On the Accuracy of Heat Storage Computations[1]

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ABSTRACT

Routinely taken oceanographic data from 55 temperature sections across the North Pacific Current along 158°W between Hawaii and Alaska are used to determine the accuracy of heat storage computations. Errors caused by the use of different instruments and their calibration are as large as those caused by environmental variability on short time and space scales and amount to $\sim 18 \times 10^7$ J m^{-2}, which is about 15% of the mean annual cycle in observed heat storage. Anomalies of heat storage and month-to-month changes of heat storage cannot be determined with any confidence, but over a complete heating or cooling season the observed changes in heat storage are systematically larger than the heat gain or loss during that season by about 50%, indicating a regular contribution by horizontal advection.

1. Introduction

Many investigators have attempted to relate changes in the observed heat storage in the ocean to the heat input at the sea surface in order to extract information about the importance of other terms in the heat budget equation such as advection and mixing (Pattullo, 1957; Bathen, 1971; Gill and Niiler, 1972; Emery, 1976; Barnett, 1981). Little attention is usually given to estimation of the errors involved in such a procedure and assessment of how they affect the final result. In this study we use temperature data from the North Pacific to estimate heat storage, its variability, and errors in its observation in order to establish the accuracy with which this important quantity can be determined. Knowledge of heat storage and its anomalies is particularly important for ocean monitoring and for climate-related studies.

A total of 55 temperature sections across the North Pacific Current between Hawaii and Alaska along 158°W could be identified between 1971 and 1977, a period when such sections were most abundant. Twenty-four of them were air XBT sections (Barnett *et al.*, 1976); the other XBT sections were taken by ships. Nine of the sections had both XBT and hydrographic station observations. All XBT and hydrographic sections were obtained from Fleet Numerical Oceanography Center at Monterey or from the National Oceanographic Data Center.

2. Heat storage

Heat storage along each section was calculated between 30 and 50°N and from the surface to a depth of 100 m. Comprehensive data are available for all sections in this interval, whereas data are often missing south and north of these limits. The lower boundary of 100 m was chosen because Barnett (1981) and Kang (1980) found that the seasonal signal is contained above 100 m and that it is indistinguishable from noise below 100 m. This does not mean that heat storage anomalies below 100 m may not occur, but they are usually of frequencies different from the annual cycle (Emery, 1976). Because of the large horizontal scales of observed sea surface temperature anomalies and heat exchange anomalies, the heat storage along such sections should be a useful parameter in ocean monitoring.

Heat storage was computed by integrating each temperature profile from the surface to 100 m and then integrating along the section from 30 to 50°N using linear interpolation. The result is expressed as an average value of heat storage along the section. The numerical values are listed in Table 1 and shown in Fig. 1. Heat storage ranges from 426×10^7 J m^{-2} in March 1977 to 635×10^7 J m^{-2} in September and October 1972. It follows a strong annual cycle, which has been computed using first and second harmonics (Fig. 2). The standard deviation of heat storage from the mean annual cycle is 17.5×10^7 J m^{-2}; the amplitude of the first harmonic is 82×10^7 J m^{-2} and the second harmonic is 12×10^7 J m^{-2}. Consequently, the variability of heat storage as expressed by the root-

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TABLE 1. Heat storage between the sea surface and 100 m and between 30 and 50°N along 158°W giving the date, heat storage, the type of observations and the number of observations (N) along each section. A for Air XBT, B for XBT and C for hydrographic station.

