intermediate isopycnal $\sigma = 27.5$ is disregarded, i.e., only two isopycnals are used. The third lines gives the exact values according to the analytical solution. As seen, the agreement is almost perfect.

5. Case of non-unique streamline patterns: the integral method

In the previous simple example the "corresponding points" that represent the two endpoints of a streamline could be paired uniquely because the potential vorticity attained the same value only twice when following an isopycnal around the section. One can construct analytical solutions for which the same P values are attained four, six, or more times along an isopycnal, giving several possibilities of pairing "corresponding points." For example, if we have numerical P-values

$$\begin{array}{ccccc} & 1 & 2 & 1 & \\ 0 & & & & 0 \\ & 1 & 2 & 1 & \end{array}$$

along an isopycnal we can obviously find two streamline patterns, one rotated 90° from the other:



with cases of crossing streamlines excluded. If the number of identical P-values is large, the number of possible streamline patterns increases rapidly, although some cases can be eliminated by continuity requirements going from one isopycnal to the next. In all cases where the analytical solution is a unique one we should, however, be able to recover it by a calculation of the minimum M-value, as described previously. Such a calculation must be made for every possible pattern; only one of these patterns should give the zero M-value (or a very small M-value when numerical errors are added).

Going to the case of real data we generally have a large number of possible streamline patterns, except when the region is small and the data are smooth. The noise comes to some extent from errors in the measurements but the main part is due to the existence of internal waves and mesoscale eddies. We may want to smooth out such features, but it is not obvious how this should be done without introducing further errors in the assumed conservation relations. In the end, we are still likely to have several possible streamline patterns. If we calculate the minimum M-values for each of these, one may be small and the others large, in which case we feel that "a solution" is found. However, it may also happen that we have several solutions with M-values of comparable magnitudes, and must accept equally many solutions.

It seems possible to avoid the problem of sorting out the different streamline patterns by using an analytical approach along the following line. The conservation of ρ, P and B along streamlines can be written in flux form as

$$\iint_L F(\rho, P, B) v_n dz ds = 0, \tag{17}$$

where F is an arbitrary function of its arguments, the three conserved variables, and v_n is the normal velocity, in geostrophic form. Incompressible flow is obviously assumed. If we integrate over a layer bounded by two neighboring isopycnals ρ_i and $\rho_i + \Delta\rho$, with thickness $\Delta h_i(s)$, we end up with the equation

$$\oint F_i(P, B) v_n \Delta h_i ds = 0 \tag{17a}$$

that can be applied to a series of such layers.

Eq. (17) or (17a) contains all the physics of the problem, and applying it to an ideal case we should be able to derive the same solution as is obtained by the M-minimization. The potential advantage of a calculation based on (17a) is that we do not need to do the pairing of "corresponding points" and thus need not examine the possible streamline patterns.

A project is under way for calculating the Bernoulli function B and thus the unknown pressure $p_0(s)$ from Eq. (17a) by using a series of piece-wise constant functions F_i:

$$\left.\begin{array}{ll} F_i = 1 & \left.\begin{array}{l} P_r \leqslant P \leqslant P_r + \Delta P \\ B_s \leqslant B \leqslant B_s + \Delta B \end{array}\right\} \\ F_i = 0 & \text{outside this interval} \end{array}\right\}. \tag{18}$$

The number of values P_r, B_s and associated intervals is determined by practical considerations. If the data are ideal or nearly ideal we use many points and fine intervals, if there is strong noise in the data, we are satisfied to use a few points and large intervals. If we have N stations and thus N unknown pressure values p_j, application of the equation to R intervals for P, and S intervals for B, at I layers, produces a total of RSI equations for the N unknowns. In one practical example we plan to use $N = 20$, $R = S = 8$, $I = 4$, producing 256 equations for the 20 (really 19, since one p_j value is arbitrary) equations. The problem is typically overdetermined. The equations are nonlinear, since both B and v_n depend on p_j, and it is suggested to solve them by an iterative method, using B^* instead of B as an argument in F_i in the first calculation, solving for a set of p_j's through a least-square fit for the resulting linear system, then replacing B^* by a better approximation using the calculated p_j-values, etc. Instead of calculating the solution by exploring a number of possible streamline patterns, we try here to iteratively steer into a solution which conserves both P and B along streamlines, which themselves are undetermined until the end of the calculation.

It is finally noted that once the boundary value problem is solved by one method or another, and the functional relation between ρ, P and B is established at the boundary, it may be possible to solve also for the density and pressure fields at interior points, that is, points in the region enclosed by the section. As mentioned in Section 2, the functional relation represents a second-order differential equation for p after the vertical coordinate z. In regions of closed streamlines every interior point is connected to boundary points through a streamline, and we therefore only need the function in the range known from the boundary values. If we further have two suitable boundary conditions for the pressure at the top and bottom (for example, the conditions $w = 0$ at $z = 0$ and $z = -H_0$ will do) the p-equation presumably could be solved along local verticals. Since the problem generally is non-linear, we may not necessarily find unique solutions.

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