# BOHR'S SEMICLASSICAL MODEL OF
# THE BLACK HOLE THERMODYNAMICS

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SUMMARY: We propose a simple procedure for evaluating the main attributes of a Schwarzschild's black hole: Bekenstein-Hawking entropy, Hawking temperature and Bekenstein's quantization of the surface area. We make use of the condition that the circumference of a great circle on the black hole horizon contains finite and whole number of the corresponding reduced Compton's wavelength. It is essentially analogous to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie's relation. It implies the standard meaning of the black hole entropy corresponding to surface of the quantum variation of the great circles on the black hole horizon surface area. We present black hole radiation in the form conceptually analogous to Bohr's postulate on the photon emission by discrete quantum jump of the electron within the Old quantum theory. This enables us, in accordance with Heisenberg's uncertainty relation and Bohr's correspondence principle, to make a rough estimate of the time interval for black hole evaporation, which turns out very close to time interval predicted by the standard Hawking's theory. Our calculations confirm Bekenstein's semiclassical result for the energy quantization, in variance with Frasca's (2005) calculations. Finally we speculate about the possible source-energy distribution within the black hole horizon.

Key words. Black hole physics

## 1. INTRODUCTION

Quantum theory, both the Old one and Quantum Mechanics, was designed to deal with microscopic phenomena, irrespective of the kind of the interaction involved. In practice, Bohr's, Heisenberg's and Schrödinger's theories deal almost exclusively with Coulombic interaction, as dominant at the atomic level. Strong and weak forces are restricted to nuclear and subnuclear levels and require a specific approach, outside the ordinary non-relativistic quantum theory, partly because these interactions are difficult to describe by the potential functions. The fourth fundamental force, gravitation, has been left out, for a number of reasons. Firstly, it appears so weak in comparison with the other three ones mentioned, that at the microscopic level can be easily ignored. Second, it is the theory of gravitation, Newtonian, Einsteinian or else which is considered relevant to study gravitating bodies and celestial phenomena in general.

### 1.1. Newtonian and Coulombian systems

Though reigning at very different scales of the physical world, Newtonian and Coulombian forces have a common formal structure which makes them

attribute to the corresponding physical systems' many common characteristics (see, e.g. Grujić 1993). These common features were revealed in a particularly remarkable way when the Quantum Mechanics was formulated and a parallel with some General Relativity phenomena established.

The first modern hydrogen atom model was contrived by Thomson as the negatively charged electron immersed in a spherical positively charged fluid. This model was to be radically changed with the later Rutherford-Bohr model, but both had one remarkable feature in common: the path an electron traced while moving inside the fluid, or around the nucleus, was the same geometrical figure, an ellipse, despite the fact that electron experiences radically different forces. In the Thomson's model, the potential function is of the form

$$V(r) = kr^2 \qquad (1)$$

characteristic for the harmonic oscillator, whereas for the Coulombian interaction one has

$$V(r) = \beta/r \qquad (2)$$

The difference was the positions of their foci. In the first case they were placed symmetrically with respect to the centre of the sphere, whereas in the motion around point-like nucleus the latter was positioned at one of the two foci. But the most remarkable similarity was revealed when comparing the semiclassical and quantum mechanical solutions of the corresponding energy spectra. It turns out that in both cases semiclassical and quantum mechanical results coincide, for all principal quantum numbers (the so-called *correspondence identities*) (see, e.g. Norcliffe 1975). Hence, the two most important interactions, harmonic and Coulombian, allow for the semiclassical and quantum mechanical treatments indiscriminately. At the same time, it is for these interactions that any single-particle trajectory is closed, irrespective of the initial conditions. Not accidentally, these (textbook) interactions appear the only ones that allow for exact analytical solutions, both classical and quantum mechanical.

The energy spectra, evaluated in either of the theories, appear however distinct. For the harmonic oscillator of Eq. (1) it reads

$$E_n = \hbar\omega(n + 3/2), n = 0, 1, 2, ... \qquad (3)$$

whereas the Coulombic case gives

$$E_n = \beta^2/2n^2, n = 1, 2, ... \qquad (4)$$

The first formula provides an equidistant distribution, whereas the Bohr's formula is the typical case for a series of discrete levels accumulating towards zero. It is this distinction that makes the investigation of the black hole spectrum of particularly interesting, as we shall see below.