	Date	Heat storage (10^7 J m^{-2})	Type	N
1	15 Apr 77	430	A	25
2	17 Mar 77	426	A	25
3	17 Feb 77	430	A	25
4	2 Jan 77	471	A	25
5	9 Dec 76	516	A	25
6	20 Oct 76	577	A	25
7	24 Sep 76	606	B	59
8	19 Sep 76	598	A	25
9	22 Aug 76	602	A	25
10	18 Aug 76	565	A	25
11	9 Jul 76	549	A	25
12	17 Jun 76	499	A	25
13	2 May 76	454	A	25
14	2 Feb 76	459	C	19
15	10 Feb 76	475	A	25
16	11 Jan 76	471	A	25
17	18 Dec 75	536	A	25
18	17 Nov 75	561	A	25
19	11 Oct 75	610	A	25
20	26 Sep 75	594	C	31
21	12 Sep 75	626	A	25
22	25 Aug 75	614	A	25
23	4 May 75	483	A	25
24	17 Mar 75	450	A	25
25	27 Feb 75	450	C	11
26	25 Feb 75	467	B	16
27	10 Feb 75	459	A	25
28	12 Jan 75	491	A	25
29	8 Dec 74	508	A	25
30	11 Nov 74	545	C	11
31	11 Nov 74	557	B	65
32	9 Nov 74	549	A	25
33	22 Aug 74	581	B	18
34	2 Aug 74	557	C	15
35	2 Aug 74	581	B	62
36	20 Apr 74	446	C	12
37	1 Nov 73	577	B	95
38	4 Sep 73	590	B	72
39	28 Jul 73	536	B	17
40	15 Jul 73	532	C	15
41	15 Jul 73	540	B	63
42	30 Apr 73	463	B	15
43	30 Jan 73	442	B	19
44	28 Dec 72	483	B	20
45	2 Dec 72	573	B	18
46	30 Oct 72	635	B	72
47	2 Oct 72	622	B	19
48	2 Oct 72	622	C	17
49	8 Sep 72	635	B	18
50	30 Aug 72	614	B	15
51	10 Jun 72	520	B	19
52	8 Apr 72	467	B	18
53	26 Nov 71	573	B	18
54	1 Oct 71	622	B	20
55	10 May 71	475	B	20

mean-square deviation of heat storage from its mean annual cycle is about 11% of the mean annual signal. This variability includes measurement errors as well as natural variability on time scales different from the annual signal.

3. Errors in heat storage determination

The uncertainty in the computation of heat storage will be assessed by evaluating measurement errors and sampling errors and indirectly by comparing nearly simultaneous sections.

Uncertainties in the temperature and depth measurements from XBT data are taken to be ±0.1°C and ±3 m, whereas the uncertainties in the AXBT measurements are taken to be ±0.3°C and ±3 m (Anderson, 1980; Barnett *et al.*, 1976; Barnett, 1981; Seaver and Kuleshov, 1979; Sessions *et al.*, 1976). The error δH in the calculated heat storage for each temperature profile is $\delta H = \rho c_p(D\delta T + \Delta T\delta D)$, where D is the depth (100 m), δT the error in temperature measurement, δD the error in depth measurement, and ΔT the temperature difference between surface and depth D; ρ is density and c_p the specific heat (Wyrtki, 1980). It is not unrealistic to assume errors of the same sign. They are usually calibration errors of the XBT, and usually the same recorder is used during one section. Different batches of XBT probes have also shown different characteristics.

Errors were determined for each temperature profile and integrated along the section. Because of the greater temperature difference between the sea surface and 100 m, the errors were larger in summer (17×10^7 J m^{-2} in August) than in winter (10×10^7 J m^{-2} in February) with a mean value of 13×10^7 J m^{-2}. It should be noted that this procedure maximizes the error, as it can be assumed that at least some of the errors will be distributed randomly and consequently will cancel each other along the section.

The errors due to variations in density ρ and specific heat c_p, which were taken constant for the determination of heat content, were also determined. Based on the observed variability of temperature and salinity along the section, the maximum possible error is 1.0×10^7 J m^{-2}, which is less than 1% of the mean annual variation of heat content and can be considered negligible compared with the other sources of error.

The spacing of temperature profiles along the section can also be a source of error. On the average there were 26 profiles per section, but some had as few as 11, others as many as 95. To estimate the error due to this discrepancy several very dense sections were split into three less dense sections, consisting of every third profile of the original section. The heat storage computed for the three decimated sections was then compared with that of the original section. This procedure was applied to four sections each having more than 59 temperature profiles. The maximum difference between a decimated section and the original was 3×10^7 J m^{-2}; and the standard deviation

