Radiation Stress Estimators

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(Manuscript received 13 December 1982, in final form 13 May 1983)

ABSTRACT

The radiation stresses S_{ij} associated with the propagation of wind-generated waves are principal driving forces for several important surf-zone processes. The accurate estimation of the onshore flux of longshore-directed mean momentum S_{yx}, using a linear array of pressure sensors, is considered here. Three analysis methods are examined: integration of two high-resolution directional-spectrum estimators [maximum likelihood (MLM) and a modified version (IMLM)], and a direct estimator of the S_{yx} directional moment (DMM,) which is developed here.

The S_{yx} estimation methods are compared using numerical simulations and field data from two experiments at Torrey Pines Beach, California. In the first field experiment, IMLM and DMM, estimates of S_{yx} (from a 3-element, 99 m long linear array) showed excellent agreement with a slope array (Higgins *et al.*, 1981) in the frequency range 0.05–0.15 Hz. In the second experiment, IMLM and DMM, estimates of S_{yx} (from a 5-element, 360 m long array) agreed with values of S_{yx} obtained from a nearby orthogonal-axis current meter for the frequency range 0.06–0.11 Hz. The integration of the MLM directional spectrum estimates yields biased (low) values of S_{yx}. Although the DMM method is used here for the estimation of S_{yx}, it can easily be adapted for the calculation of any arbitrary directional moment. While conventional methods are shown to be deficient in S_{yx} estimation, they provide accurate estimates of S_{xx}, the onshore flux of onshore-directed momentum.

1. Introduction

The flux of wave momentum, represented by the radiation stress tensor (Longuet-Higgins and Stewart, 1964), drives many important processes in the nearshore environment. The two dominant terms of the radiation stress tensor are the onshore flux of longshore momentum, S_{yx} and the onshore flux of onshore momentum S_{xx}. These terms can be expressed as a function of frequency, and assuming linear wave dynamics, they are related to the following moments of the frequency-directional spectrum, $E(f, \alpha)$:

$$\left.\begin{aligned} S_{xx}(f) &= n(f)\int_\alpha E(f,\alpha)\cos^2\alpha d\alpha \\ S_{yx}(f) &= n(f)\int_\alpha E(f,\alpha)\sin\alpha\cos\alpha d\alpha \end{aligned}\right\}. \quad (1)$$

The total stresses are

$$\left.\begin{aligned} S_{xx} &= \rho g\sum_f S_{xx}(f)\Delta f \\ S_{yx} &= \rho g\sum_f S_{yx}(f)\Delta f \end{aligned}\right\}, \quad (2)$$

where f is the wave frequency, α the propagation direction, $n(f)$ the ratio of group to phase speeds expressed as a function of frequency, and ρ the density of seawater. From relations (1) it is apparent that directional wave measurements are required to estimate the wave-radiation stress.

The generation of longshore currents and the longshore transport of sand by waves is a phenomenon of considerable importance in the dynamics of shoreline evolution. The problem of deriving a relationship between the energy supplied by the waves and the resulting longshore currents and sediment transport has been approached both theoretically and empirically. Lundgren (1963) and Bowen (1969) relate longshore currents to on–offshore variations in the onshore flux of longshore-directed momentum S_{yx}. Several investigators, including Komar and Inman (1970), have worked with the relationship for longshore transport

$$I_l = K_o(CS_{yx})_b, \quad (3)$$

where I_l is the immersed-weight transport rate, C the wave phase speed, K_o a constant, and b denotes evaluation of the parameters at depth of wave breaking. Good-quality field data are fairly rare because both the sediment transport rate and S_{yx} are difficult quantities to measure.

The onshore flux of onshore directed momentum, S_{xx} is also an important forcing function in surf zone dynamics. For example, Bowen *et al.* (1968) showed that wave set-up and set-down, with steady amplitude waves, are related to on–offshore gradients in S_{xx}. Longuet-Higgins and Stewart (1962) suggest that surf beat results from the time variation of S_{xx} associated with groups of high and low waves. However, accurate estimates of S_{xx} are readily obtained from integration of crude directional-spectrum measurements when the waves are near normal incidence to a linear array (generally true for nearshore arrays) because of the smooth and symmetric behavior of the $\cos^2\alpha$ weighting. Therefore, attention here is focused on the more difficult problem of making accurate estimates of S_{yx}.

There are significant sampling problems involved in the measurement of the directional spectrum (e.g., Barber, 1963) and many approximate forms have been used for the estimation of $S_{yx}(f)$. Komar and Inman (1970) used the phase difference between pairs of wave sensors for the estimation of breaker angle in the estimate

$$\hat{S}_{yx} = \rho g n(f_o) E \cos\alpha_b \sin\alpha_b, \qquad (4)$$

where E is the variance of the water-level fluctuation of the wave field, and f_o the observed "average" wave frequency. The weighting of this average angle, however, is not the same as the directional weighting in the spectral moment of (1) and significant errors are expected with complicated or broad directional spectra, although reasonably accurate results are expected if the waves are approximately unidirectional.

Many analysis techniques have been used for the calculation of the directional spectrum from wave-array data. Dean (1974) compared the use of the Barber window (Barber, 1963) with the W2 smoothed scheme (Panicker, 1971) for the estimation of $S_{yx}(f)$ [Eq. (1)]. His results indicated the superiority of the Barber window with expected errors of less than 10% for unidirectional waves. However, Pawka (1977) demonstrated $S_{yx}(f)$ estimation errors in the range 5–40% for the Barber window with directional spectra of finite width, and found better relative performance of the Maximum Likelihood Method [MLM (Capon, 1969)]. This data-adaptive estimator was shown to be particularly accurate in the estimation of $S_{yx}(f)$ with the conditions of narrow directional spectra.

The directional moments given in (1) can be obtained directly (independent of a directional-spectrum estimate) from sensor systems that measure orthogonal components of wave velocity or sea-surface slope (e.g., Longuet-Higgins *et al.*, 1963). Examples of these systems include the pitch-and-roll buoy, orthogonal-axis current meter, and the slope array introduced by Seymour and Higgins (1977). Although these "orthogonal component" systems yield a direct estimate of $S_{yx}(f)$, they give relatively poor resolution of the directional properties of the spectrum. The poor resolution would render these systems inadequate for detailed studies of directional wave properties (e.g., Pawka, 1983). Also, the directional spectra obtained from these systems are not generally adequate for the accurate transformation of the radiation-stress estimates to other onshore or longshore localities.