## 1.2 Black hole

It was only with the appearance of the concept of gravitational collapse and the model of black hole, that the gravitational force becomes dominant and even exclusively present (see, e.g. Bekenstein 1994). In view of the formal similarity of the asymptotic behaviour of Newtonian and Coulombic forces, one may expect that the properties of atomic systems with charged constituents and black-hole like gravitational objects should share a number of common features. As noted by Bekenstein (1998), black hole is a hydrogen atom in the field of strong gravity regime. In particular, quantum effects may be present on the black hole surface and one may expect that some quantization rules are valid.

One of the essential ingredients of the statistical mechanics was the observation that the number of degrees of freedom of a quantum system should be proportional to the surface of the system, rather than to the volume. In fact, it was this assumption which led to the Bekenstein's linking of the black hole entropy and the area of its horizon.

Semiclassical quantization of the black hole has been attempted by various authors. In a recent work, Frasca (2005) calculated the semiclassical energy spectrum of the Schwarzschild black hole making use of the Hamilton-Jacobi formalism. For the stable circular orbits he derived the formula

$$E_n \approx M - \frac{2G^2M^5}{n^2\hbar^2} \qquad (5)$$

which is, up to an additive constant M, Bohr's formula for the Coulombian interaction. In addition, the partition function turned out to coincide with that derived by the loop quantum gravity formalism (see, e.g. Nicolai et al. 2005).

## 1.3 Black hole characteristics

Thermodynamical characteristics of a black hole is one of the most important subjects of contemporary physics (see, e.g. very recent paper by Samuel and Chowdhury 2007). In a sense, it plays the role of the black body studies around the turn of $19^{th}$ century, linking the thermodynamics and statistics, more precisely the gravitation and information theories. First, Bekenstein (1973) suggested that a black hole contains the entropy $S_{BH}$ proportional to the horizon surface area, $A$. For the Schwarzschild's black hole one has:

$$S_{BH} = \frac{k_B c^3}{4\hbar} A \qquad (6)$$

where $k_B$ is Boltzmann's constant, $c$ - speed of light and $\hbar$ - reduced Planck constant. Also, Bekenstein suggested that the horizon surface area is quantized, and can be changed only discretely

$$\Delta A = n8\frac{G\hbar}{c^3} \equiv n8L_P^2, n = 1, 2, .. \qquad (7)$$

where $L_P = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}}$ is the Planck length. Bekenstein's analysis is, on the one hand, based on the characteristics of corresponding, complex quantum measurement procedures, i.e. Heisenberg's uncertainty relations and Ehrenfest's adiabatic theorem.

On the other hand, it relies on general relativistic and quantum field theoretical requirement on the stability of the capture of a quantum system within black hole. According to this requirement, roughly speaking, Compton's wavelength of a given quantum system must be smaller than double Schwarzschild's radius. (Otherwise, a quantum system can escape from the black hole by means of quantum tunneling.)

After Bekenstein, Hawking (1975, 1976) showed that black hole can be considered as a black body which radiates at the temperature

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8 \pi k_{\mathrm{B}} G M} \qquad (8)$$

where $M$ is the black hole mass. This Hawking's temperature, according to usual rules of the thermodynamics, appears compatible to Bekenstein-Hawking entropy (6). Roughly speaking, Hawking's analysis is physically based on the non-invariance of the quantum field dynamics according to general transformations of coordinates, which implies that a wave can be considered as a complex mixture of the plane waves (this mixture can be effectively treated as the spectrum of the black body). Simplified, black hole can gravitationally interact with fluctuated quantum vacuum near horizon. Then black hole can absorb one member of particle-antiparticle virtual pair, while the other member of the pair can be effectively considered as the radiation. Mathematically, Hawking's analysis is based on a complex formalism of the quantum fields in the curved space (in a quasi-classical approximation). Later it has been proved, by Hawking (1979) and others, that Hawking's results can be reproduced even by more complex formalism, i.e. the quantum field dynamics, without quasi-classical approximations (see, for example, review articles by Wald (1997, 1999), and Page (2004), and references therein).

