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# Application of Mathematical Methods to Designing a Cam-Driven Loom Batten

**Abstract**
*The mathematical model of a cam-driven batten mechanism is presented. The set of differential equations describes the behaviour of the system, taking into account the elasticity of its elements and the electric motor. The results of numerical calculations are plotted. General suggestions for designers are given.*

**Key words:** *batten, loom, cam, computer modelling.*

## Introduction

The selection of a machine's optimal parameters is possible at the design stage, but doing so requires the construction of a mathematical model of the machine as well as the application of computer methods to study its behaviour. Methods for formulating computer models of textile machines are explained in work [1]. The batten of the loom can be modelled as a mechanism composed of two shafts coupled together by mechanisms. The problem of torsional vibrations in such shafts has been studied in papers [2-6]. The purpose of this paper is to present a mathematical description of the system.

## Mathematical Description of the Batten Mechanism

The batten, which performs an oscillating rise-return-dwell motion, is driven by two parallel cam mechanisms, as shown in Figure 1. Each cam mechanism is composed of two conjugate cams, one convex and the other concave. The system is driven by an electric motor through an elastic belt. The batten beam with the reed attached to the batten shaft is not shown in Figure 1.

Separating the rigid elements by cutting the elastic shafts and belt, using the d'Alembert and virtual work principles, the dynamic behaviour of the system can be described by the set of equations (1).

Here, ($\Phi$, $\Psi$) are rotation angles, ($A$, $B$) the mass moments of inertia, ($S$, $K$) the stiffnesses, ($D$, $H$) the damping coefficients, and M the external torque.

On the basis of what is known about motor parameters or its characteristic dependencies the driving torque for a three-phase induction motor [7] can be

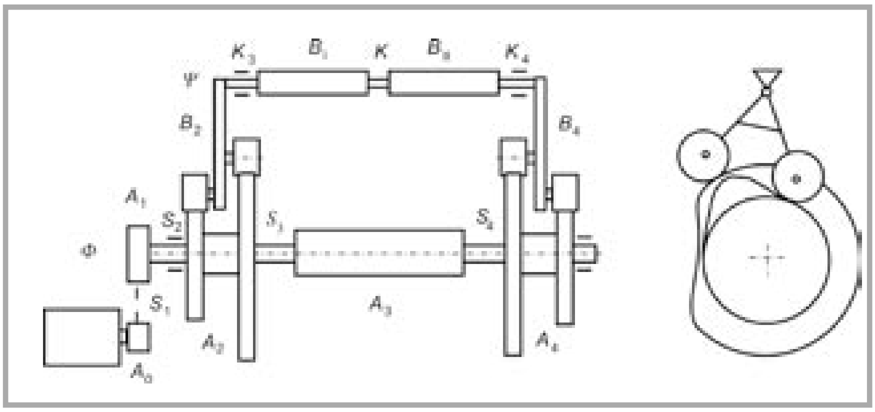

***Figure 1.*** *Dynamic model of loom batten.*

$$A_0\frac{\mathrm{d}^2\Phi_0}{\mathrm{d}t^2}+D_1\left(\frac{\mathrm{d}\Phi_0}{\mathrm{d}t}r_0-\frac{\mathrm{d}\Phi_1}{\mathrm{d}t}r_1\right)r_0+s_1\left(\Phi_0 r_0-\Phi_1 r_1\right)r_0-M_0=0,$$

$$A_1\frac{\mathrm{d}^2\Phi_1}{\mathrm{d}t^2}+D_1\left(\frac{\mathrm{d}\Phi_1}{\mathrm{d}t}r_1-\frac{\mathrm{d}\Phi_0}{\mathrm{d}t}r_0\right)r_1+S_1\left(\Phi_1 r_1-\Phi_0 r_0\right)r_1+D_2\left(\frac{\mathrm{d}\Phi_1}{\mathrm{d}t}-\frac{\mathrm{d}\Phi_2}{\mathrm{d}t}\right)+S_2\left(\Phi_1-\Phi_2\right)=0,$$