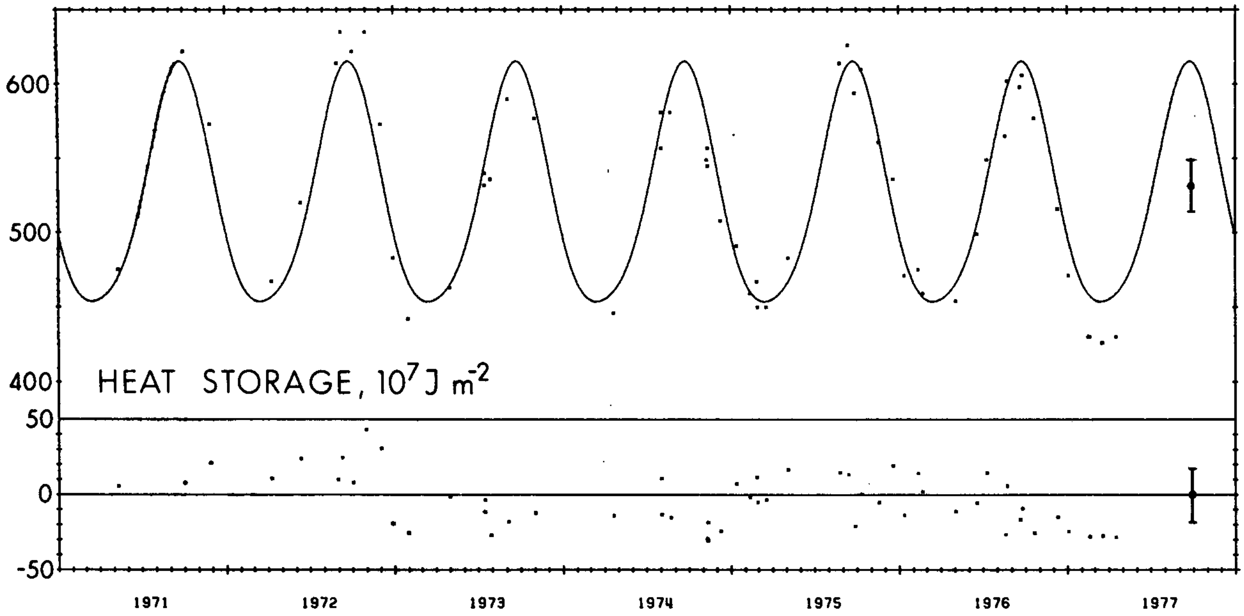

FIG. 1. Heat storage (10^7 J m^{-2}) derived from temperature sections in the North Pacific between 30 and 50°N from 1971 to 1977. The dots represent individual sections; the curve is the mean annual signal; the lower panel gives the anomalies of heat storage and their standard deviation.

was 1.6×10^7 J m^{-2}. This error represents the uncertainty in the determination of heat content introduced by the arbitrary choice of station location and the influence of small-scale features in the thermal structure. Because observations along these high-density sections were probably made with the same instrument, systematic errors are most likely not involved. Uncertainties caused by station spacing are small compared to instrumental errors, and consequently a station density of 20–25 profiles or one station per degree of latitude is sufficient to reduce this error to about 1% of the annual variation in heat storage.

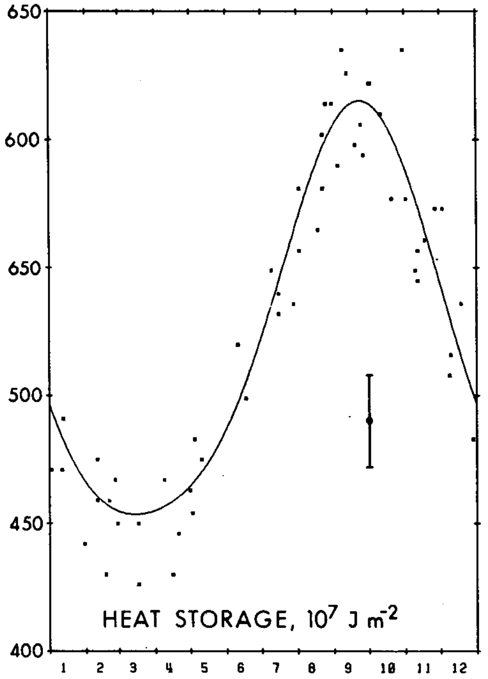

FIG. 2. Mean annual variation of heat storage (10^7 J m^{-2}) in the North Pacific between 30 and 50°N (curve) and individual values for each section. The standard deviation of individual values from the mean annual cycle (17.5×10 J m^{-2}) is also shown.

To establish the temporal variability we have compared all sections taken within less than 18 days of each other. The average time difference between sections was 10 days. The standard deviation was 18×10^7 J m^{-2}. The differences in heat content between these pairs of sections could not be accounted for by the mean annual signal. Instead they represent the combined errors introduced by different instruments and by the temporal variability of the thermal structure.

On five occasions both hydrographic stations and XBT drops were made along the same section, but the number of hydrographic stations was only between 11 and 17, whereas the number of XBT drops was between 16 and 65. Heat content derived from the XBT sections was systematically higher by 12×10^7 J m^{-2}, and the standard deviation of the two sets of data was 9×10^7 J m^{-2}. This indicates that the use of different instrumentation is a major contributing factor to the uncertainty of heat storage computations.