Hawking also predicted time of the black hole evaporation. Namely, Stefan-Boltzmann law applied at the black hole surface area, according to Hawking temperature (8) and relativistic equivalence relation $E = Mc^2$, has the form

$$
\begin{aligned}
-dE/dt &= -c^2 dM/dt = \sigma_{SB} T_{\mathrm{H}}^4 A = \\
&= \frac{hc^6}{15360 \pi G^2 M^2},
\end{aligned}
\qquad (9)
$$

where $\sigma_{\mathrm{SB}}$ is the Stefan-Boltzmann constant. It yields, after simple integration, the following expression for black hole evaporation time

$$\tau_{\mathrm{ev}} = 5120 \frac{\pi G^2}{\hbar c^4} M_0^3 \qquad (10)$$

where $M_0$ denotes the black hole initial mass.

Detailed analysis of the quantum and thermodynamical characteristics of a black hole requires a very complex (in this moment incomplete) theoretical formalism including application of different string theories (Strominger and Vafa 1996, Proline

2006). Nevertheless, there are many attempts of analysing the quantum and thermodynamical characteristics of a black hole by means of the relatively simple (approximate) theoretical concepts. For example, in Ram (2000), Ram et al. (2005) it is shown that a black hole can be consistently considered as a Bose-Einstein condensate, while in Nagatani (2007) a conceptual analogy between the so-called minimum black hole and Bohr's model of the hydrogen atom is considered. Even in these cases, mathematical formalism is based on various differential (e.g. Schrödinger's) equations solved within certain (e.g. mean field) approximations.

## 2. THEORY

In this work we shall determine, in a simple way, the three most important, thermodynamical characteristics of a Schwarzschild's black hole: Bekenstein-Hawking's entropy, Hawking's temperature and Bekenstein's quantization of the surface area. We shall use an original, simple and intuitively transparent (quasi-classical) condition. We demand that circumference of a great circle at the black hole horizon contains finite (statistically averaged) number of corresponding reduced Compton's wavelength. It is essentially analogous to Bohr's quantization postulate in his Old quantum theory, interpreted by de Broglie's ontology, according to which circumference of an electron circular orbit comprises an integer number of corresponding de Broglie's wavelengths. It implies the simple usual meaning of the black hole entropy as corresponding to the surface of the quantum variation of the great circles at black hole horizon surface area. Finally, we express the black hole radiation in the form conceptually analogous to Bohr's postulate on the photon emission by discrete quantum jump of the electron in his atomic model. It, in accordance with Heisenberg's energy-time uncertainty relation and a correspondence principle conceptually analogous to Bohr's one, allows a rough estimate of the time interval for black hole evaporation. This time interval is very close to the time interval of the black hole evaporation obtained via Hawking's radiation.

Thus, in this work we shall make the most simplified but non-trivial description of the quantum and thermodynamical characteristics of a Schwarzschild's black hole, which we simply call Bohr's black hole.

### 2.1 Bohr's black hole

Making use of de Broglie's relation

$$\lambda = \frac{h}{mv} \qquad (11)$$

and Bohr's quantization postulate

$$m v_n r_n = n \frac{h}{2\pi}, n = 1, 2, ... \qquad (12)$$

it follows

$$2 \pi r_n = n \lambda_n, n = 1, 2, ... \qquad (13)$$

where $\lambda_n$ represents the electron $n$-th de Broglie's wavelength, $m$ - electron mass, $v_n$ - $n$-th electron speed, $r_n$ - radius of the $n$-th electron circular orbit and $h$ - Planck constant. Expression (13) simply means that circumference of electron $n$-th circular orbit contains exactly $n$ corresponding $n$-th de Broglie's wavelengths, for $n = 1, 2, ...$