$$A_3\frac{\mathrm{d}^2\Phi_3}{\mathrm{d}t^2}+D_3\left(\frac{\mathrm{d}\Phi_3}{\mathrm{d}t}-\frac{\mathrm{d}\Phi_2}{\mathrm{d}t}\right)+S_3\left(\Phi_3-\Phi_2\right)+D_4\left(\frac{\mathrm{d}\Phi_3}{\mathrm{d}t}-\frac{\mathrm{d}\Phi_4}{\mathrm{d}t}\right)+S_4\left(\Phi_3-\Phi_4\right)=0,$$

$$\left(A_2+B_2\left(\frac{\mathrm{d}\Psi_2}{\mathrm{d}\Phi_2}\right)^2\right)\frac{\mathrm{d}^2\Phi_2}{\mathrm{d}t^2}+B_2\frac{\mathrm{d}\Psi_2}{\mathrm{d}\Phi_2}\frac{\mathrm{d}^2\Psi_2}{\mathrm{d}\Phi_2{}^2}\left(\frac{\mathrm{d}\Phi_2}{\mathrm{d}t}\right)^2+D_2\left(\frac{\mathrm{d}\Phi_2}{\mathrm{d}t}-\frac{\mathrm{d}\Phi_1}{\mathrm{d}t}\right)+$$
$$+D_3\left(\frac{\mathrm{d}\Phi_2}{\mathrm{d}t}-\frac{\mathrm{d}\Phi_3}{\mathrm{d}t}\right)+H_3\left(\frac{\mathrm{d}\Phi_2}{\mathrm{d}t}\frac{\mathrm{d}\Psi_2}{\mathrm{d}\Phi_2}-\frac{\mathrm{d}\Psi_I}{\mathrm{d}t}\right)\frac{\mathrm{d}\Psi_2}{\mathrm{d}\Phi_2}+$$
$$+S_2\left(\Phi_2-\Phi_1\right)+S_3\left(\Phi_2-\Phi_3\right)+K_3\left(\Psi_2-\Psi_I\right)\frac{\mathrm{d}\Psi_2}{\mathrm{d}\Phi_2}=0, \qquad (1)$$

$$\left(A_4+B_4\left(\frac{\mathrm{d}\Psi_4}{\mathrm{d}\Phi_4}\right)^2\right)\frac{\mathrm{d}^2\Phi_4}{\mathrm{d}t^2}+B_4\frac{\mathrm{d}\Psi_4}{\mathrm{d}\Phi_4}\frac{\mathrm{d}^2\Psi_4}{\mathrm{d}\Phi_4{}^2}\left(\frac{\mathrm{d}\Phi_4}{\mathrm{d}t}\right)^2+D_4\left(\frac{\mathrm{d}\Phi_4}{\mathrm{d}t}-\frac{\mathrm{d}\Phi_3}{\mathrm{d}t}\right)+$$
$$+H_4\left(\frac{\mathrm{d}\Phi_4}{\mathrm{d}t}\frac{\mathrm{d}\Psi_4}{\mathrm{d}\Phi_4}-\frac{\mathrm{d}\Psi_{II}}{\mathrm{d}t}\right)\frac{\mathrm{d}\Psi_4}{\mathrm{d}\Phi_4}+S_4\left(\Phi_4-\Phi_3\right)+K_4\left(\Psi_4-\Psi_{II}\right)\frac{\mathrm{d}\Psi_4}{\mathrm{d}\Phi_4}=0,$$

$$B_I\frac{\mathrm{d}^2\Psi_I}{\mathrm{d}t^2}+H_3\left(\frac{\mathrm{d}\Psi_I}{\mathrm{d}t}-\frac{\mathrm{d}\Psi_2}{\mathrm{d}t}\right)+K_3\left(\Psi_I-\Psi_2\right)+H\left(\frac{\mathrm{d}\Psi_I}{\mathrm{d}t}-\frac{\mathrm{d}\Psi_{II}}{\mathrm{d}t}\right)+K\left(\Psi_I-\Psi_{II}\right)+M_I=0,$$

$$B_{II}\frac{\mathrm{d}^2\Psi_{II}}{\mathrm{d}t^2}+H\left(\frac{\mathrm{d}\Psi_{II}}{\mathrm{d}t}-\frac{\mathrm{d}\Psi_I}{\mathrm{d}t}\right)+K\left(\Psi_{II}-\Psi_I\right)+H_4\left(\frac{\mathrm{d}\Psi_{II}}{\mathrm{d}t}-\frac{\mathrm{d}\Psi_4}{\mathrm{d}t}\right)+K_4\left(\Psi_{II}-\Psi_4\right)+M_{II}=0.$$