Therefore, an investigation was made into the methods of extraction of accurate values of $S_{yx}(f)$ from a linear array of sensors. A slope and linear array were positioned together in the field for the comparison of the $S_{yx}(f)$ estimates from the systems. A similar test of a linear array with a current meter was also conducted. Although the orthogonal-component systems theoretically yield nearly unbiased local estimates of $S_{yx}(f)$, they have not been previously field-tested.

Initial comparisons of the linear and slope arrays were reported by Higgins *et al.* (1981). A strong bias between the two systems was observed for medium-to -high wave frequencies ($f > 0.08$ Hz). The MLM was used in the array analysis and the authors suspected that the bias between the two systems was due to deficiencies in this method in the calculation of the sensitive $S_{yx}(f)$ directional moment [Eq. (1)]. Subsequent simulation tests (Pawka, 1982) showed the MLM consistently to bias (low) the value of $S_{yx}(f)$, particularly in response to multi-modal directional forms. The spectra observed in the field test did display a distinct bimodal form, for $f > 0.08$ Hz, explaining the poor performance of the MLM. The simulation tests indicated extremely accurate performance of a modified MLM estimator (IMLM) which is discussed by Pawka (1982, 1983). An alternative method (DMM_v), which uses the linear-array cross spectra to make a direct estimate of the appropriate directional moment [Eq. (1)], is developed here. The $S_{yx}(f)$ estimates of the MLM, IMLM and DMM_v methods are all compared to the orthogonal-component systems in the field tests.

2. Directional spectrum estimators

The MLM was first applied to wavenumber spectrum analysis by Capon (1969) in an investigation of seismic waves. The methodology employed in this data-adaptive method includes the minimization of estimate variance subject to the constraint that the amplitude of a pure unidirectional wave (no noise) is estimated without bias. A complete treatment of one derivation of the estimator is given by Regier (1975). The MLM was chosen for the original data analysis because it is a widely used high-resolution estimator.

A method was introduced by Pawka (1982) that modifies the MLM directional spectrum estimates in an iterative fashion to yield a spectrum that is more consistent with the cross-spectral moments. The modified estimate (IMLM) is a possible solution for the true directional spectrum while this is generally not true for the MLM estimate. The mechanics of the IMLM estimator revolve around solving for a spectrum which transforms through the array's smearing filter to yield the original MLM estimate. The details of this method are discussed by Pawka (1982, 1983).

3. Directional-moment method

An arbitrary moment of the directional spectrum is

$$M = \int_\alpha M(\alpha)E(\alpha)d\alpha, \tag{5}$$

where $E(\alpha)$ is the directional spectrum and $M(\alpha)$ the desired weighting function. The directional-spectrum estimate $\hat{E}(\alpha)$ can be used as an approximation of $E(\alpha)$ in (5) to yield an estimate of the moment M. However, a more direct estimate of this moment can be obtained by making use of the smearing inherent in the directional-spectrum estimate

$$\hat{E}(\alpha) = \int_{\alpha'} W(\alpha, \alpha')E(\alpha')d\alpha', \tag{6}$$

where $W(\alpha, \alpha')$ is the spectral window, and α' is a dummy variable. If the spectrum estimate is formed from a linear combination of the cross-spectra from a linear array

$$\hat{E}(\alpha) = \sum_l \sum_m \beta_{lm}(\alpha)X_{lm}, \tag{7}$$

then the window is

$$W(\alpha, \alpha') = \sum_l \sum_m \beta_{lm}(\alpha) \exp(ikx_{lm} \sin\alpha'), \tag{8}$$

where β_{lm} are complex weights, k the wavenumber, X_{lm} an element of the complex cross–spectral matrix, x_{lm} a sensor lag size, and l and m denote the sensor pair. If the coefficients β_{lm} are now constants over the look angle α, then the smearing of (6) is

$$\hat{M} = \int_{\alpha'} E(\alpha')W(\alpha')d\alpha'. \tag{9}$$

It was suggested by R. E. Davis (personal communication) that $W(\alpha')$ be shaped, by manipulation of the coefficients, to resemble the function $\sin\alpha' \cos\alpha'$ for the estimation of S_{yx}. Although with a limited number of sensors it may be difficult to shape $W(\alpha')$ accurately for all angles, it is only necessary to make the window approximate the function $\sin\alpha' \cos\alpha'$ for angles where $E(\alpha')$ is relatively large.

The window can be expressed in the form

$$W(\alpha') = \sum_l \sum_m a_{lm} \exp(ikx_{lm} \sin\alpha') = \sum_l \sum_m a_{lm}h_{lm}(\alpha'), \tag{10}$$

where $a_{lm} = a^*_{ml}$ (the asterisk denotes the conjugate) to ensure a real-valued estimate. The shaping of the window, for the best S_{yx} estimate in the least-squares sense, requires the minimization of the parameter

$$I = \sum_{\alpha'} [W(\alpha') - \tfrac{1}{2} \sin2\alpha']^2E(\alpha')\Delta\alpha' \tag{11}$$

with respect to the coefficients a_{lm}. The coefficients for a similar method (MSEM) were shown to be relatively insensitive to the spectral estimate used for the weighting function (Pawka, 1983). Therefore, the MLM spectra were used as estimates of $E(\alpha')$ in (11). The problem is then reduced to solution by the method of least squares.

It was observed that the coefficients a_{lm} were very sensitive to noise in the cross spectra with this form of I. The variance of the estimate $\hat{M}$, in the presence of noise, is proportional to $\sum_l \sum_m a^2_{lm}$ (Davis and Regier, 1977). Therefore, the term $\nu \sum_l \sum_m a^2_{lm}$ is added to the RHS of Eq. (11) to lend stability to the estimates. The system of equations to solve for a_{lm} is then

$$\frac{\partial I}{\partial a_{lm}} = 0 = \sum_j \sum_k a^*_{jk} \sum_{\alpha'} h_{lm}(\alpha')h^*_{jk}(\alpha')E(\alpha') + \nu a^*_{lm} - \sum_{\alpha'} \tfrac{1}{2} \sin2\alpha' E(\alpha')h_{lm}(\alpha'). \tag{12}$$

This estimator will be referred to as the Directional Moment Method (DMM_ν), where ν is the noise suppression coefficient. The solution of (12) is straightforward and follows closely the mechanics of the MSEM method (Pawka, 1983).