We shall now apply similar analysis to a Schwarzschild's black hole with mass $M$ and Schwarzschild's radius

$$R_S = \frac{2GM}{c^2}. \qquad (14)$$

We write down the following expression analogous to (12)

$$m_n c R_S = n \frac{\hbar}{2\pi}, n = 1, 2, ... \qquad (15)$$

that implies

$$2\pi R_S = n \frac{\hbar}{m_n c}, n = 1, 2, ... \qquad (16)$$

analogous to (9). Here $2\pi R_S$ represents the circumference of the black hole while

$$\lambda_{rn} = \frac{\hbar}{m_n c}, n = 1, 2, ... \qquad (17)$$

is $n$-th reduced Compton's wavelength of a quantum system captured at the black hole horizon surface expression (16) simply means that *circumference of the black hole horizon holds exactly n reduced Compton's wavelengths of a quantum system captured at the black hole horizon surface*. Obviously, it is essentially analogous to above mentioned Bohr's quantization postulate interpreted via de Broglie's relation. However, there is a principal difference with respect to Bohr's atomic model: in Bohr's atomic model different quantum numbers $n = 1, 2, ...$, correspond to different circular orbits (with circumferences proportional to $n^2$): here, any quantum number $n = 1, 2, ...$ corresponds to the same circular orbit (with the circumference $2\pi R_S$).

According to (16), it follows

$$m_n = n \frac{\hbar}{2\pi R_S} = n \frac{\hbar c}{4\pi GM} \equiv nm_1, \ n = 1, 2, ... \ , \qquad (18)$$

where

$$m_1 = \frac{\hbar c}{4\pi GM} = \frac{M_P^2}{4\pi M} \ , \qquad (19)$$

and $M_P = (\hbar c/G)^{1/2}$ is the Planck mass. Obviously, $m_1$ depends on $M$ so that $m_1$ decreases when $M$ increases and vice versa. For a macroscopic black hole, i.e. for $M \gg M_P$ it follows $m_1 \ll M_P$.

Suppose now that the black hole mass equals

$$M = \sigma m_1 = \frac{\sigma \hbar}{4\pi c R_S} = \frac{\sigma \hbar c}{4\pi GM} \qquad (20)$$

where $\sigma$ denotes some integer (or approximately integer) number. According to (14,18) it follows

$$\sigma = \frac{M}{m_1} = \frac{4\pi GM^2}{\hbar c}. \qquad (21)$$

It means that the number $\sigma$, for a fixed black hole mass $M$, is finite.

After multiplying (21) by Boltzmann constant $k_B$ we have

$$k_B \sigma = \frac{4\pi k_B GM^2}{\hbar c} \qquad (22)$$

Obviously, right-hand side of (22) represents Bekenstein-Hawking's entropy of the Schwarzschild's black hole (6). It is, therefore, reasonable to assume

$$S_{BH} = k_B \sigma = \frac{4\pi k_B GM^2}{\hbar c} \ . \qquad (23)$$

This assumption implies that $\sigma$ must have a statistically appropriate form to be considered later on.

Differentiation of (23) yields

$$dS_{BH} = k_B d\sigma = 8\pi k_B GM/(\hbar c^3) dE \qquad (24)$$

where

$$E = Mc^2 \qquad (25)$$

is the black hole energy. According to second law of thermodynamics, expression (24) implies that term

$$T = \hbar c^3/8\pi k_B GM = m_1 c^2/(2k_B) \qquad (26)$$

represents the black hole temperature. Evidently, this temperature is identical with the Hawking's black hole temperature (8). According to (23), (24) it follows

$$dA = \frac{32\pi G^2}{c^3} M dM \qquad (27)$$

or, in a corresponding finite difference form,

$$\Delta A = \frac{32\pi G^2}{c^3} M \Delta M, \Delta M \ll M. \qquad (28)$$

Further, we assume

$$\begin{aligned} \Delta M &= m_n - m_k = (n - k)m_1 = \\ &= \frac{\hbar c}{4\pi GM}, n, k < n = 1, 2, ... \end{aligned} \qquad (29)$$

which, after substitution in (28), yields

$$\begin{aligned} \Delta A_{nk} &= (n - k)8\frac{G\hbar}{c^3} = \\ &= (n - k)8L_P^2, (n - k) = 1, 2, ... \ . \end{aligned} \qquad (30)$$

Obviously, expression (30) represents Bekenstein's quantization of the black hole horizon surface area (12).

In this way we have reproduced, i.e. determined in an independent way, three most important characteristics of Schwarzschild's black hole thermodynamics: Bekenstein-Hawking's entropy, Hawking's

temperature and Bekenstein's quantization of the surface area.