***Equation 1.***

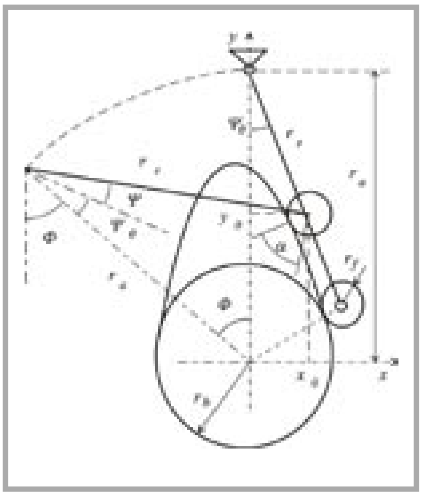

*Figure 2. Cam follower motion reversed.*

$$M_0 = \frac{1}{2} i^T \frac{\partial L}{\partial \Phi_0} i, \quad \frac{\mathrm{d}}{\mathrm{d}t}(Li) + Ri = u, \quad L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix},$$
$$L_{11} = L_{sr} diag(1) + L_{sg} c(0), \quad L_{12} = L_M c(\Phi_0),$$
$$L_{21} = L_{12}^T, \qquad L_{22} = L_{wr} diag(1) + L_{wg} c(0), \tag{2}$$
$$c(\Phi_0) = \begin{bmatrix} \cos\Phi_0 & \cos(\Phi_0 + \alpha_0) & \cos(\Phi_0 + 2\alpha_0) \\ \cos(\Phi_0 - \alpha_0) & \cos\Phi_0 & \cos(\Phi_0 + \alpha_0) \\ \cos(\Phi_0 - 2\alpha_0) & \cos(\Phi_0 - \alpha_0) & \cos\Phi_0 \end{bmatrix},$$

$$\frac{\mathrm{d}M_0}{\mathrm{d}t} = \frac{M_{st} - M_0}{T}, \quad M_{st} = \frac{(2 + s_k b)\, M_{max}}{\dfrac{1 - \dfrac{\mathrm{d}\Phi_0}{\mathrm{d}t}\dfrac{1}{\omega_s}}{s_k} + \dfrac{s_k}{1 - \dfrac{\mathrm{d}\Phi_0}{\mathrm{d}t}\dfrac{1}{\omega_s}} + b s_k}, \tag{3}$$

$$C = \frac{M_{zn}}{\omega_s - \omega_{zn}}, \quad b = -2 + \frac{C\omega_s}{M_{max}} - 2\sqrt{\left(\left(\frac{M_{max}}{M_r} - 1\right)\frac{C\omega_s}{M_{max}}\right)}, \quad s_k = \frac{2}{\dfrac{C\omega_s}{M_{max}} - b}, \tag{4}$$

$$\mu_{zn} = \frac{M_{zn}}{M_{max}}, \quad \mu_r = \frac{M_r}{M_{max}}, \quad s_{zn} = 1 - \frac{\omega_{zn}}{\omega_s},$$
$$a_1 = \frac{\mu_{zn}}{s_{zn}}(\mu_r - 1) - \mu_r \mu_{zn} + \mu_r, \quad b_1 = 2\mu_{zn} - 2(\mu_r - 1) - 2,$$
$$c_1 = \mu_{zn} s_{zn}(\mu_r - 1) - \mu_r \mu_{zn} + \mu_r, \quad \text{for } \mu_r = 1: s_k = 1,\ b = 0, \tag{5}$$
$$\text{for } \mu_r \neq 1: \quad s_k = \frac{-b_1 - \sqrt{(b_1^2 - 4a_1 c_1)}}{2a_1}, \quad b = \frac{2s_k - (1 + s_k^2)\ \mu_r}{s_k^2(\mu_r - 1)}.$$