The performance of the DMM_o ($\nu = 0.0$) method in the estimation of $S_{yx}(f)$ was tested in response to several deterministic directional spectrum forms. A 1–2–4–5 configuration linear array (unit lag = 33.0m) employed in the slope-array comparison experiment was used in the test analysis. The results for a variety of test spectra, shown in Fig. 1, are excellent for wave frequencies 0.05–0.13 Hz. The performance of the DMM_o method drops off at 0.13 Hz for the broad test spectrum. The high-frequency estimates are the most accurate in response to the narrow directional forms, falling off only with the onset of spatial aliasing, $f \approx 0.17$ Hz.

The various estimators were subjected to simulated random cross spectra, following Borgman (1973). As anticipated, the DMM_o estimates were relatively noise sensitive. The values of ν_n are normalized by the spectrum

$$\nu_n = 10\nu[\sum_\alpha E(\alpha)\Delta\alpha]^{-1} \tag{13}$$

to compare the results with different spectral forms. Table 1 shows the bias and variance of the DMM_ν estimates as a function of ν_n. This analysis was done with a relatively noise sensitive 2–2–2–5 array (because of side-lobe problems) which was used in the current-meter comparison experiment. A marked drop in variance occurs with values of ν_n between 0.01 and 0.5.

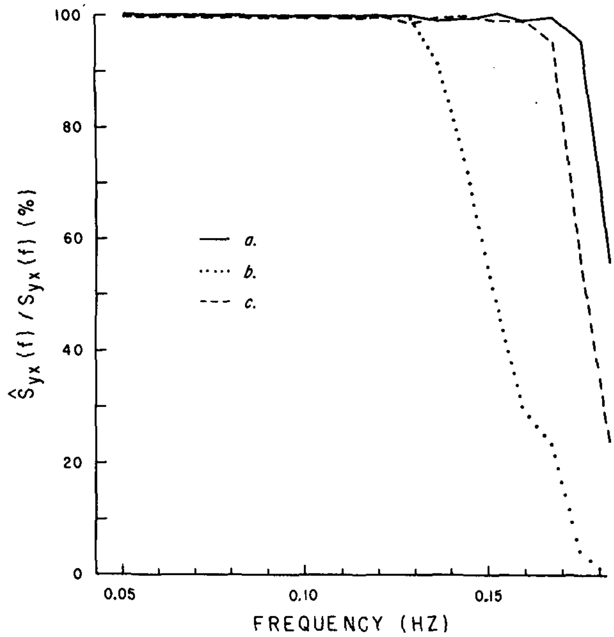

FIG. 1. Estimates of $S_{yx}(f)$ obtained from the DMM_ν method in response to deterministic test spectra. The estimates are normalized by the true test value of $S_{yx}(f)$. The 1-2-4-5 array of length 396 m was used for the analysis. The curves are for the test spectra: (a) narrow, $\cos^{200}(\alpha + 20°)$, (b) broad, $\cos^4(\alpha + 20°)$, (c) bimodal, $\cos^{200}(\alpha + 10°) + 2.0 \cos^{200}(\alpha - 10°)$.

The location of this jump varies as a function of directional-spectrum form and frequency. The estimator bias also varies but remains in the 10% range for the tested values of ν_n. The DMM_ν bias is similar to the accuracy of the IMLM estimates while the MLM estimates of S_{yx} are biased very low in magnitude (Table 1). The DMM_ν estimates at 0.098 Hz are biased high at low values of ν_n and biased low at high values of ν_n. This effect is seen later in the comparative field data.

The directional spectrum estimators and the DMM_ν method assume a linear wave field in the analysis. These assumptions will break down as the field becomes strongly nonlinear. An analysis by Freilich (1982) indicated that wave fields, similar to those treated here, are quasi-linear at the array depth (~10 m) at this site. The effects of nonlinear waves, perhaps due to significantly more energetic waves, on this analysis are unclear. Evaluation of these effects requires a finite-depth, two-dimensional nonlinear shoaling model, which is not presently available.

4. Field comparison of a slope and linear array

In June 1977 a slope array was aligned within a 1-2-4-5 linear array, shown in Fig. 2, at Torrey Pines Beach, California. Although the bottom contours at the site are roughly plane-parallel, the 1-2 configuration subarray was used for the primary comparisons to the slope array to minimize bias from possible spatial homogeneities in the wave field. However, comparisons for selected sets of field data and various numerical tests showed roughly equivalent accuracy of the 3- and 5-sensor systems. For example, the MLM, IMLM and DMM_o (no noise rejection) estimates of $S_{yx}(f)$ with a strongly bimodal deterministic test spectrum are shown in Fig. 3. The IMLM and DMM_o methods show similar good accuracy while the MLM results are biased very low.

The slope array uses short sensor lags (~6 m) for the estimation of the orthogonal slope components. The directional moment associated with $S_{yx}(f)$ is directly proportional to the cross-spectrum of the orthogonal components of sea-surface slope:

$$C_{n_x n_y}(f) = k^2 \int_\alpha E(f, \alpha) \cos\alpha \sin\alpha d\alpha, \qquad (14)$$

where $C_{n_x n_y}(f)$ is the co-spectrum of the x and y components of slope. The only fundamental bias in this measurement is the finite-difference approximation of the sea-surface slopes. The expected errors associated with this approximation are a function of k versus the size of the sensor lags (Higgins *et al.*, 1981) but should be very small (<1%) for the wave frequencies considered in these comparisons.

A high-quality continuous data run lasting 2.6 h was taken on 10 June. The average frequency spectrum for this run is dominated by a very narrow low-frequency peak (Fig. 4). There is a wide variety of

TABLE 1. Bias and standard deviation of the $S_{yx}(f)$ estimators in response to numerically simulated random cross-spectra associated with a bimodal spectrum of the form: $\cos^{500}(\alpha - 10°) + 2.0 \cos^{500}(\alpha + 10°)$. The bias is given as the ratio of the mean estimate value $\langle \hat{S}_{yx}(f) \rangle$ to the true value $S_{yx}(f)$; σ_n is the standard deviation of the estimates normalized by the true mean value. Each $\hat{S}_{yx}(f)$ estimate is based on a cross-spectrum with 16 degrees of freedom. Mean and variances of $\hat{S}_{yx}(f)$ are based on 50 realizations. The analysis was performed with a 2-2-2-5 array configuration (unit lag = 33 m). Bias values are in percent.

