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# Computer Simulation of the Kinematics and Dynamic Load of Selected Elements in the Needle Mechanism of a Warp-Knitting Machine

## Abstract

This work presents a computer analysis of the kinematics (analysed by means of a mathematical model) of the mechanism of a cam-lever drive for a needle rail in a K13 warp-knitting machine. Functions connecting the movement of mechanism elements, i.e. a system of non-linear, algebraic equations, were solved by using the iterative Newton-Brent method. The results of these solutions in the discrete form were replaced by a continuous function using Lagrange's multinomial. This allows the parameters of the chosen mechanism movement to be determined as derivatives of these functions. An attempt was made at computer simulation of slipper surface wear, locally acting on the rising part of the cam outline in this mechanism. An analysis was carried out of the influence of simulated wear on the parameters concerning movement of the pusher and the needle mechanism's point connected with the needle ending, as well as on the quantity and character in changes of the load at the contact point of the cam and the pusher roller.

**Key words:** cam mechanism, computer simulation, mathematical model of mechanism, warp-knitting machine.

In case of kinematic pair closure, the movable connections of the cam mechanism elements (kinematic pairs) are usually made with some clearances. These clearances are necessary to secure the correct work of the mechanism, but their presence causes additional dynamic loads to arise, together with vibrations connected with them and the noise that accompanies them.

Each time the pusher is turned, the acceleration sign (direction) changes, which causes a temporary loss of contact in the kinematic pairs, as well as a repeated encounter of elements in a different place than before, connected with an impact collision. It is desirable to change the acceleration direction in such a way that it should not appear as a rapid stroke.

The aim of this work is a theoretical analysis of the kinematic and dynamical load of selected elements in the needle mechanism of a warp-knitting machine. An attempt at computer simulation of local wear of the surface of part of the cam outline of this mechanism was made by means of theoretical considerations. An evaluation of the influence of the estimated wear on the loads of selected mechanism elements was carried out.

## Equations of Movement

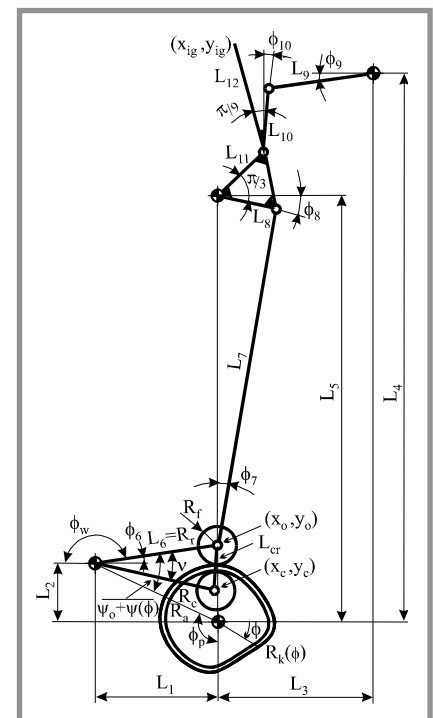
A single power unit of the needle mechanism of the K13 warp-knitting

machine was chosen to carry out the kinematic and dynamic analysis. The lack of a working drawing specification of the geometrical parameters of the links' configuration of the warp-knitting machine's needle mechanism forced a