In summary, we can state that the uncertainty of the determination of heat storage is chiefly caused by instrumental errors and by environmental variability on time scales of the order of weeks and less. Instrumental errors result largely from the use of different instruments and their calibration. The largest contribution to the error comes probably from the uncertainty of depth determination. The instrumental error that must be attributed to routinely collected data is between 10 and 15 × 10^7 J m^{-2}. The uncertainty due to environmental variability is even larger, about 18 × 10^7 J m^{-2}, and is apparently caused by changes in the thermal structure along the section on time scales of one to two weeks, for which comparisons were made. On the other hand, station spacing of 1° of latitude is adequate to sample heat storage with a precision better than 2 × 10^7 J m^{-2}. The standard deviation of heat storage from the mean annual cycle was previously determined as 17.5 × 10^7 J m^{-2}; consequently we can state that most of the calculated heat storage values are not significantly different from the mean annual cycle, as is obvious from Figs. 1 and 2. Only the very largest heat storage anomalies, such as those in the winter of 1977 and in the fall of 1972, are significant.

Anomalies of heat storage have also been computed as the difference between the observed heat storage and its mean annual cycle (Fig. 1). Although the scatter of these anomalies is large, an obvious low-frequency signal exists. Anomalies are positive in 1971 and 1972, consistently negative in 1973 and 1974, near zero in 1975 and early 1976, and again negative after the summer of 1976. The amplitude of these anomalies is about the same as the error of individual heat storage determinations, but their persistence in time is an indication that they are real.

4. Surface heat exchange

Values of monthly mean surface heat exchange were obtained from the NORPAX data bank; they are a continuation of data published by Clark *et al.* (1974). The data are means for areas of 5° of latitude and longitude and eight such squares were averaged to give the mean monthly heat input into the area bounded by 30°N, 50°N, 160°W and 170°W, which is situated just upstream of the sections studied. As the mean geostrophic flow across the temperature sections is from west to east with the North Pacific Current, variations in the heat storage observed along the section should be related to the heat input immediately to the west of it. Moreover, east-west gradients of heat input are small in the central North Pacific (Clark *et al.*, 1974).

Heat input exhibits an annual signal as regular as that of heat storage (Fig. 3). The standard deviation of heat input from the mean annual cycle, which was computed by using first and second harmonics is 21 W m^{-2}, only 10% of the mean annual signal. Thus the relative variability of heat input is about the same as that of heat storage. Hastenrath (1980) estimates the error of heat budget computations as 7 W m^{-2} for long term means, but the error for individual monthly means may be twice as large. Hastenrath (1980, p. 169) estimates the "total uncertainty" of heat input to be of the order 20–30 W m^{-2}.

Anomalies of heat exchange have also been computed and are shown in Fig. 3. They do not show a systematic low-frequency signal, and it is obvious from Fig. 3 that most of the fluctuations are of high

