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# The Analysis and Applicability Conditions of Tshebyshev's Theory Concerning Garment Patterns

**Abstract**
*Many publications related to the garment industry refer to Tshebyshev's works about garment patterns. This is because his point of view allows us to hope that an engineering method for garment design can someday be created. This work is an attempt to verify the equations Tshebyshev obtained, and to compare their precision with a numerical solution. To this end, the value of the deformation angle of the network determined in accordance with Tshebyshev's theory was compared with the deformation angle obtained by the same method but using modern computer systems. The results obtained form the conclusions to this work.*

**Kay words:** *garment patterns, Tshebyshev theory clothing design, network deformation, numerical solution*

## Introduction

In 1878 the world-famous Russian mathematician Tshebyshev published a paper concerning the transformation of a two-dimensional structure onto a curvilinear surface, giving an application of this transformation for garment patterns. The available source paper is just a shortened description, which includes only the presented results. This work aims at a full description of Tshebyshev's concept with special attention to the physical interpretation of the assumption made, and to the degree of accuracy in the formulas quoted. Because Tshebyshev's results are often referred to, the research attempt, which we have undertaken, seemsadvisable .

Moreover, the development of a method of designing garment patterns based on Tshebyshev's assumption would give us a chance to eliminate the first sewing during the preparation stage of the production. The possibilities of designing clothing forms in an unambiguous way starting from the material features and size characteristics of the human body, and using computer-aided engineering have been explored worldwide (as is proved by numerous publications). Work on garment patterns conducted in recent years has led us to think that textile simulation in a form of proper net works and precise digital characteristics of the human body which records not only sizes, but also shapes, could result in positive effects. On one hand, a complete transformation from a three-dimensional model of the human body to two-dimensional forms characterised by the fabrics from which the clothing is made is likely expected. On the other hand, computer research has been conducted to simulate clothing on the basis of the expected appearance, resulting from an appropriate combination of joined fabric patterns produced using the fabric of familiar parameters.

## Angles of form deformation described by Tshebyshev

A non- deformed plane structure consisting of two mutually perpendicular groups of fibres (weft, warp) is deformed onto a curvilinear surface.

The assumptions made by Tshebyshev have the following physical interpretations:

- The plane structure is placed onto a curvilinear surface,
- Fibres do not elongate in two mutually perpendicular directions,
- Groups of fibres cannot shift at the points of mutual crossing.

The assumptions written above are an approximate description of the fabric, and are more accurate if the fabric deformations are smaller.