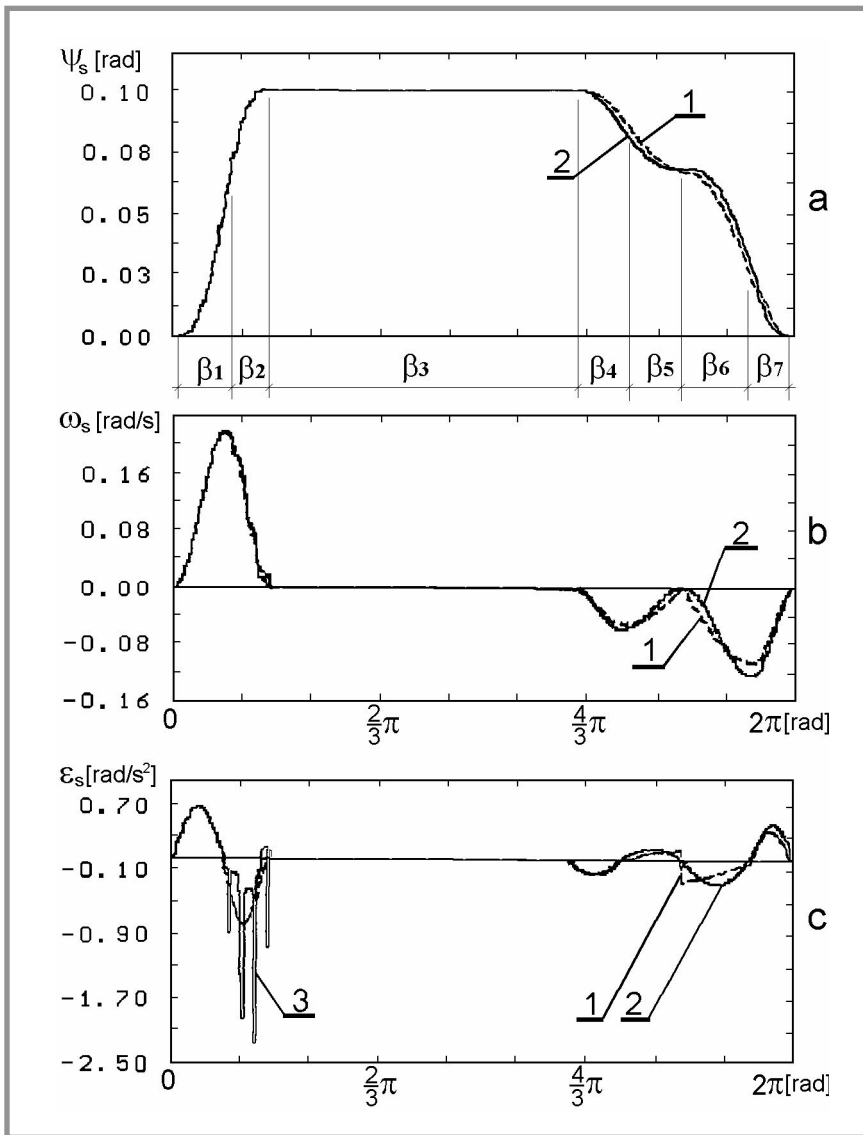
## Introduction

The creation and analysis of mathematical models for selected complex mechanisms of knitting machines have been the main focus of the authors' research works on the subject of the dynamics of textile machines.

The production of warp-knitted fabric takes place thanks to so-called warp-knitting machines. The mechanisms which play a direct part in the formation of knitted fabric loops are as follows: the needle-, the sinker-, and the guide needle-mechanisms, together with a press or trick plate. Their construction is highly complex. Cam and multi-link lever mechanisms usually co-operate together. Cams are connected to the main shaft of the machine. Pushers co-operating with cams make the links of the lever mechanism move. The precision of the reciprocal location of the elements working at the same time, in every single cycle of knitted fabric loop formation, is a characteristic feature of these complex kinematic systems.



**Figure 1.** Diagram of a cam-lever mechanism for a drive of the needle rail in a K13 warp-knitting machine;  $L_i$  - length of links or the mutual positions of fixed knots measured along the axis of the  $x$ - $y$  co-ordinate system;  $\phi_i$  - momentary inclinations of links towards the co-ordinate system axis.



**Figure 2.** Parameters of pusher movement; a - track, b - speed, c - angular acceleration as a function of cam rotational angle  $\phi$  for connections of partial functions: 1 - inverted cosines; 2 - cycloidal, 3 - locally used on the rising part of the cam outline, including a function which simulates slipper surface wear.

reconstruction, which was very labour-consuming, in the preliminary stage of this investigation.

The location of fixed knots and the length of some elements, which were determined by measurements, may be slightly different from the real location and size. Considerations concerned with the kinematics of the mechanism, the calculation results obtained and their evaluation are based on the created model of the mechanism which is very similar to the real solution.

The mechanism under consideration is shown in Figure 1. Two coaxial cams (the first with an external, the second with internal outlines) form one driving link. They interact with a stiff, forked, pendulous self-aligning pusher

during the kinematic pair closure. The parameters  $L_i$ , by which the model is characterised, indicate the length of links or the mutual positions of fixed knots measured along the axes of a rectangular co-ordinate system. The angles  $\phi_i$  indicate the momentary inclinations of links towards the chosen axis of this co-ordinate system.