$$\Psi = \frac{\Psi_A}{2}\left(1 - \cos\frac{\pi\Phi}{\beta}\right), \tag{6}$$

$$\Psi(\Phi) = \psi_1\left(\frac{\Phi}{\beta_1} - \frac{1}{\pi}\right)\sin\left(\frac{\pi\Phi}{\beta}\right) \quad \text{for } \Phi \in [0, \beta_1],$$
$$\Psi(\Phi) = \psi_2 \sin\left(\frac{\pi}{2}\frac{(\Phi - \beta_1)}{\beta_2}\right) + \psi_1 \quad \text{for } \Phi \in [\beta_1, \beta], \tag{7}$$
$$\rho = \frac{\beta_2}{\beta_1}, \ \beta_1 = \frac{\beta}{\rho + 1}, \ \beta_2 = \frac{\rho\beta}{\rho + 1}, \psi_1 = \frac{\pi\Psi_A}{4\rho + \pi}, \psi_2 = \frac{4\rho\Psi_A}{4\rho + \pi},$$
$$\Psi(\Phi) = \Psi(2\beta - \Phi) \text{ for } \Phi \in [\beta, 2\beta], \ \Psi(\Phi) = 0 \ \text{ for } \ \Phi \in [2\beta, 2\pi],$$

$$\psi(\Phi) = \Psi_A \begin{pmatrix} 6.09755\left(\dfrac{\Phi}{\beta}\right)^3 - 20.7804\left(\dfrac{\Phi}{\beta}\right)^5 \\ + 26.73155\left(\dfrac{\Phi}{\beta}\right)^6 - 13.60965\left(\dfrac{\Phi}{\beta}\right)^7 + 2.56095\left(\dfrac{\Phi}{\beta}\right)^8 \end{pmatrix}, \tag{8}$$
$$\Psi(\Phi) = \psi(\Phi) \ \textit{for} \ \Phi \in [0, \beta], \ \Psi(\Phi) = \psi(2\beta - \Phi) \ \textit{for} \ \Phi \in [\beta, 2\beta],$$
$$\Psi(\Phi) = 0 \ \textit{for} \ \Phi[2\beta, 2\pi]$$

***Equation 2 - 5.***

calculated from differential equations (2) or (3, 4) or (3, 5).

The following notation is used: $i$ is a column matrix of current intensities, $R$ is a diagonal matrix of winding resistances, $L$ is an inductance square matrix, $u$=[$U cos(\omega_s t)$, $U cos(\omega_s t - \alpha_0)$, $U cos(\omega_s t - 2\alpha_0)$, 0, 0, 0]$^T$ is a column matrix of supply voltage, $\alpha_0$=2π/3, $\omega_s$ is the angular velocity of the rotation of the electrical field. For the parameters given by equations (4, 5), the motor characteristic $M_{st}$ (3) passes through the starting torque $M_r$, the maximum torque $M_{max}$ and the synchronous angular speed $\omega_S$. Either the characteristic (5) or its tangent (4) passes through nominal torque $M_{zn}$ for nominal angular velocity $\omega_{zn}$.

Let us denote the working angle of cam by $2\beta$, and the angle between the extreme positions of the batten by $\Psi_A$. The dependence of the motion $\Psi$ of the batten shaft on the rotary motion of the cam $\Phi$ can be described by any of the following functions: a cosine (6), a cycloid (7) or a polynomial (8), shown in Figure 3a ($\Psi$) and Figure 3b ($d^2\Psi/d\Phi^2$).

In order to determine the co-ordinates of the cam profile, we follow the procedure described in [8,1]. The coordinates of the cam profile ($x=x_k$, $y=y_k$) (Figure 2) and the dimensions of cam mechanism can be determined from the set of equation (9).

Here, $\sigma$ denotes the stress acting on the cam; $M_K$ denotes the follower torque, which exerts a force on the cam. Other symbols are defined in Figure 2, or by equation (9).