	Frequency* (Hz)	DMM_ν $\nu = 0.0$	$\nu = 0.1$	$\nu = 0.5$	$\nu = 1.0$	MLM	IMLM
$\langle \hat{S}_{yx}(f) \rangle / S_{yx}(f)$	0.067	96.3	95.4	92.7	91.3	56.4	100.9
	0.098	107.8	105.5	98.6	91.7	50.5	92.9
σ_n	0.067	0.68	0.60	0.58	0.58	0.41	0.67
	0.098	0.82	0.79	0.70	0.62	0.27	0.54

* Results for two frequencies are shown.

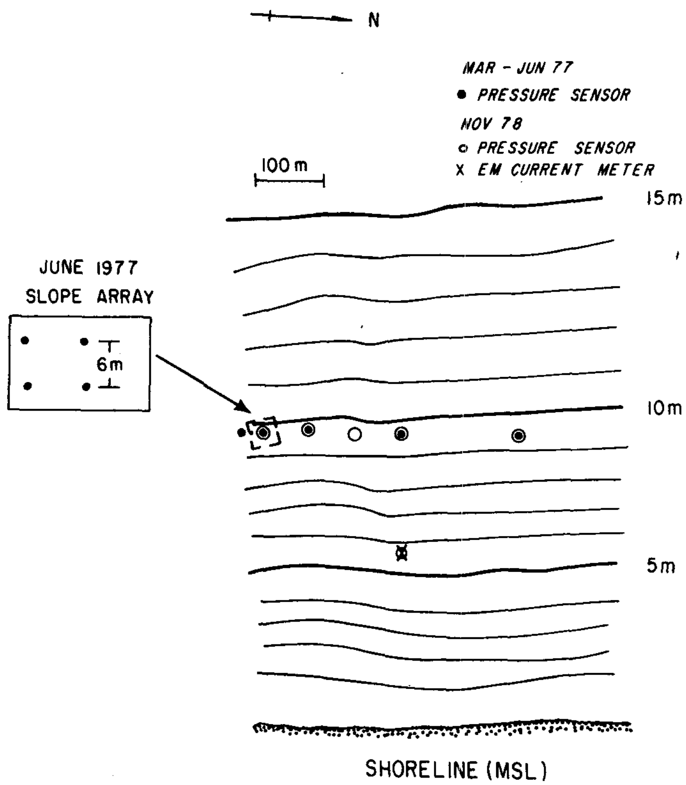

FIG. 2. Local bathymetry of Torrey Pines Beach and sensor locations for the two experiments. The contours were sampled in November 1978 and reported in Gable (1981).

directional-spectrum shapes in the wind–wave frequency range. Fig. 5 is a plot of the MLM and IMLM estimates of the directional spectra for two wave frequencies on 10 June. The data set was divided into nine segments of length 1024 s for the analysis of the frequency spectra, directional spectra and $S_{yx}(f)$. Spectral smoothing within each segment yielded 16 degrees of freedom (DOF) with a bandwidth of 0.0078 Hz.

The comparison of the various linear-array estimates of $S_{yx}(f)$ with the slope-array results are shown in Fig. 6. The discrepancy between the MLM and slope estimates, discussed first by Higgins *et al.* (1981), are shown to be particularly large for medium- to-high wave frequencies (0.17 Hz $\geqslant f \geqslant$ 0.08 Hz). The IMLM and DMM_o methods show a dramatic improvement over the MLM in the comparisons to the slope array in this broad frequency range, although the DMM_o estimates appear to be slightly biased high.

A normalized rms deviation of any two estimates is defined as

$$\text{nrms deviation} = 100\left[\frac{4}{N}\sum_{j=1}^{N}\left(\frac{E1_j - E2_j}{E1_j + E2_j}\right)^2\right]^{1/2}, \quad (15)$$

where $E1_j$ and $E2_j$ are the two estimates for segment j, N is the number of data segments, and the values are in percent. The quantitative comparisons of the various $S_{yx}(f)$ estimates for wave frequencies 0.098–0.145 Hz are listed in Table 2. The DMM_ν estimates show increasing correlation with the slope results for increasing values of ν_n up to 3.5. However, the nrms deviation of these estimates shows a minimum for $\nu_n = 1.0$. This is in rough agreement with the results from the simulation tests discussed above. Too much noise rejection degrades the resolution to the extent that $S_{yx}(f)$ is not well estimated. The $S_{yx}(f)$ estimates of the DMM_1, slope and IMLM methods in the frequency range 0.098–0.145 Hz agree to an average of roughly 7–8%. Note that the MLM estimates are biased significantly low with nrms deviations of 40–100% throughout this frequency range.

The average sample variance of the various estimates is also shown in Table 2. The IMLM, slope and DMM_1 estimates show approximately the same variability. The MLM estimates are much more stable; however, they show less correlation with the slope results than either the IMLM or DMM_ν estimates.

The correlation of the estimates from the various methods is very high ($r \approx 0.99$) at the peak of the frequency spectrum (f = 0.059 Hz). However, the various linear array methods show significant bias relative to the slope estimates. The average directional