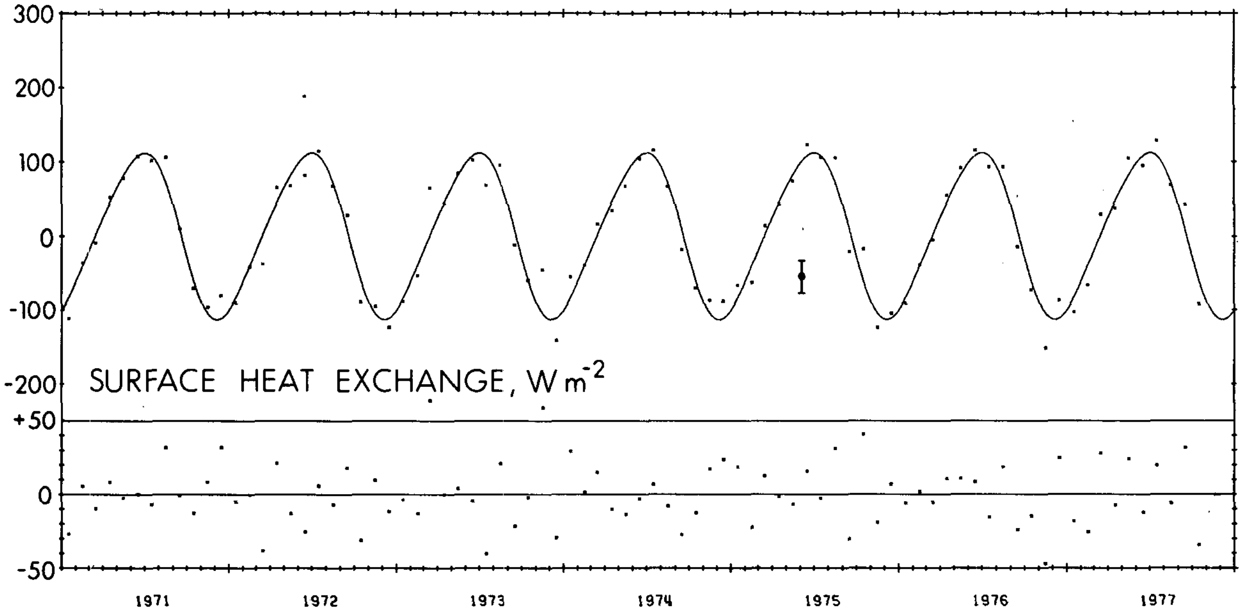

FIG. 3. Surface heat exchange (W m^{-2}) for the area between 30 and 50°N and 160 and 170°W. The dots are monthly means, the curve is the mean annual cycle. Anomalies and their standard deviation are shown in the lower panel.

frequency as indicated by a change of sign of the anomalies from month to month. Consequently, the low-frequency signal of the heat storage anomalies cannot be caused by consistently anomalous heat exchange.

During periods of maximum heating and cooling in summer and winter, heat storage changes by about 40×10^7 J m^{-2} in one month. Because its value can only be established to about $\pm 18 \times 10^7$ m^{-2} by monthly sampling along a section, it is not meaningful to compare the observed month-to-month changes of heat storage with the heat input determined from synoptic meteorological data. Consequently, from such a comparison no meaningful conclusions can be drawn about the contribution of horizontal and vertical advection to the changes in heat storage on the monthly time scale.

On the other hand, it should be possible to relate the total heat input during an entire heating and cooling season to the observed change in the heat storage. Over such a time frame the uncertainties in the determination of heat storage amount to only about 15% of the observed difference.

5. Comparing heat storage and heat input

Heat storage H and heat exchange Q are related through the equation $\partial H/\partial t = Q + R$, where the residual R is the combined contribution of horizontal and vertical advection and diffusion. From the mean annual cycle of heat storage the mean annual cycle of the rate of change of heat storage can be computed; it varies from 167 W m^{-2} in July to -196 W m^{-2} in December (Fig. 4) and has a shape similar to that of heat input, but a larger amplitude. The error for the mean annual cycle of H is 7.3×10^7 J m^{-2}, the error for $\partial H/\partial t$ is 36 W m^{-2}, and the error of heat input is about 7 W m^{-2}. The residual is positive in summer, indicating a heat gain in the North Pacific between 30 and 50°N, which is most likely due to meridional advection of warmer water. During the winter the residual is negative, representing an additional heat loss, which is probably the result of advection of cooler water from the north. This analysis confirms the earlier findings of Wyrtki and Haberland (1968) and of Bathen (1971) that horizontal advection is a major contributing factor to the annual signal of heat storage and that the observed changes in heat storage are larger than can be accounted for by local heat exchange.

In view of the large error in determining the heat storage ($\sim 18 \times 10^7$ J m^{-2}), the change of heat storage from month to month can only be established with an uncertainty of $\pm 18\sqrt{2} \times 10^7$ J m^{-2}. Over a period of one month $\partial H/\partial t$ is therefore known to only ± 100 W m^{-2}, which is as large as the heat exchange during the peak of the heating or cooling season. Consequently it is impossible to estimate the contributions of advection or mixing with any accuracy over a monthly interval. Over one complete heating or cooling season, however, such an estimate may become feasible.

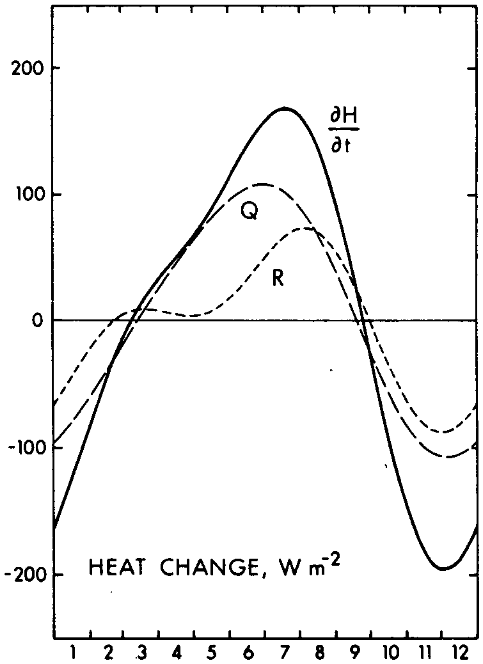

FIG. 4. Mean annual cycles (W m^{-2}) of the change of heat storage $\partial H/\partial t$, of surface heat exchange Q, and of the residual R.