By appropriate connection of the mechanism's knots shown in Figure 1, two closed polygons of vectors were formed. A system of four non-linear, algebraic equations was formed after projecting these vectors of the rectangular co-ordinate system (1). The following system of equations describes the temporary state of balance for the mechanism's links:

$$\begin{aligned}
 &L_1 - L_6 \sin(\phi_w) - L_7 \sin(\phi_7) + \\
 &+ L_8 \cos(\phi_8) = 0 \\
 &L_2 + L_6 \cos(\phi_w) + L_7 \cos(\phi_7) + \\
 &+ L_8 \sin(\phi_8) - L_5 = 0 \\
 &-L_3 + L_9 \cos(\phi_9) + L_{10} \sin(\phi_{10}) + \\
 &+ L_{11} \cos(\pi/3 - \phi_8) = 0 \\
 &L_4 - L_5 - L_9 \sin(\phi_9) - L_{10} \cos(\phi_{10}) + \\
 &- L_{11} \sin(\pi/3 - \phi_8) = 0
 \end{aligned} \quad (1)$$

The system of these equations was solved by using the iterative Newton-Brent method [2,3]. The vector  $\Psi(\phi_7, \phi_8, \phi_9, \phi_{10})$  of the momentary angle positions in request of the mechanism elements is the solution result of the consecutive positions of the cam shaft. The calculated values of these parameters are defined as discrete functions and join the movements of the mechanism's elements. Their discrete form was replaced by a continuous function by means of Lagrange's multinomial. It allows us to determine the movement parameters of the selected mechanism elements as derivatives of these functions.

Equations (2) present the momentary co-ordinate positions of the cam follower axis of the internal self-aligning pusher as a function of the cam rotation angle  $\phi$ . These co-ordinates are centres of a circle family which determines the envelope of the external cam outline

$$\begin{aligned}
 x_o &= R_r \sin[\phi + \Psi_o + \Psi(\phi) + \phi_p] + \\
 &- R_a \sin(\phi + \phi_p) \\
 y_o &= R_a \cos(\phi + \phi_p) - R_r \cos[\phi + \\
 &+ \Psi_o + \Psi(\phi) + \phi_p]
 \end{aligned} \quad (2)$$

where:

- $\Psi_o$  - the angle of pusher diversion in the zone of the bottom stop, while the roller with radius  $R_r$  is in contact with the cam at the cam's basic radius  $R_b$ , and
- $\Psi(\phi)$  - the angle of the pusher deflection; a function determining the cam follower movement during the cycle of cam rotation.

The straight line which connects the knots of the rotational fastening of the pusher with the cam fastening in a frame form an angle  $\phi_p = \frac{3}{4}\pi$  with the x axis of the co-ordinate system.

The co-ordinates of the momentary centres of the family of circles which determine the cam's internal outline in dependence on the cam's rotational angle  $\phi$  are described by the following equations (3):

$$\begin{aligned}
 x_c &= R_c \sin[\phi + \Psi_o + \Psi(\phi) + \phi_p + \\
 &- v] - R_a \sin(\phi + \phi_p) \\
 y_c &= R_a \cos(\phi + \phi_p) - R_c \cos[\phi + \\
 &+ \Psi_o + \Psi(\phi) + \phi_p - v]
 \end{aligned} \quad (3)$$

The angle  $\nu$  of the pusher arms' spacing is given by equation (4):