## Example results

The results shown the Figure 3 were obtained on the basis of the following data considering a construction developed by the author in 1989 (the data was based on constructions existing in ......, but adapted to the project realised); loom width 2m; motor power $P_{zn}$ = 15kW; angular velocities: $\omega_{zn}$ = 910π/30rad/s, $\omega_S$ = 1000π/30rad/s; torque ratios: $M_{max}/M_{zn}$ = 3.9, $M_r/M_{zn}$ = 3.9; mass moments of inertia [in kgm$^2$]: $A_0$ = 0.246, $A_1$ = 0.006, $A_2$ = $A_4$ = 0.25, $A_3$ = 0.32, $B_I$ = $B_{II}$ = 0.16, $B_2$ = $B_4$ = 0.12; belt pulleys $r_0$ = $r_1$; stiffnesses [Nm]: $S_1 r_0^2$ = 6250, $S_2$ = 4172427, $S_3$ = $S_4$ = 4074988, $K_3$ = $K_4$ = 867752, $K$ = 0; damping coefficients [Nms]: $D_1 r_0^2$ = 0.035, $D_2$ = 235,

$$x_k = x_0 \pm \frac{r_f \operatorname{sgn}\left(\frac{\partial y_0}{\partial \Phi}\right)}{\sqrt{1+\left(\frac{\partial x_0}{\partial \Phi} \Big/ \frac{\partial y_0}{\partial \Phi}\right)^2}}, \quad y_k = y_0 - \frac{(x_k - x_0)\frac{\partial x_0}{\partial \Phi}}{\frac{\partial y_0}{\partial \Phi}},$$

$$\boldsymbol{x_0 = r_r \sin(\Phi + \Psi_0 + \Psi) - r_a \sin\Phi, \quad y_0 = r_a \cos\Phi - r_r \cos(\Phi + \Psi_0 + \Psi),}$$

$$\boldsymbol{(r_b - r_f)^2 = r_a^2 + r_r^2 - 2 r_a r_b \cos\Psi_0,}$$

$$\sigma = \sqrt{\frac{1}{\pi}\left|\left(\frac{F_n}{s_z}\right)\frac{\frac{1}{r_f} \pm K_k}{\frac{1-\nu_f^2}{E_f} + \frac{1-\nu_k^2}{E_k}}\right|}, \quad K_k = \frac{\frac{d^2 y_k}{d\Phi^2}\frac{dx_k}{d\Phi} - \frac{dy_k}{d\Phi}\frac{d^2 x_k}{d\Phi^2}}{\left(\left(\frac{dx_k}{d\Phi}\right)^2 + \left(\frac{dy_k}{d\Phi}\right)^2\right)^{\frac{3}{2}}}, \qquad (9)$$

$$F_n = \frac{M_K}{r_r \cos\alpha}, \quad \alpha = \Phi + \Psi_0 + \Psi - \pi/2 - \alpha_s,$$

$$\tan\alpha_s = \frac{dy_k}{dx_k} = \frac{dy_k / d\Phi}{dx_k / d\Phi}.$$

***Equation 9.***

$D_3 = D_4 = 229$, $H_2 = H_4 = 48$, $H = 0$; batten oscillation angle $\Psi_A = 24\pi/180$rad; cam working angle $2\beta = 110\pi/180$rad; cam mechanism dimensions [m]: $r_r = 0.11$, $r_a = 0.21$, $r_f = 0.05$, $r_b = 0.1$ (convex cam) and $r_b = 0.15$ (concave cam); cam thickness $s_z = 0.038$; Young's modulus $E = 200{,}000$ MPa; Poisson ratio $\nu = 0.3$. The initial conditions were assumed to be: $t = 0$, $M_0 = 0$, $\Phi_0 = 0$, $d\Phi_0/dt = 0$, $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_3 = 0$, $d\Phi_1/dt = d\Phi_2/dt = d\Phi_3/dt = d\Phi_3/dt = \omega_S$, $\Psi_I = \Psi_2 = \Psi_{II} = \Psi_4$, $d\Psi_I/dt = d\Psi_2/dt = d\Psi_{II}/dt = d\Psi_4/dt$. The torques and stresses shown in Figure 3 were calculated by taking the parameters of the motor characteristic according to equations (5), the motion of the batten according to function (7) and the moments of external forces $M_I = M_{II} = 0$.