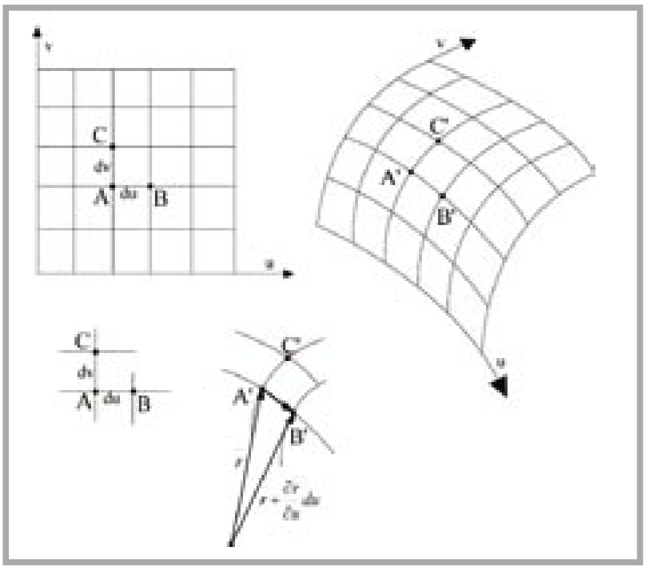

*Figure 1. Deformation of a network on a curvilinear surface.*

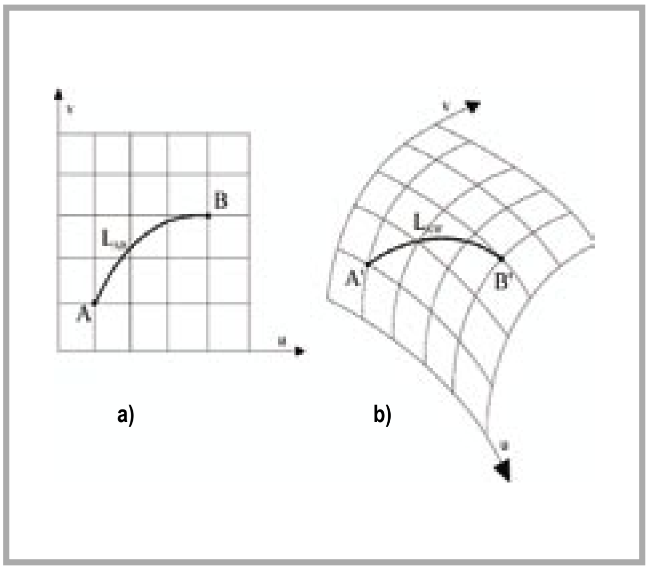


*Figure 2. Deformation of line AB on a curvilinear surface.*

## Dependence between the curvature and the deformation angle

As the result of deformation, point A with co-ordinates u, v in a non-deformed configuration, is moved to point A' (Figure 1), the location of which is described by vector *r (u,v)*. The next point B (*u+du,v+dv*), lying on the line *v*=const., is moved to point B', the location of which is described by vector

$$\overline{r_v} = \bar{r} + \frac{\partial \bar{r}}{\partial u} du \, .$$

The increment of vector $\bar{r}$, which is the distance of the measure of the two points, equals

$$\overline{A'B'} = \frac{\partial \bar{r}}{\partial u} du$$

The equality $\overline{AB} = \overline{A'B'}$ results from the assumed non-elongation of fibres, which leads directly to the equality:

$$\bar{r}_{,u} \cdot \bar{r}_{,u} = 1 \qquad (1)$$

and similarly for direction v:

$$\bar{r}_{,v} \cdot \bar{r}_{,v} = 1 \qquad (2)$$

where:

$$\frac{\partial \bar{r}}{\partial u} = \bar{r}_{,u} \qquad \frac{\partial \bar{r}}{\partial v} = \bar{r}_{,v}$$

The angle φ between tangents to the line AB is described by:

$$\cos\varphi = \bar{r}_{,u} \cdot \bar{r}_{,v} \qquad (3)$$

The Gaussian curvature in the curvilinear co-ordinate system is described by the formula:

$$K = \frac{LN - M^2}{EG - F^2} \qquad (4)$$

where:

$$E = \overline{r_{,u}} \cdot \overline{r_{,u}}$$
$$G = \overline{r_{,v}} \cdot \overline{r_{,v}}$$
$$E = \overline{r_{,u}} \cdot \overline{r_{,v}}$$
$$L = \frac{\overline{r_{,uu}} \cdot \left(\overline{r_{,u}} \times \overline{r_{,v}}\right)}{\sqrt{EG - F^2}}$$
$$M = \frac{\overline{r_{,uv}} \cdot \left(\overline{r_{,u}} \times \overline{r_{,v}}\right)}{\sqrt{EG - F^2}}$$
$$N = \frac{\overline{r_{,vv}} \cdot \left(\overline{r_{,u}} \times \overline{r_{,v}}\right)}{\sqrt{EG - F^2}}$$

The deformation of line AB on a curvilinear surface is presented in Figure 2.

Using the equalities [1,2,3] and substituting them to formula [4], the following relation was obtained:

$$K \sin^2\varphi = \frac{\partial^2 \cos\varphi}{\partial u \partial v} + \frac{\partial \varphi}{\partial u}\frac{\partial \varphi}{\partial v}\cos\varphi \qquad (5)$$

We need to notice that in Tshebyshev's original work, on the right side of the equation [5] we subtract the two values, yet this difference does not change the further considerations due to the approximation accepted by Tshebyshev of one element to zero.The form presented above allows us to further simplify the formula [5], which in consequence gives the following equation:

$$K \sin\varphi = -\frac{\partial^2 \varphi}{\partial u \partial v} \qquad (6)$$

## Properties of deformation angles on a geodesic line

We choose two points A and B in the non-deformed co-ordinate system on Tshebyshev's previously discussed net work (Figure 2a). Next, we deform this system onto a curvilinear surface, and as a result of this deformation points A and B take the new positions A'B' (Figure 2b).