We now evaluate the necessary statistical form of $\sigma$. We suppose that the black hole can be considered as a canonical statistical ensemble of Bose-Einstein quantum systems. Then the statistical sum, Z, according to (18), equals

$$Z = \sum_{n=0} \exp[-\frac{E_n}{k_B T_H}] \, , \qquad (31)$$

where

$$
\begin{aligned}
E_n &= m_n c^2 = n \frac{\hbar c^3}{4\pi GM} = n \frac{M_P}{M} \frac{E_P}{4\pi} = \\
&= n E_1, n = 0, 1, 2, .... \qquad (32)
\end{aligned}
$$

and where $E_P = M_P c^2$ is Planck energy. (It is supposed, implicitly, that $n$ can be zero. Or, precisely, it can be shown by a more detailed analysis, that $n$ can be substituted by $(l(l+1))^{\frac{1}{2}}$ for $l = 0, 1, ...$)

Hence, our calculations provide harmonic-oscillator-like spectrum, supporting Bekenstein's result, and in variance with Frasca's (2005) calculations. The difference between the latter two approaches concerns not only the mere spectrum of the black hole energy, but may shed light onto the possible spatial energy-distribution within black hole. As described above, equidistant energy level distribution signals a uniform source-matter distribution, as the case of Thomson's atomic models shows. If the harmonic-oscillator-like spectrum proves correct, this would imply the uniformity of the gravitational field within the horizon.

According to (8), (32), it follows

$$\frac{E_1}{k_B T_H} = 2 \qquad (33)$$

which, introduced in (31), yields

$$Z = \sum_{n=0} \exp[-2n] = \exp[2]/(\exp[2] - 1) \qquad (34)$$

Then

$$
\begin{aligned}
w_n &= \exp[-\frac{E_n}{k_B T_H}]/Z = \\
&= (\exp[2] - 1)\frac{\exp[-2n]}{\exp[2]} \qquad (35)
\end{aligned}
$$

represents probability of quantum (eigenenergy) state $n$.

Further, from (31) we have

$$
\begin{aligned}
M &= -c^{-2} \frac{\partial(\ln[Z])}{\partial(1/(k_B T_H))} = \sum_{n=0} w_n m_n = \\
&= \sum_{n=0} w_n n m_1 = m_1 \sum_{n=0} w_n n \qquad (36)
\end{aligned}
$$

which implies

$$\sigma = \sum_{n=0} w_n n = <N> . \qquad (37)$$

Obviously, $\sigma$ can be regarded as the statistical average value, $<N>$, of the number of the quantum (eigenenergy) states. On the other hand, $\sigma$ considered as statistically determined entropy (in $k_B$ units), must have the form

$$\sigma = -\sum_{n=0} w_n \ln[w_n] \qquad (38)$$

Consistency of the analysis requires that (37) and (38) be equivalent which implies that condition

$$n = \ln[w_n], n = 0, 1, 2, ... \qquad (39)$$

must be satisfied. However, according to (35), it follows

$$
\begin{aligned}
\ln[w_n] &= 2n - \ln[\frac{\exp[2] - 1}{\exp[2]}] \approx 2n - 0.145, \\
n &= 0, 1, 2, ... \qquad (40)
\end{aligned}
$$

and this reveals that condition (39) is not satisfied. Nevertheless, we note that left- and right-hand sides of (39) have the same order of magnitude, precisely that for large $n$ right hand side of (38) is twice greater than left-hand side of (41), what appears to be an interesting result.

We assume now that black hole represents a great statistical ensemble of Bose-Einstein systems with statistical sum

$$Z = \sum_{n=0} \exp[-\frac{E_n - \mu n}{k_B T_H}], \qquad (41)$$

where $\mu$ represents the chemical potential, while $n$ in $\mu n$ can be considered as statistical average value of Bose-Einstein systems in quantum (eigenenergy) state $n$.