## Final remarks

1. The system is very sensitive to the magnitude of the mass of its elements. A small decrease in the mass of the working elements makes it possible to decrease all the masses in the chain of mechanism elements, which in turn leads to a decrease in the forces acting on the driving cam.
2. The inertia forces resulting from the oscillatory motion of the batten cause the speed of driving the cams to fluctuate, while the flywheel attached to the cam shaft has a tendency to rotate at a constant speed. The collision of these two tendencies results in the torsional vibration of the shaft which connects them. In order to avoid these vibrations, the moment of inertia of the masses directly attached to the cam should be increased, while the moment of inertia of the masses attached to the cam by the shaft should be decreased.
3. In order to keep stress at as low a level as possible, the period of natural vibration associated with shaft stiffness should be shorter than the duration of the cam stress pulse.
4. Since the motor's inertia is great, it should drive the camshaft via an elastic belt in order not to oppose the fluctuation of speed of camshaft.

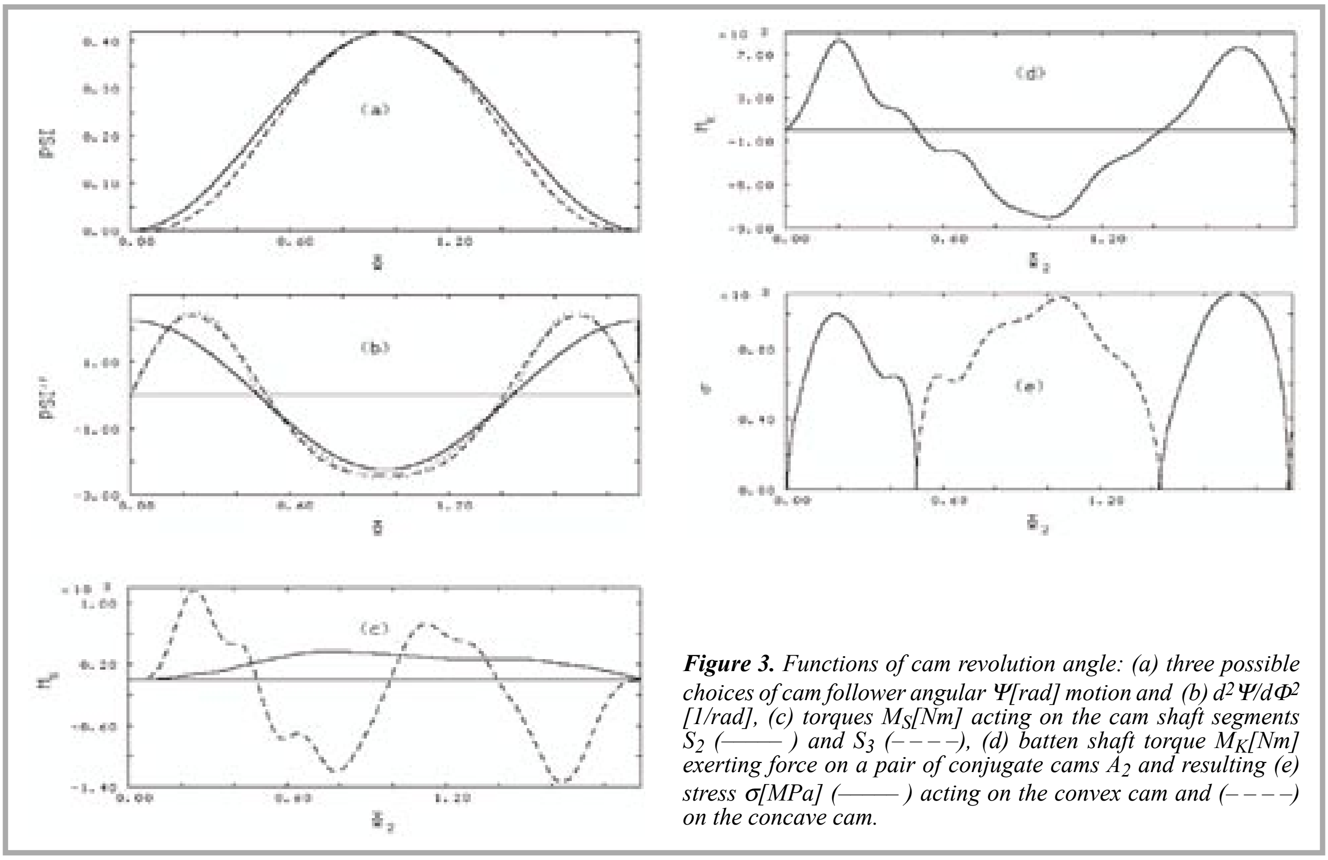


***Figure 3.*** *Functions of cam revolution angle: (a) three possible choices of cam follower angular $\Psi$[rad] motion and (b) $d^2\Psi/d\Phi^2$ [1/rad], (c) torques $M_S$[Nm] acting on the cam shaft segments $S_2$ (——— ) and $S_3$ (– – – –), (d) batten shaft torque $M_K$[Nm] exerting force on a pair of conjugate cams $A_2$ and resulting (e) stress $\sigma$[MPa] (——— ) acting on the convex cam and (– – – –) on the concave cam.*

5. Torsional vibration of the batten shaft is a serious problem. Any increase in the batten shaft diameter results in both the increase of its cross-section inertia and the increase of its mass moment of inertia. As a consequence, the increase of the shaft diameter does not result in any decrease of the amplitude of torsional vibration. In order to decrease those vibrations, the batten shaft should be divided into segments which have a common axis but are driven separately.
6. The batten beam and reed attached to the shaft makes it very unbalanced, which results in bending vibration. however, balancing alone does not solve the problem because, although it decreases the centrifugal forces, at the same time it increases the moment of inertia of the batten, and therefore increases the tangent forces. In order to avoid bending vibration, the batten shaft should be supported on a sufficient amount of properly spaced bearing.