$$\nu = \arctan[(1 - C^2)^{1/2} / C] \quad (4)$$

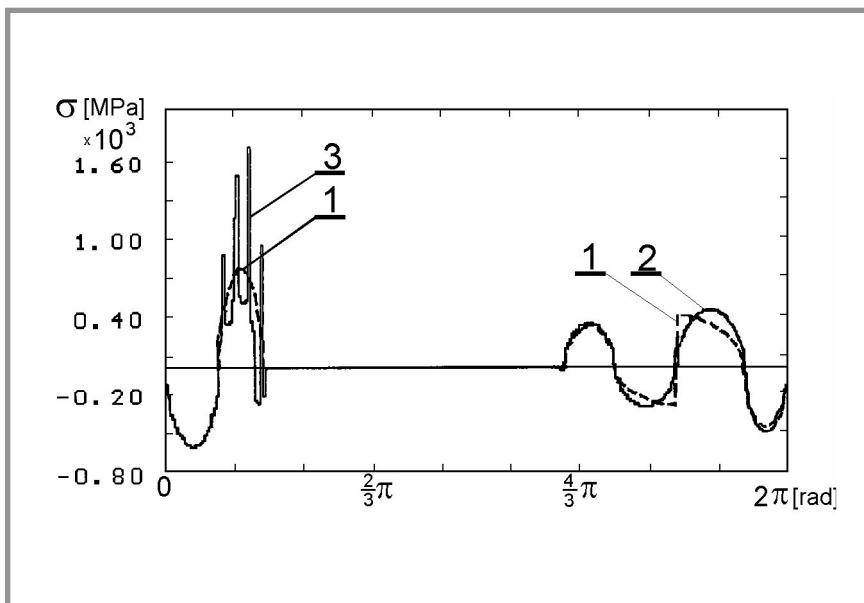
and the auxiliary quantity  $C$  occurring therein is presented by equation (5):

$$C = (R_c^2 + R_r^2 - L_{cr}^2) / 2R_cR_r \quad (5)$$

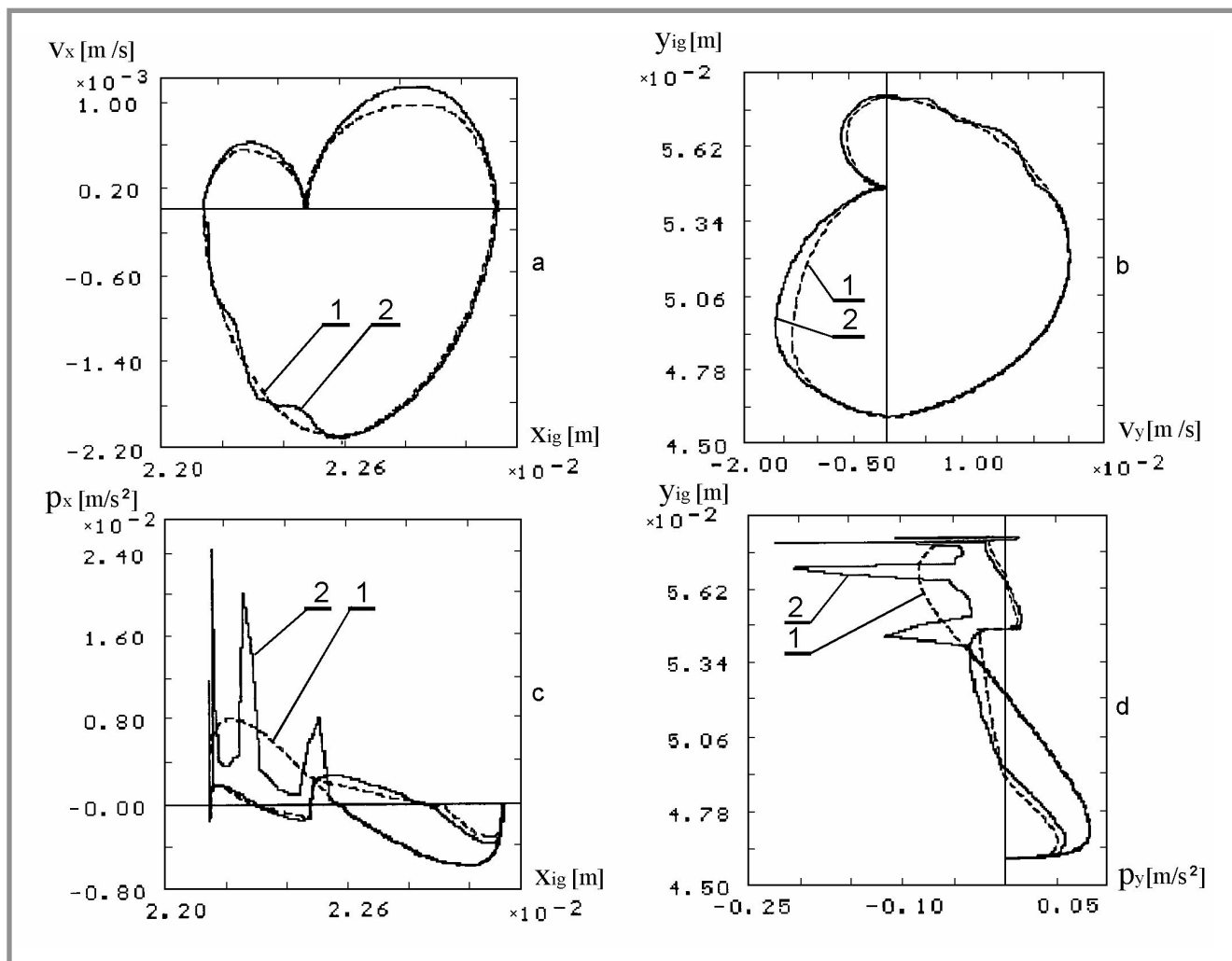
The displacing co-ordinates  $L_9$  of this end of the needle, which is the farthest towards the centre of the stable knot of the rotational link fastening, are described by (6):