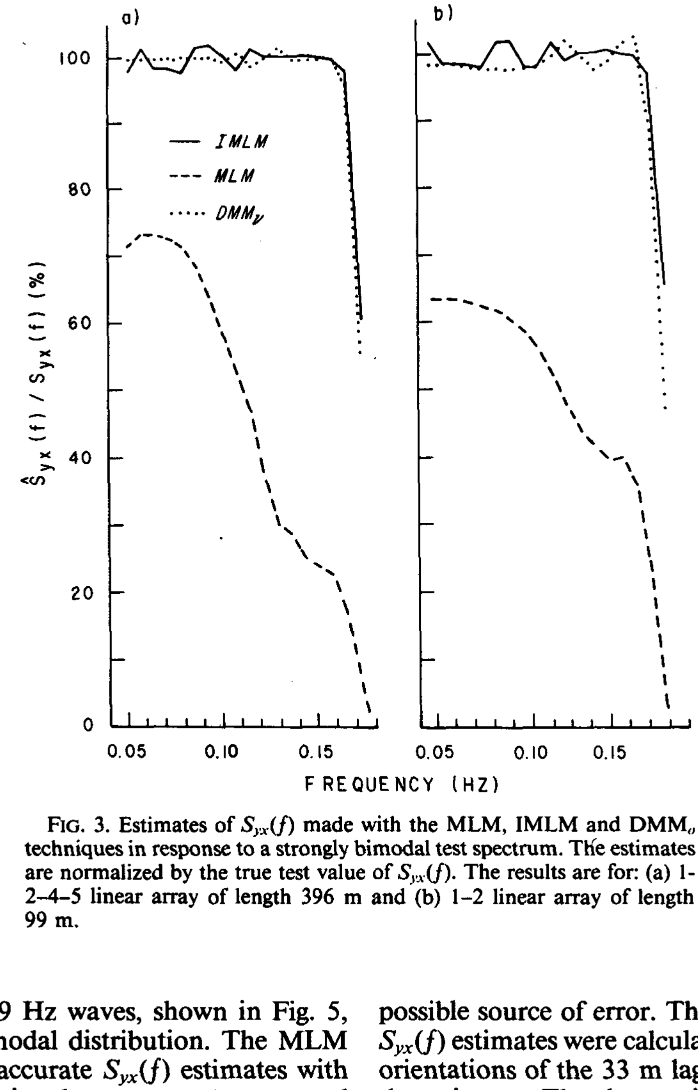

FIG. 3. Estimates of $S_{yx}(f)$ made with the MLM, IMLM and DMM$_\nu$ techniques in response to a strongly bimodal test spectrum. The estimates are normalized by the true test value of $S_{yx}(f)$. The results are for: (a) 1-2-4-5 linear array of length 396 m and (b) 1-2 linear array of length 99 m.

spectrum for the 0.059 Hz waves, shown in Fig. 5, is a very narrow unimodal distribution. The MLM should provide fairly accurate $S_{yx}(f)$ estimates with this form of the directional spectrum. As expected with these conditions, there is not much relative change in the IMLM and DMM$_\nu$ results compared to the MLM estimates (Fig. 6). The correlation of the DMM$_\nu$ and slope estimates at 0.059 Hz peaks at a value of ν_n equal to 0.3. The DMM$_{0.3}$ estimates fall between the MLM and IMLM values at this frequency.

Deterministic and random tests of unimodal spectral forms (Pawka, 1982) showed the MLM estimates of $S_{yx}(f)$ always to be biased low. The fact that the MLM estimates of $S_{yx}(f)$ are greater than the slope estimates for 0.059–0.067 Hz (Fig. 6), coupled with the very close agreement of the various array estimates in this range, suggests possible errors in the experimental set-up creating a bias between the slope and linear systems. One possible source of bias between the sensor systems is the relative orientation of the sensor lags. The slope array was physically aligned with the 66 m lag of the linear array (see Fig. 2), leaving only the orientation of the 33 m lag as a possible source of error. The directional spectra and $S_{yx}(f)$ estimates were calculated for different assumed orientations of the 33 m lag to test the sensitivity of the estimates. The changes in the $S_{yx}(f)$ estimates for the IMLM estimates for a 2° rotation are shown in Fig. 6 and indicate relatively strong sensitivity in the low-frequency region. This rotation significantly improves the low-frequency comparisons while causing relatively small changes in the high-frequency tail. It is possible that this rotation explains the discrepancies in the low-frequency range. An error of roughly 1 m in the location of either sensor in the 33 m lag can cause a discrepancy of 2° in orientation. This is the approximate accuracy of the sensor locations.

Relatively low values of $S_{yx}(f)$ result when the directional spectrum is strongly bimodal and centered about normal incidence to the coast. This spectral condition exists for the frequency range 0.082–0.090 Hz where the emphasis in the spectrum is shifting from southern to northern dominance (Fig. 5). The IMLM and MLM methods have particular problems in the estimation of $S_{yx}(f)$ with these spectral forms. The IMLM estimates in this range deviate from the slope results by values up to 30%. The MLM results

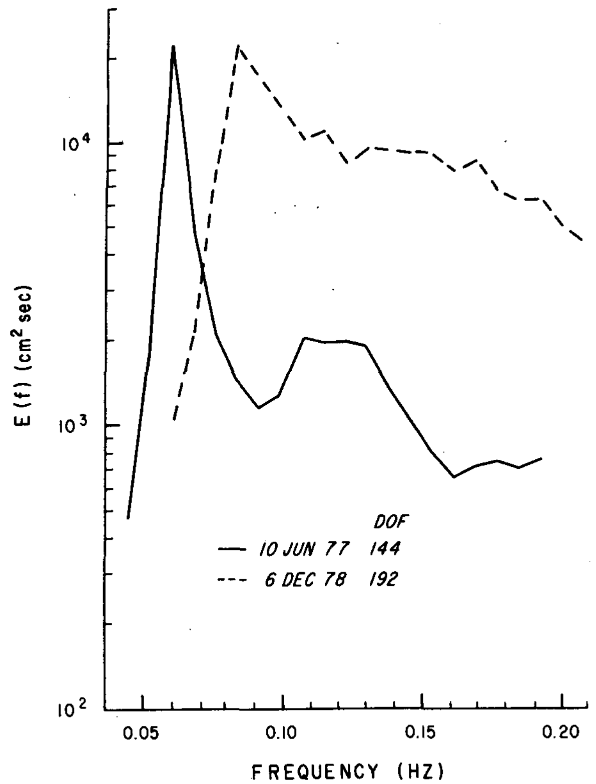

FIG. 4. Average frequency spectra for the $S_{yx}(f)$ estimation comparison data sets. The 10 June data were sampled at the 9.6 m array depth while the November–December 1978 runs were obtained from the 5.7 m depth pressure gage; DOF indicates the degrees of freedom resulting from the spectral averaging.

have deviations up to 100%. The DMM_o and DMM_1 estimates show significantly improved agreement with deviations within the values of 20 and 10%, respectively.

There is a problem in the decision of what value of ν_n to use in the general application of the DMM_ν method. The numerical simulation tests and field data comparisons indicate that the value of ν_n which results in minimum bias relative to the slope system varies with wave frequency and directional spectrum form. However, the value of ν_n which yields the best agreement with the slope estimates was close to 1.0 for most of the medium- to-high frequency range. Also, the DMM_1 results at a low frequency differ from the DMM_o estimates by only 10%. Therefore, a constant value of ν_n can result in accurate estimates of $S_{yx}(f)$ for a variety of wave frequencies and spectral forms with this array configuration.