7. In order to carry its load, the reed must move a required distance. This can be achieved either by increasing the length of the arms which carry it, or increasing the angle of oscillation. An optimal solution exists, which must be found by minimising the stress acting on the driving cam. It is noteworthy that placing the cam followers with their rollers opposite to the arms carrying the reed decreases the unbalancing of the shaft.
8. Excessive transverse vibration of the batten beam which carries the reed must be avoided. This can be done either by increasing the beam thickness or decreasing the distance between the arms carrying it. An optimal solution has to be found by minimising the stress acting on cam.
9. The basic criterion which must be considered when designing the driving cam is the stress acting on the cam. The function describing the motion of the cam follower versus the cam revolution angle is composed from segments of elementary functions. At the point where the functions are connected, the continuity of the function and its first derivative must be assured. Any continuity of the second derivative may lead to an increase of the maximum acceleration, and this is not required. It should be noted that equality of the maximum values of acceleration and deceleration does not guarantee the equality of the maximum stresses. The design of the cams should be such that the maximum forces act in the area of the largest radius of the cam curvature, or in places where the cam is concave.
10. The roller should have a barrel shape with the barrel radius properly chosen to compensate for the nonparallel axis of the roller and the cam.
11. To maintain the contact of the roller and cam, two rigidly connected cams should be used, one convex and the other concave. A spring cannot be used for this purpose, because it would requires a great deal of motor power to start the motion of the system.
12. A journal bearing lubricated with oil under proper pressure should be used. Roller bearings are not appropriate, especially for the batten, because it will not rotate in a full angle, and only a few rollers will work.
13. In order to minimize the stress acting on the cam, the dimensions of the triang formed from the segments $r_r$, $r_a$, $r_f$+$r_b$ should be so chosen as to make the angle between $r_r$ and $r_f$+$r_b$ equal to $\pi/2$.

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