$$\begin{aligned} x_{ig} &= L_9 \cos(\phi_9) + L_{10} \sin(\phi_{10}) + \\ &+ L_{12} \sin(\pi/9 - \phi_{10}) \\ y_{ig} &= -L_9 \sin(\phi_9) - L_{10} \cos(\phi_{10}) + \\ &+ L_{12} \cos(\pi/9 - \phi_{10}) \end{aligned} \quad (6)$$

Computer functions and procedures in Pascal, as well as synthesis programmes of cam outlines [1,4,5] were used for calculating the cam mechanism kinematics. A calculation procedure for determining the normal force  $N$  between



**Figure 3.** Contact stresses between cam and roller in one working cycle of the mechanism for some connections of partial functions: 1- inverted cosines, 2 - cycloidal, 3 - locally used on the rising part of the cam outline, including a function which simulates slipper surface wear.



**Figure 4.** Parameters of needle ending point movement; a, b - speed; c, d - acceleration of this point as a function of displacements in directions of axes  $x, y$  for: 1 - connection of cycloidal and inverted cosine functions; 2 - cam outline locally considering a function which simulates the wear of slipper surface.

the roller and the cam, and a procedure of determining the cam curve radius  $\rho_k$  in a full cycle of the mechanism movement, both devised at the Department of Textile Machines, were also used. The procedure for calculating contact stresses  $\sigma_{\max}$  according to the Hertz formula (7) was applied for analysis of loads at the contact point of the cam and the pusher roller:

$$\sigma_{\max} = 0.564 \cdot \sqrt{\frac{N \left| \frac{1}{R_f} \pm \frac{1}{\rho_k} \right|}{S_c \left( \frac{1-\nu_f^2}{E_f} + \frac{1-\nu_k^2}{E_k} \right)}} \quad (7)$$

The quantities occurring in this formula are described in the chapter concerning calculation assumptions.

### Assumptions for Calculations and the Results Obtained

The probable, partial, angular segments of the outline of the catenation function synthesis were adapted on the basis of calculations carried out, the mapping of the external cam profile and the pusher roller co-operating with it. For the raising part of the pusher roller, this might be the sum of angles  $\beta_1=1/6\pi$  and  $\beta_2=5/36\pi$ . The upper stop might take place at the cam rotational angle  $\beta_3=35/36\pi$ . The sum of the other four angle  $\beta_4=1/6\pi$ ,  $\beta_5=7/36\pi$ ,  $\beta_6=2/9\pi$ ,  $\beta_7=5/36\pi$  determines the range of partial function catenation for the descent zone. The sum of the angles mentioned above, on which the cam outline is represented, closes one whole cycle of the needle mechanism movement. Different combinations of basic functions such as the cycloid (8)

$$\Psi_i = A_i (\pi\phi/\beta_i - \sin(\pi\phi/\beta_i)) \quad (8)$$

and the function of the inverted cosine (9)