The 1–2–4–5 sensor array was also used for the analysis of directional spectra for selected frequency bands. On the average, the estimates from the 3- and 5-sensor systems agreed to within 10%. There is a small amount of spatial inhomogeneity indicated as the 5-sensor directional results are always shifted south relative to the 3-sensor estimates. The measured bias in $S_{yx}(f)$ estimates corresponds to angular differences of less than 1°.

5. Field comparisons of a linear array and current meter

The Nearshore Sediment Transport Study (NSTS) conducted an intensive field experiment at the TPB site during the month of November 1978. Approximately 60 instruments, including pressure sensors, wave staffs and current meters covered the shelf region from 10 m depth to the swash zone. The experiment was designed for the investigation of fluid dynamics and water-sediment interaction in the surf zone. Gable (1981) gives the details of the experimental setup, sensors employed, and the data recorded.

Detailed statistics of the incident wave field were necessary for support data to the surf-zone study. These "offshore" wave measurements were made by a linear array of pressure sensors in 10 m of water and an orthogonal-axis electromagnetic current meter and pressure sensor in a mean depth of 5.7 m. The layout of the sensors is shown in Fig. 2. The array had a 2–2–2–5 configuration with an integral lag equal to 33

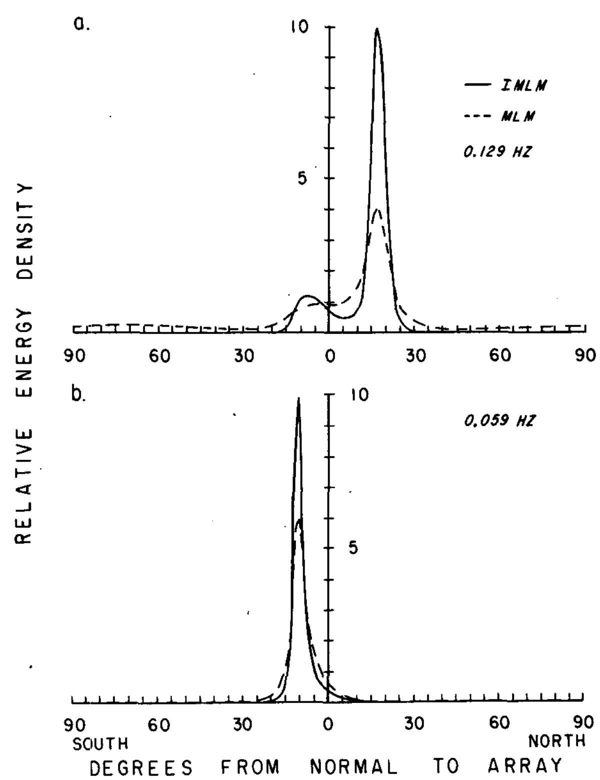

FIG. 5. Directional spectrum estimates from the 1–2 linear array for the average of 9 consecutive 17.1 min data segments on 10 June 1977 (144 degrees of freedom). The estimates are for a mean depth of 9.6 m.

TABLE 2. Quantitative comparison of various linear-array estimates of $S_{yx}(f)$ with slope-array (S) results for 10 June 1977. NRMS deviation is defined in Eq. (15). The statistics were calculated from estimates of nine consecutive 1024 s runs. All quantities are then averaged over the frequency range 0.106–0.153 Hz.

	Estimator pairs					
	MLM–S	IMLM–S	DMM_o–S	DMM_1–S	DMM_2–S	DMM_1–IMLM
NRMS deviation	69.2	4.7	17.9	6.8	14.2	4.9
Correlation	0.77	0.91	0.81	0.83	0.84	0.93
	Estimator					
$\hat{S}_{yx}(f)$	S	MLM	IMLM	DMM_o	DMM_1	DMM_2
Mean (10^2 cm^2 s)	2.36	1.27	2.35	2.78	2.35	2.08
Variance (10^4 cm^4 s^{-2})	0.63	0.43	0.58	0.68	0.63	0.42

m. A shorter lag was planned but not realized because of errors in sensor placement.

The expected directional performance of the 2-2-2-5 array is seriously degraded in the higher-frequency bands relative to the 1-2-4-5 array. Fig. 7 is a plot of the $S_{yx}(f)$ estimates obtained from the MLM, IMLM and DMM_o methods with the NSTS array configuration for two test spectra. The performance of all the estimators drops off at a relatively low frequency for this array configuration (compare to Fig. 1). The analysis indicates a slight superiority of the IMLM method over the DMM_o in the frequency range where the general performance of both methods begins to drop (0.113–0.153 Hz).

The average wave conditions during the November experiment had significant wave energy at frequencies above the usable range for directional analysis of the 2-2-2-5 array. It was necessary to use the current-meter/pressure-sensor pair in 5.7 m for the complete directional coverage of the energetic wave frequencies. The co-spectrum of the two orthogonal components of orbital velocity is related to the frequency-directional spectrum:

$$C_{uv}(f) = \left(\frac{gk}{2\pi f}\right)^2 \frac{\cosh^2 kz'}{\cosh^2 kh} \times \int_o E(f, \alpha) \cos\alpha \sin\alpha d\alpha, \quad (16)$$

where z' is the height of the sensor off the bed, h the mean water depth, and $C_{uv}(f)$ the co-spectrum of the cross-shore velocity, u and the longshore velocity v. This moment is directly proportional to $S_{yx}(f)$, which was a desired parameter for the longshore current studies.

The major problem with the current meter was a relatively large uncertainty in its absolute orientation. The estimated confidence in the current-meter orientation was roughly 3–5° compared to the uncertainty of, at most, 1° for the larger lags in the array. The orientation of the current meter was checked in a preliminary way by comparing the mean direction, at a particular frequency, with the modal direction of the directional spectrum estimated at the 10 m array. These comparisons are only meaningful for a very narrow, unimodal directional spectrum. The directional spectra of the array were refracted to 5.7 m depth for the directional comparisons with the current meter. The results showed a consistent discrepancy of 5° in the assumed orientation of the current meter.