Suppose, further,

$$\mu = \frac{E_1}{2} \qquad (42)$$

which, according to (33), implies

$$\mu n = \frac{E_n}{2}, n = 0, 1, 2, ... \qquad (43)$$

and, according to (32)

$$Z = \sum_{n=0} \exp[-\frac{E_n}{2k_B T_H}]. \qquad (44)$$

In (41) $Z$ can be considered as the statistical sum of a canonical ensemble with

$$w_n = \exp[-E_n/2k_B T_H](\exp[1] - 1)\exp[-n]/(Z \exp[1]),$$

$$n = 0, 1, 2, ... \qquad (45)$$

This implies that:

$$
\begin{aligned}
\ln[w_n] &= n - \ln[\frac{\exp[1]-1}{\exp[1]}] \approx n - 0.45, \\
n &= 0,1,2,...
\end{aligned} \tag{46}
$$

and

$$
\ln[w_n] \approx n, n \gg 1. \tag{47}
$$

Relation (47) implies that condition (40) concerning the consistent statistical interpretation of $\sigma$ is well satisfied for probabilities effectively defined by (46).

Thus, we can simply and intuitively (quasi-classically) clearly explain Schwarzschild's black hole entropy. Namely, we assume that the circumference of the black hole horizon contains a (statistical) mixture of the reduced Compton's waves, any of which, multiplied by corresponding integer quantum number, is equivalent to the given circumference. On the one hand, entropy represents the statistical average value of given quantum numbers, i.e. numbers of the quantum (eigenenergy) states. On the other hand, in full agreement with usual rules of statistical mechanics or thermodynamics, this entropy can be considered as the entropy of a typical (great) canonical statistical waves, i.e. "micro-states", which are placed on the horizon, but not inside it. Also, as it will be shown in the following, there is a process of the black hole radiation. Accurate detection of given radiation in any individual case allows a precise distinction of the "micro-states", while inaccurate detection of given radiation at a statistical ensemble corresponds to the mixture of "micro-states", i.e. to the black hole entropy. For this reason, contrary to Hawking's assumption [3], given "micro-states" are not obscure, i.e. unobservable in principle.

There is an additional interesting possibility for analysis of the Schwarzschild's black hole entropy. Suppose that black hole horizon surface area, representing without quantum effects a Schwarzschild's sphere, contains, by means of the quantum effects, a more complex (curved) structure. Suppose that instead of a great circle with radius $R_S$ at Schwarzschild's sphere, there is a great quasi-circle, precisely, a deformed great circle representing formally a static Compton's wave with reduced Compton's wavelength $\lambda_{rn}$. For

$$
\lambda_{rn} \ll 2\pi R_S, \ n = 1,2,... \tag{48}
$$

this wave can be approximated by

$$
\begin{aligned}
u_n &= u_{n0} \sin[2\pi x/\lambda_{rn}], \ x \in [0, 2\pi R_S], \\
n &= 1,2,...
\end{aligned} \tag{49}
$$

Here $u_{0n}$ represents $u_n$ constant amplitude that will be determined later for $n = 1,2,...$ Also, the great circle on the black hole horizon area is linearized approximately, i.e. approximated by a finite direction with the length equivalent to the circumference $2\pi R_S$ of a given circle. Then $x$ represents a variable that changes along a given direction from 0 towards $2\pi R_S$. As it is not difficult to see, it follows

$$
\int_0^{2\pi R_S} u_n dx = 0, \ \ n = 1,2,... \tag{50}
$$

It means that surface area within great quasi-circle is equivalent to surface area within great circle.

However, the following relation holds

$$
\begin{aligned}
\alpha_n &= \int_0^{\frac{\lambda_n}{4}} u_n dx = u_{n0} \frac{\lambda_{rn}}{2\pi}, \\
n &= 1,2,\ ...
\end{aligned} \tag{51}
$$

which, according to (13)-(15), implies

$$
\begin{aligned}
a_n &= n 4\alpha_n = u_{n0} \frac{8MG}{c^2} = u_{n0} \frac{8L_P^2}{\lambda_M}, \\
n &= 1,2,...\ ,
\end{aligned} \tag{52}
$$

where

$$
\lambda_M = \frac{\hbar}{Mc} \tag{53}
$$

is the Compton's wavelength of the black hole.