$$\Psi_j = A_j (1 - \cos(\pi\phi/2\beta_j)) \quad (9)$$

were adopted for the representation of the cam slipper outline as partial functions of the outline catenation. Minimal local wear traces of the working surface were observed at the cam with external outline, in the part of the outline which is related to lifting the pusher and the elements of link mechanism connected with them. They were probably made as a result of dynamic local load acting and long-lasting exploitation. For the purpose of this analysis, it was assumed that the described wear does not exceed five-hundredths of a millimetre. The function (10) presented below, based on a cycloid, was created for carrying out an attempt at computer simulation of this local wear:

$$\Psi_k = A_i (\pi\phi/\beta_i - \sin(\pi\phi/\beta_i)) + |\Psi_s \sin(25\phi)/160| \quad (10)$$

The basic geometrical parameters of the cam mechanism, determined by using the direct and the indirect methods, are as follows: the basic cam radius  $R_b=0.083\text{m}$ , the roller radius  $R_f=0.026\text{m}$ , the distance between the pusher axis and the cam axis  $R_a=R_a(L_1, L_2)=0.1485\text{m}$ , the distance between the pusher axis and the roller axis  $R_r=0.105\text{m}$ , the distance between the pusher axis and the counter-roller  $R_c=0.118\text{m}$ , the distance between axes of both rollers  $L_{cr}=0.063\text{m}$ , and the roller width  $S_c=0.018\text{m}$ . In addition to these quantities, the maximum rocker stroke  $\Psi_s=1/30\pi$  was established by measurements. Other parameters needed for calculating the dynamic influence of the roller on a cam are as follows: angular speed of the main shaft and the cam  $\omega=24\text{ rad/s}$ , Young's modulus for roller and cam materials respectively  $E_f=2 \cdot 10^{11}\text{ Pa}$ ,  $E_k=2 \cdot 10^{11}\text{ Pa}$ , and the Poisson ratio for roller and cam  $\nu_f=0.3$ ,  $\nu_k=0.3$ . The inertia of the single power unit of the lever and rocker system was estimated as  $B=0.12\text{ kgm}^2$ .

A kinematic analysis of the needle mechanism's model was performed on the basis of the following lengths of the link system elements (presented in metres) measured by the indirect method:  $L_1=0.105$ ,  $L_2=0.105$ ,  $L_3=0.230$ ,  $L_4=0.645$ ,  $L_5=0.510$ ,  $L_6=R_r=0.105$ ,  $L_7=0.400$ ,  $L_8=0.0725$ ,  $L_9=0.1865$ ,  $L_{10}=0.0585$ ,  $L_{11}=0.078$ ,  $L_{12}=0.127$ .

The calculations were made by a  $\Delta\phi$  step which equals one degree for the whole cycle of a cam turn. Figure 2 shows the catenation function of the rocker movement, which consists of the pusher's angular trace, the pusher's speed and the angular acceleration, in a full cycle of the external cam outline movement for two different combinations of the catenation outline function.

Stresses between the roller and the cam for one cycle of the mechanism movement, resulted from the calculations described above, are presented in Figure 3. Parameters of the needle ending movement are shown in Figure 4.

Extreme values of the displacements  $x_{ig}$ ,  $y_{ig}$  in the plane perpendicular to the cam shaft axis of the needle end point in this mechanism's model may be slightly different than the displacements in a real system, such as occurs in the machine.

### Conclusions

On the basis of the calculation results and the graphically determined parameter courses of pusher movement, movement of the end of needles in the needle mechanism, and the pressure of the cam on the roller, the following conclusions were drawn:

- The application of cycloidal partial functions or inverted cosine functions for the catenation of slipper outline synthesis does not considerably influence the rocker course (Figure 2a).
- Using an inverted cosine function for a project of the cam outline results in a rapid, stroke-like change of acceleration in the place of partial function connection (Figure 2c), which significantly influences the loads in knots.
- The suggested simulation of local wear of the cam surface demonstrates how great changes of contact stresses can occur at the place of pusher roller and cam connection.
- By comparing curves 1 and 2 from Figure 4 (which presents the courses of the component speeds and accelerations of the point related to needle endings), we can note how such apparently small changes of the outline of the cam slipper surface may significantly influence the momentary values of these movement quantities.

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