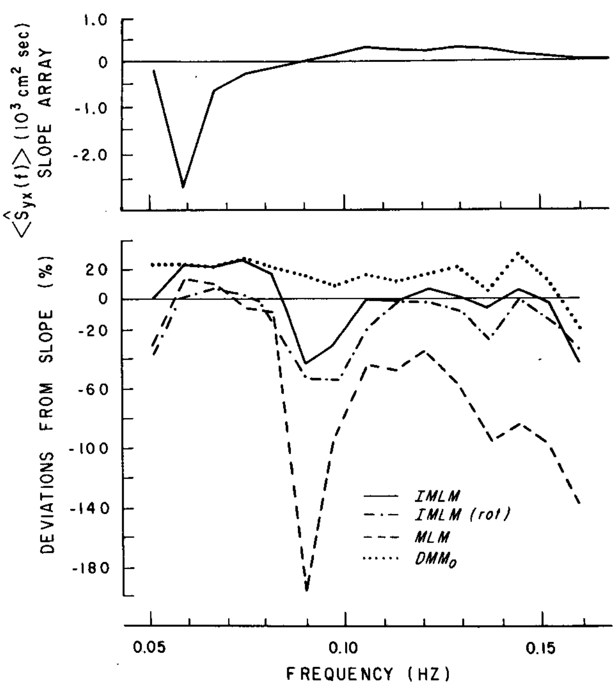

FIG. 6. (a) Slope-array estimates of $S_{yx}(f)$ for the average of 9 consecutive 17.1 min data runs on 10 June 1977. (b) Deviations of the various linear array $S_{yx}(f)$ estimates from the slope results. The deviations are defined as (array estimate minus slope estimate)/(average estimate) and are given in percent. The results are shown for the MLM, IMLM and DMM_o methods. Also shown are the IMLM deviations with a 2° rotation in the 33 m lag of the array.

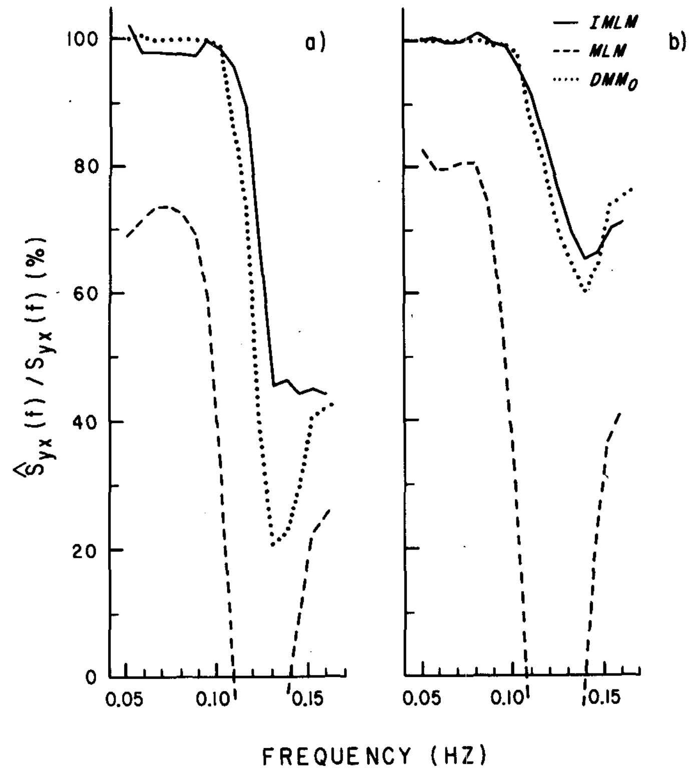

FIG. 7. Linear array estimates of $S_{yx}(f)$ for a 2-2-2-5 configuration with (a) strongly bimodal test spectrum and (b) unimodal test spectrum. The deterministic response of the various methods is normalized by the true test value of $S_{yx}(f)$, and given in percent.

There was only a small number of cases available for the modal directional comparisons. Therefore, another approach was adopted for the orientation computations. Array estimates of $S_{yx}(f)$ were made using the IMLM spectra and transformed to the 5.7 m contour by numerical refraction analysis. The assumed orientation of the current meter was then rotated to yield the best fit, in the least-squares sense, to the $S_{yx}(f)$ values in the frequency range 0.051–0.106 Hz. The array has a relatively good expected performance in this range.

Data runs from five separate days spanning the month-long experiment were selected for the orientation analysis. The "optimal" rotation angles of all five runs fell within a range of 1.7° and the mean of these angles was only 0.5° different from the result of the modal directional analysis. The mean rotation angle was then used for a constant current-meter orientation angle for all the data runs of the November experiment.

A relatively energetic northern swell dominated the wave field on 6 December. The average frequency spectrum and sample directional spectra obtained from the linear array are shown in Figs. 4 and 8, respectively. The comparative $S_{yx}(f)$ estimates ob-

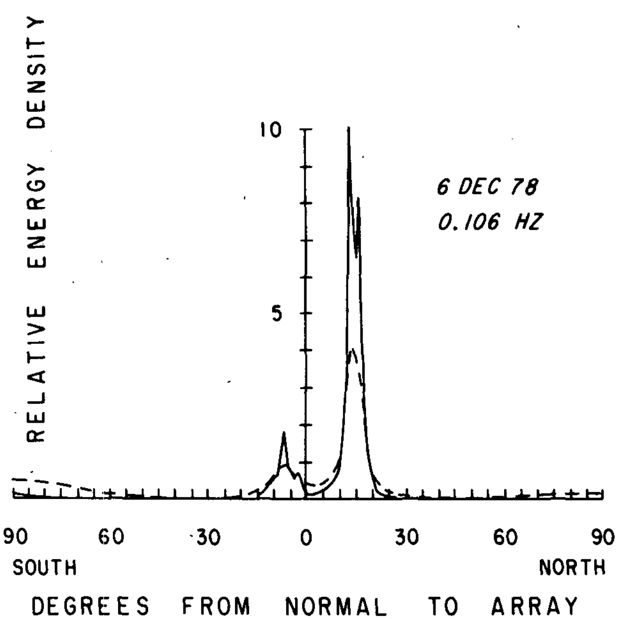

FIG. 8. MLM (dashed) and IMLM (solid) directional-spectrum estimates from the 2-2-2-5 linear array for 12 consecutive 17.1 min segments on 6 December 1978. The estimates are for a mean depth of 9.6 m.