It is obvious that $a_n$ in (52) is proportional to $\Delta A_{n+1n}$ in (31) for $n = 1,2,....$. Moreover, suppose

$$
u_{n0} = \lambda_M, n = 1,2,... \tag{54}
$$

Now, we assume

$$
u_{n0} \ll R_S, n = 1,2,... \tag{55}
$$

or, equivalently,

$$
M \gg M_P/\sqrt{2}\ , \tag{56}
$$

that is satisfied for macroscopic black holes. Then (52) becomes

$$
a_n = 8L_P^2 = \Delta A_{n+1n}, n = 1,2,.... \tag{57}
$$

It represents the minimal change of the black hole surface area corresponding, according to (23), to the minimal change of the black hole entropy

$$
\begin{aligned}
\Delta S_{BH} &= \frac{kBc^3}{4\hbar} \Delta A_{n+1n} = 8\frac{kBc^3}{4\hbar} L_P^2, \\
n &= 1,2,...
\end{aligned} \tag{58}
$$

All this can be interpreted in the following way. Quantum field effects cause small deformations of the great circle at the black hole horizon surface, more precisely, they change the unique classical (non-quantum) great circle into different quantum (quasi-classical) great quasi-circles corresponding to different quantum states. Given changes, i.e. variations, according to (48), (51), (55)-(57), are classically effectively unobservable, but they are observable from the point of view of the quantum field theory (quantum mechanics). Difference (58) between the classical effective non-observability and the quantum observability of given changes in any quantum state can be considered as the minimal entropy (59). It represents a typical, usual interpretation of the entropy according to which entropy corresponds to micro-states unobservable by a rough (statistical) analysis, but observable by a more accurate account. Of course, since given black hole contains (statistically averaged) $\sigma$ quantum states, the total entropy of a black hole can be roughly estimated by

$$
S_{BH} = \sigma \Delta S_{BH} \tag{59}
$$

as an equivalent to Bekenstein-Hawking entropy.

## 2.2 Energy spectrum and evaporation time

In Bohr's atomic model we have the postulate on the energy emission by discrete, spontaneous, quantum jump of the electron from a higher onto a lower circular orbit. This quantum jump represents an effective final result (or simplified description) of the electromagnetic self-interaction of the atom. Also, according to Bohr's correspondence principle, emission of the photon appears most probably by quantum jump of the electron from an initial, sufficiently high quantum state $n$ onto the neighbouring final quantum state $(n-1)$.

In conceptual analogy with Bohr's atomic model, suppose that black hole, considered as Bose-Einstein quantum system, in some initial quantum state $n$, can spontaneously and discretely (by means of gravitational self-interaction) pass, i.e. jump, to some final, lower quantum state $k$, for $k < n = 1, 2, ...$. Suppose, also, that by this quantum jump an effective final emission of a quantum of energy takes place which propagates far away from the black hole. Of course, black hole, according to its classical definition, captures any physical system near horizon by means of the gravitational interaction. Nevertheless, *according to principles of the quantum theory, (quantum mechanics and quantum field theory alike) gravitationally self-interacting black hole passes from an initially non-stable quantum state $n$ in the final, stable quantum state $k < n$ by emitting one energy quantum outside horizon.* This is, of course, a simplified, phenomenological description of the black hole gravitational self-interaction.

Energy of given energy quantum, according to (32), equals

$$E_n - E_k = \hbar\omega_{nk}, \ k < n = 1, 2, ... \qquad (60)$$

where $\omega_{nk}$ is the circular frequency of a given energy quantum. Then, according to (32), it follows

$$\begin{aligned} E_n - E_k &= E_{n-k} = (n-k)\frac{c^3}{4\pi GM}, \\ k &< n = 1, 2, ... \end{aligned} \qquad (61)$$

Here we assume that a correspondence principle, conceptually similar to Bohr's, holds. More precisely, suppose that for initial, large quantum state $n$, there is most probable quantum jump to the final state $k = n - 1$, with corresponding emission of the one energy quantum

$$E_n - E_{n-1} = E_1 = \frac{\hbar c^3}{4\pi GM}, \ n = 1, 2, ... \qquad (62)$$

Of course, given quantum jump can be considered definitive, i.e. irreversible, if and only if condition

$$\Delta E_n + \Delta E_{n-1} \ll E_n - E_{n-1} = E_1, \ n = 1, 2, ... \qquad (63)$$

is satisfied. Here $\Delta E_n$ and $\Delta E_{n-1}$ represent the energy natural widths of quantum states $n$ and $n-1$ and, for sufficiently large $n$, we assume

$$\Delta E_n \approx \Delta E_{n-1} . \qquad (64)$$

For a more accurate form of (64), a more rigorous form of the quantum gravitation is necessary. Nevertheless, we shall simply suppose, according to (63), (64)