Circumscribing the curvilinear co-ordinate describing the length of a line lying on a surface by s(u,v), and the vector radius of that line by $\bar{r}(s)$, the length of the line joining points A'B' is given by [7].

$$L_{A'B'} = \int_{A'}^{B'} ds = \int_{A'}^{B'} \sqrt{d\bar{r} \cdot d\bar{r}} = \int_{A'}^{B'} \sqrt{\bar{r}_{,s} \cdot \bar{r}_{,s}}\, ds \qquad (7)$$

where:

$$r_{,s} = r_{,u}\, u_{,s} + r_{,v}\, v_{,s}$$
$$v_{,u} = v_{,s}/u_{,s}$$

The shortest line joining points A'B', is called the geodesic line, and is defined by the condition:

$$L_{A'B'} \rightarrow Min \, ,$$

or by using the perquisite condition of the extremum existing, the variance $L_{A'B'}$:

$$\partial L_{A'B'} = 0 \qquad (8)$$

By substituting length $L_{A'B'}$ (7) to condition (8) we obtain:

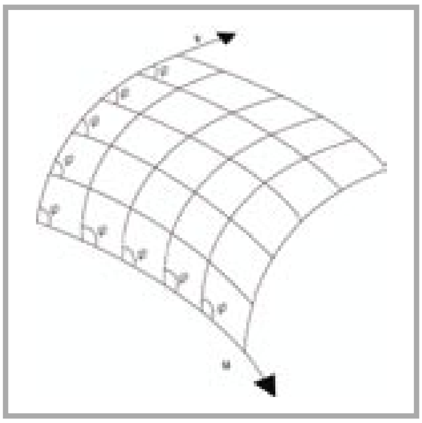

*Figure 3. Stability of angles on geodesic lines overlapping the lines of the network.*

$$\delta L_{A'B'} = \int_{A'}^{B'} \frac{\bar{r}_{,s}\, \delta \bar{r}_{,s}}{\sqrt{\bar{r}_{,s} \cdot \bar{r}_{,s}}} ds = 0 \qquad (9)$$

By means of numerous transformations we obtain the final equation (10) circumscribing the geodesic line of the system analysed. The differential equation presented in (10) describes the location of the geodesic line in the form of the function v(u) searched for.

The assumption made during the analysis was that the line *v=v(u)* is being sought. If we assume that the axis *u* lies on the geodesic line, then *v*=constant, and the derivative

$$\frac{dv}{du} = 0$$

It results from formula (10) that

$$\frac{\partial \cos\varphi}{\partial u} = 0$$

Similarly, if we accept the reverse relation, that is *u=u(v)*, we will obtain

$$\frac{\partial \cos\varphi}{\partial v} = 0$$

It allows us to state that the angle φ lying at one of the geodesic lines which overlaps with one of the net lines is always constant.

$$\sin^2\varphi \frac{d^2 v}{du^2} + \frac{\partial \cos\varphi}{\partial v}\left(\frac{dv}{du}\right)^2 \left(\cos\varphi + \frac{dv}{du}\right) + \frac{\partial \cos\varphi}{\partial u}\left(\cos\varphi \frac{dv}{du} + 1\right) = 0 \qquad (10)$$

$$K = K_0 + K_1 u + K_2 v + K_3 u^2 + K_4 uv + K_5 v^2 + \ldots \qquad (12)$$

$$\varphi = -uv\left(K_0 + \frac{1}{2}K_1 u + \frac{1}{3}K_3 u^2 + \frac{1}{2}K_2 v + \frac{1}{4}K_4 uv + \frac{1}{3}K_5 v^2\right) + \varphi_0 \qquad (13)$$

***Equation 10, 12, and 13.***

Figure 3 presents a special case where the geodesic lines lie on two mutually intersecting network lines. Then the angle φ lying at a point where the two directions intersect is a constant angle in the direction of the net works line *u*, as well as in the direction of *v*.