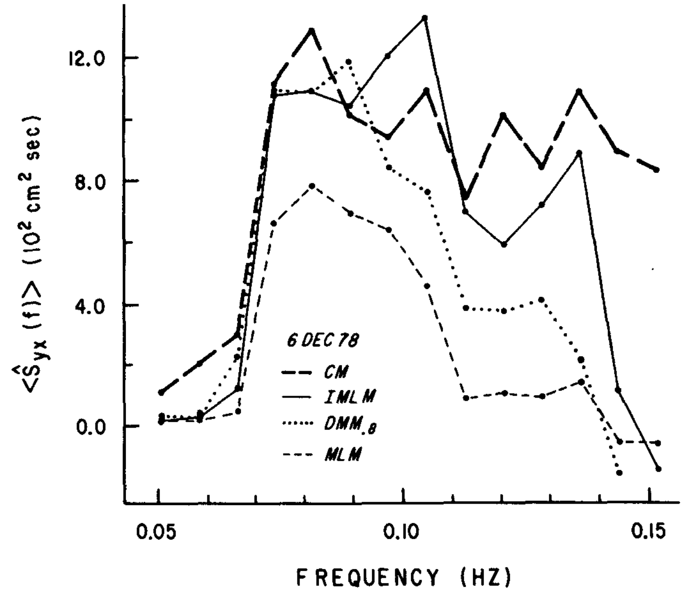

FIG. 9. Estimates of $S_{yx}(f)$ for three methods with the 2-2-2-5 linear array and the biaxial current meter (CM). The data are averaged over 12 consecutive 17.1 min runs. The value of ν_n = 0.8 yielded the best fit of the DMM$_\nu$ estimates to the current-meter results in the frequency range 0.074–0.106 Hz.

tained from the DMM$_{0.8}$ and IMLM methods and the current meter (CM) system are shown in Fig. 9 and compare well in the frequency range 0.074–0.106 Hz. The DMM$_{0.8}$ estimates drop off sharply for wave frequencies > 0.106 Hz. This result was demonstrated in the deterministic model testing of the estimators. No tested value of ν_n suppressed this sharp cutoff. The $S_{yx}(f)$ estimates obtained from the IMLM method agree well with the CM results for frequencies up to 0.129 Hz. Additional data are presented be Pawka (1982) which show similar (and sometimes better) comparisons of the IMLM, DMM$_\nu$ and CM estimates. The MLM estimates show a considerable bias (low), particularly at higher frequencies, for all data sets analyzed.

The DMM$_o$ estimates are very unstable but the sample variance for values of ν_n = 0.1–1.0 is comparable to that of the CM and IMLM estimates (Table 3). The mean $S_{yx}(f)$ values in the frequency range 0.074–0.106 Hz obtained from the DMM$_\nu$ method, also shown in Table 3, have a small bias relative to the CM results for values of ν_n = 0.5–1.0.

6. Conclusions

Results from deterministic and random computer simulation tests and field comparisons with a slope array and current meter, show the MLM method to be deficient in the estimation of $S_{yx}(f)$. The MLM performs well only for very narrow, unimodal distributions with low background noise levels.

The IMLM and DMM$_\nu$ methods show significant improvement over the MLM method in the deterministic and random simulation tests of the $S_{yx}(f)$ estimates. The response of the 1-2-4-5 array to random cross-spectra showed no instabilities for either the IMLM or DMM$_\nu$ estimates in the frequency range 0.051–0.160 Hz.

The field comparisons of a 1-2 linear array of length 99.0 m with a slope array show roughly equivalent comparisons of the IMLM and DMM$_\nu$ estimates with the slope-array results. Estimates from either method deviate from the slope-array estimates by about 10–20% compared to values up to 100% for the MLM comparisons. The DMM$_\nu$ showed the clos-

TABLE 3. Quantitative comparisons of various linear-array estimates of $S_{yx}(f)$ with current-meter (CM) results for 6 December 1978. The statistics were calculated from 12 consecutive 2048 s runs. All quantities are then averaged over the frequency range 0.074–0.106 Hz.

	CM	IMLM	DMM$_o$	DMM$_{0.1}$	DMM$_{0.5}$	DMM$_{0.8}$	DMM$_1$
Mean (10^2 cm^2 s^{-1})	11.1	11.5	3.14	9.02	10.2	10.1	9.95
Variance (10^4 cm^4 s^{-2})	35.2	43.4	232.	37.8	29.4	27.7	26.3

est and most stable comparisons with the slope array with low values (relative to band energy) of $S_{yx}(f)$. The useful range of the 1–2 array was shown to be approximately 0.051–0.153 Hz.

Simulation tests showed degraded performance of the array methods with the 2–2–2–5 array configuration, particularly at higher wave frequencies. The IMLM estimates of $S_{yx}(f)$ compared well with the current-meter results in the range 0.059–0.129 Hz. The DMM$_\nu$ (ν_n = 0.5–1.0) estimates compared well with the other methods in the range 0.059–0.106 Hz. The severe degradation of the DMM$_\nu$ estimates above 0.106 Hz was demonstrated in the simulation test and shown to be independent of ν_n.

The variance of the DMM$_\nu$ estimates drops with increased values of ν_n. However, the value of ν_n which minimized the bias of the DMM$_\nu$ estimates relative to the results from the other methods varied from 0.1 to 1.0. The simulation tests and field comparisons show the minimum bias ν_n to vary as a function of array configuration, directional spectrum and wave frequency. However, values of ν_n from 0.1 to 1.0 yield reasonable bias and estimator variance for most of the frequency range and both arrays. Some attention will have to be paid to this parameter in the application of the DMM$_\nu$ method with other array configurations and directional spectral forms.

These experiments also represent the first attempts at field verification of the $S_{yx}(f)$ measurement capability of the slope array and biaxial current meter. The comparative results yield confirmation of accurate performance of these systems over the entire frequency range of usable data provided by the linear array. However, the orthogonal-component systems measure only limited low-order directional moments. In general, more detailed directional-spectrum information is desirable. Linear arrays can be designed for much higher resolution of the directional spectrum. We have shown here that if the required wave information can be stated in terms of specific directional moments, these data may be accurately extracted from linear arrays.

Acknowledgments. This study was funded by the Office of Naval Research, Code 422CS (Coastal Sciences), under Contract N00014-75-C-0300. The November 1978 data collection was sponsored by Sea Grant project R/CZ-N-40-1. The staff of the Shore Processes Laboratory supervised by R. Lowe installed and maintained the linear arrays and current meter. Richard Seymour designed the slope array and supervised its operation and associated data analysis. Russ Davis provided valuable ideas and critical review. Michael Clark drafted the figures and Joan Semler typed the manuscript. A special note of appreciation is given to Richard Anderson, M.D. and Mark Green, M.D. who helped the first author to complete this manuscript.

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