We have computed the change in heat content ΔH over a complete heating or cooling season as defined by the maximum and minimum of heat storage observed at the end of each such season (Table 2). For the same time interval we also determined the total heat input or loss ΣQ by surface heat exchange. The two sets of values correlate extremely well ($r = 0.99$), but their magnitude differs considerably. Whereas the mean absolute rate of change of heat storage is 155×10^7 J m^{-2}, the mean absolute heat exchange is only 97×10^7 J m^{-2}, and cannot account for the observed changes in heat storage. Advection will chiefly account for the difference of about 57×10^7 J m^{-2}. This calculation implies that about one-third of the heat storage change in the area considered is due to seasonal advection, horizontal or vertical.

During the seven years of observations, the heating season is between 124 and 179 days long, and the cooling season is substantially longer, between 190 and 233 days (Table 2). This is due to the sparsity of observations defining the maxima or minima of heat storage in each year. In each season the residual has the same sign as the change in heat content and as the total heat exchange, and consequently the warm or cold advection must be very systematic in each year and not only in the average annual signals. The accuracy of ΔH over one season is about $\pm 18\sqrt{2}$

TABLE 2. Changes of heat content over complete heating and cooling seasons. H is heat content, ΔH the change of heat content, ΣQ the total heat exchange and R the residual—all in 10^7 J m^{-2}.

Date	Duration (days)	H	ΔH	ΣQ	R
10 May 1971		475			
	144		+147	+92	+55
1 Oct 1971		622			
	190		−155	−105	−50
8 Apr 1972		467			
	153		+168	+103	+65
8 Sep 1972		635			
	234		−172	−84	−88
30 Apr 1973		463			
	127		+127	+93	+34
4 Sep 1973		590			
	228		−144	−83	−61
20 Apr 1974		446			
	124		+135	+91	+44
22 Aug 1974		581			
	207		−131	−84	−47
17 Mar 1975		450			
	179		+176	+123	+53
12 Sep 1975		626			
	233		−172	−88	−84
2 May 1976		454			
	145		+152	+100	+52
24 Sep 1976		606			
	174		−180	+123	−57
17 Mar 1977		426			

$\times 10^7$ J m^{-2} or 25×10^7 J m^{-2} and the accuracy of ΣQ is about 10×10^7 J m^{-2}; therefore, the possible error in the residual R is $\pm 35 \times 10^7$ J m^{-2}, which is more than half of its actual value. Thus, from the kind of observations used here, it will not be possible to determine whether or not heat advection is anomalous in a particular season.

6. Conclusions

We have examined the accuracy with which heat storage can be determined from routine oceanographic temperature sections for the purpose of ocean monitoring and for heat budget estimates.

1) Errors in the determination of heat storage are large and amount to more than 15% of its mean annual signal in the area studied. These errors are caused by the use of different instruments and their calibration ($\sim 15 \times 10^7$ J m^{-2}) and by the environmental variability on short time and space scales ($\sim 18 \times 10^7$ J m^{-2}). In contrast, the density of observations contributes little to the error as long as at least one temperature profile is available per degree of latitude.

2) Anomalies of heat storage cannot be determined from individual sections with any confidence, but time series of the anomalies indicate that heat storage undergoes systematic long-term changes that can be detected by frequent sampling in spite of the large errors of the individual determinations.

3) The errors in determining heat storage are large enough to make it impossible to estimate advective or diffusive contributions to the heat budget over a monthly interval.

4) Over one complete heating or cooling season, such estimates can be made with some confidence.

5) The errors in determining heat storage are most severe when data obtained by different instrumentation are used.

We feel compelled to add a remark by one of the reviewers: "This paper can teach a lesson to optimists that hope to assess the interannual variability of the oceanic heat budget. On a positive note, the evidence is encouraging concerning multi-annual mean conditions and the average annual cycle. Perhaps the authors may wish to underline this aspect."

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