$$2\Delta E_n \leq \frac{E_1}{100}, \ n \gg 1. \qquad (65)$$

According to Heisenberg's energy-time uncertainty relation

$$\tau\Delta E_n \approx \frac{\hbar}{2}, \ n \gg 1, \qquad (66)$$

where $\tau$ represents the time of the one-energy quantum emission or life-time of the Bose-Einstein system in the initial quantum state, it follows

$$\Delta E_n \approx h/(2t), \ n \gg 1. \qquad (67)$$

Then, according to (53), (55), one has:

$$t = 100h/E1 = 100 \cdot 4\pi GM/c3, \ n \gg 1. \qquad (68)$$

Suppose now that a black hole is initially in the (statistically averaged) quantum state $\frac{M}{m_1}$. Let the black hole, according to previous discussion, emit, by quantum jump, energy quantum $E_1$ within time interval $\tau$. It implies that initial black hole with mass $M$ will entirely evaporate by means of its gravitational self-interaction after a time interval $\tau_{ev}$. Given time interval can be roughly estimated, according to (19), (68), by

$$\begin{aligned} \tau_{ev} &\geq \frac{M}{m_1}100\frac{\hbar}{E_1} = \\ &= 100(16\pi)\frac{\pi G^2 M^3}{c^4} \approx 5027\frac{\pi G^2 M^3}{c^4}. \end{aligned} \qquad (69)$$

We see that the result is very close to Hawking time for black hole evaporation (10).

## 3. CONCLUSION

We carried out a simplified but non-trivial quasi-classical analysis of quantum and thermodynamical characteristics of a Schwarzschild's black hole. Our analysis is conceptually analogous to the formalism of Bohr's atomic model and, for this reason, black hole we consider can be simply called Bohr's black hole. We start by a condition, analogous to Bohr's quantization postulate, via de Broglie relation. This condition states that circumference of a great circle at the black hole horizon contains a finite (statistically averaged) number of corresponding reduced Compton's wavelength. It implies simple determination of three most important thermodynamical characteristics of a black hole: Bekenstein-Hawking entropy, Hawking's temperature and Bekenstein's quantization of the surface area. In particular, it allows a simple interpretation of the black hole Bekenstein-Hawking entropy. Finally, we present the black hole radiation in the form conceptually analogous to Bohr's postulate on

photon emission by discrete quantum jump of the electron in Bohr's atomic model. In accordance with Heisenberg's energy-time uncertainty relation and a correspondence rule conceptually analogous to Bohr's correspondence principle, it allows a rough estimate of the time interval for black hole evaporation. This time interval is very close to the time interval of the black hole evaporation obtained via Hawking's radiation.

Finally, we speculate about the relevance of the energy spectrum for the evidence of the source-field distribution within the horizon, which is, otherwise, an unobservable quantity.

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# БОРОВ СЕМИКЛАСИЧНИ МОДЕЛ ТЕРМОДИНАМИКЕ ЦРНЕ РУПЕ

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Предложен је једноставан поступак за израчунавање основних атрибута Шварцшилдове црне рупе: Бекенштајн-Хокингове ентропије, Хокингове температуре и Бекенштајнове квантизације површине хоризонта. Коришћен је услов да обим великог круга хоризонта садржи коначан број редукованих Комптонових таласних дужина. Поступак је аналоган Боровом постулату за квантизацију атома водоника преко де Брољеве релације. Поступак имплицира уобичајено значење ентропије црне рупе, у односу на површину квантне варијације великих кругова на хоризонту. Зрачење црне рупе презентирано је у облику аналогном Боровом концепту емисије фотона путем дискрених квантних скокова у окиру Старе Квантне теорије. То омогућава, према Хајзенберговим релацијама неодређености и Боровом принципу кореспонденције, процену временског интервала за испарење црне рупе, за који је нађено да је веома близак интервалу према стандардној Хокинговој формули. Наш рачун потврђује Бекенштајнов семикласични резултат за квантовање енергије, а не слаже се са рачуном Фраске (Frasca 2005). Најзад, дискутоване су последице израчунате енергијске расподеле на процену расподеле енергије унутар црне рупе.