## Angles of form deformation according to Tshebyshev

In order to check the approximation accepted by Tshebyshev, the equation

$$K \sin\varphi = -\frac{\partial^2\varphi}{\partial u \partial v}$$

was solved, accepting (after Tshebyshev) the small increments of the angle φ. Then for φ ≅ π/2, sin φ ≅ 1, and equation (6) takes a form suitable for integration:

$$K = -\frac{\partial^2\varphi}{\partial u \partial v} \qquad (11)$$

The acceptance of the co-ordinate system *u, v* lying on the curvilinear surface,

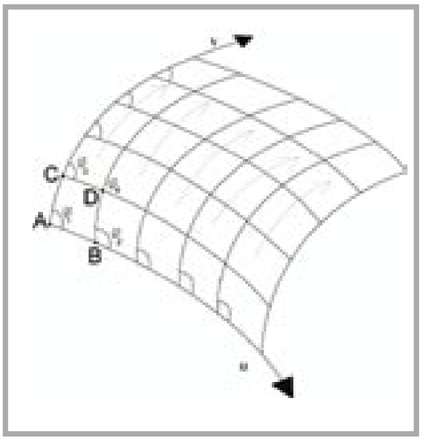


***Figure 4.*** *The determination of the deformation angles by schema of finite differences.*

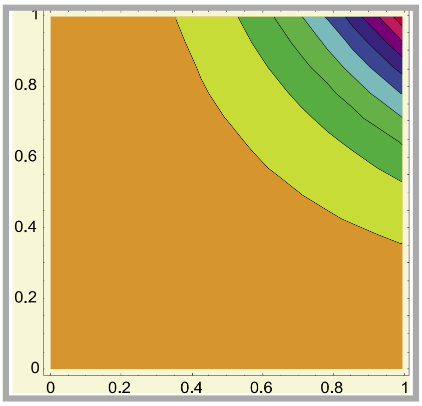


***Figure 5.*** *Comparison of the changes of the deformation angles for the numerical solution and Tshebyshev's method: a contour line visualisation..*

and overlapping with the geodesic lines, causes the boundary conditions resulting from the property (10) to take the following form:

- for $u = 0 \rightarrow \varphi = \varphi_0$
- for $v = 0 \rightarrow \varphi = \varphi_0$

By the development of the curvature *K* into a series within the surroundings of the beginning of the co-ordinate system (*u*=*v*=0) presented on equation (12), and by the integration of equation 11, taking into consideration the boundary conditions, we obtain equation (13).

## Numerical verification of Tshebyshev's approximations

The received equations were verified for a sphere, for which the Gaussian curvature is constant and equals: $K = 1/R^2$, where *R* is the radius of this sphere.

Then the solution of equation 11 takes the form:

$$\varphi = -\frac{uv}{R^2} + \varphi_0 \qquad (14)$$

The above solution is the result of the acceptance of small increments of deformation angles, and at the same the approximation sin φ ≅ 0. The verification carried out was based on the numerical solution of the full equation 6. This equation defines the value of the deformation angle on the surface of the sphere, and the angle φ is equal to the angle between the two intersected geodesic lines. By carrying out the analysis of the above-mentioned equation, the assumptions consistent with Tshebyshev were made; that is, that both directions of the network overlapped with the geodesic lines of the sphere, and that the angle between them was 90˚. The boundary co-ordinates of the network were covered with two meridians of the sphere. Through the approximation of the second derivative by the schema of finite differences:

$$\left.\frac{\partial^2\varphi}{\partial u \partial v}\right|_D = \frac{\varphi_D - \varphi_C - \varphi_B + \varphi_A}{\Delta u \Delta v} \qquad (15)$$

the equation [6] for point D takes the form:

$$K \sin\varphi_D \cong \frac{\varphi_D - \varphi_C - \varphi_B + \varphi_A}{\Delta u \Delta v}.$$

If in the contiguous points A, B and C, the deformation angles φ are known, then it is possible to determine the angle $\varphi_D$. We start the calculations from the determination of the deformation angles in nodes next to the node lying on the boundary lines, overlapping with the geodesic lines (Figure 4). The findings obtained were compared with the solution of equation 14, and are presented on Figure 5.

## Summary

The comparison of these two methods allows us to statethe following:

- For deformation angles only slightly different from 90°, the disproportion resulting from the use of both methods is not large. The farther we move into the network, the greater the difference is.
- The greatest discrepancy between both values is at the extreme node of the network, and equals about 3°. This value allows us to suppose that if we change the deformation angle from 90˚ to about 40˚, the difference is not significant, and we can accept that Tshebyshev's method sufficiently approximates the values of the deformation angles way.

## Conclusion

- As a result of completing all the calculations, it can be unequivocally stated that the method developed by Tshebyshev can be used when it is possible to describe the curvature of the surface on which the deformation is made.
- By comparing Tshebyshev's method with the numerical solution, we can state that for a body with known curvature the divergence resulting from both methods is small, and we can accept that both solutions are correct.

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