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Modelling the Batten Lever Mechanism's Kinematics with a Non-circular Toothed Gear

Abstract
The modelling of the kinematic characteristics of a rocker of the batten lever mechanism by means of an elliptic toothed gear is presented. This gear, introduced between a motor and a crank of the lever mechanism, alters the uniform motion of the crank into variable rotary motion. The modelling of the rocker's kinematic characteristics consists thus in the determination of a directed angle of the crank and the passive pinion of the gear at which the highest increase in the angular acceleration of the rocker occurs. This is one way to increase the weft beating-up forc, and in turn, to improve the working conditions of the loom.

Key words: *modelling, kinematics, lever mechanism, elliptic toothed gear.*

■ Introduction

The kinematic characteristics of lever mechanisms that include links formed by lower pairs characterised by a surface contact are inflexible. A change in the nature of the motion of the link being driven can at this point be achieved by connecting the quadrilateral mechanism with another lever mechanism in which the passive link of the quadrilateral is simultaneously the driving link of the next mechanism (Figure 1). As a result, for instance, an additional extreme position of the rocker is provided in one cycle of the crank motion. It is a new quality of the motion of this rocker, but this motion cannot be changed in a continuous way. A further alternation, for example of the angular acceleration of the passive link (rocker) that consists in an increase or a decrease in extreme values of acceleration, can be obtained by introducing the variable motion of the crank as the driven link. This can be done if such a mechanism is introduced between the driving motor and the crank that will change the former uniform motion of the crank into the variable rotary motion. The mechanism that alters the character of the crank motion, which is the subject of this paper's considerations , is formed by a toothed gear with a variable gear ratio, which consists of two non-circular pinions. An active pinion with a constant angular speed puts a passive pinion, rigidly connected to the lever mechanism crank, into the variable rotary motion. The essence of modelling the rocker motion consists in an angular orientation of the crank with respect to the passive pinion in which the expected effect is achieved. As an example, if the loom batten is the above-mentioned rocker, then an increase in the rocker maximum acceleration in its extreme position (which means, when the weft is beaten up) will contribute to an increase in the weft beating-up force.

The aim of this work is to propose a mechanical solution which would allow the loom's working conditions to be improved, and to present a method which would enable the best parameters of the device used to be selected. This in turn will contribute to better formation of the woven fabrics, especially those characterised by very high weft density.

■ Characteristics of the non-circular gear

The toothed gear that drives the lever mechanism crank is composed of two, non-circular pinions, whose task is to alter the uniform rotary motion of the active pinion into the variable rotary motion of the passive pinion. This motion transformation is achieved by means of non-circular

toothed gears that perform the continuous motion. The shape of the rolling lines of the pinions that form this toothed gear can be various, depending on the expected character of the possible passive link motion and on the technological preferences recognised by the designer. Elliptic gears with axes of revolution in the focuses of ellipses have been known and used for a long time in textile machines. Circular-non-circular gears with a circular active pinion with an axis of revolution displaced with respect to the geometrical centre and a non-circular passive pinion represent another solution [2]. Both the above- mentioned types of gears share the same disadvantage, namely that the axis of revolution of the pinion is displaced with respect to the geometrical centre. This fact is followed by some consequences of a dynamic character that limit any
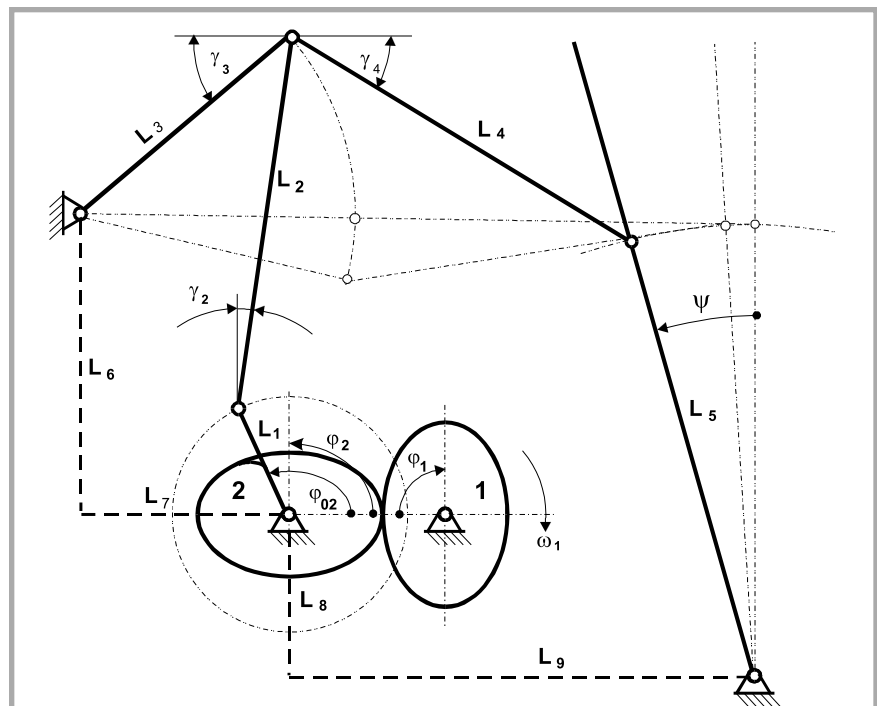

Figure 1. *Schematic view of the toothed-lever mechanism with a double swing of the link L_5.*
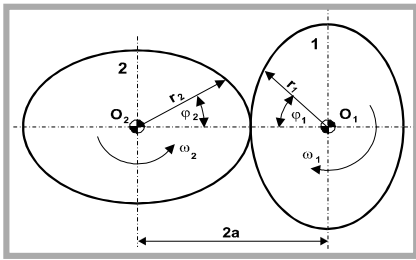
Figure 2. *Rolling lines of the non-circular elliptic toothed gear under investigation.*

possibility of employing higher rotational speeds to a great extent. Regarding the elliptic gear, the rolling lines (ellipses) are described by polar coordinates in the following form:

$$r = \frac{p}{1 + e\cos\varphi} \qquad (1)$$

As follows from [1], in order to obtain the rolling lines of non-circular pinions with the axes of revolution in geometrical centres, it is enough to increase the argument of the cosine function by k times, in equation (1).

Thus, a following family of rolling lines, described by the equation:

$$r = \frac{p}{1 + e\cos k\varphi} \qquad (2)$$

has been obtained, where $k \in N$. The pinions, whose rolling lines are described by equation (2), have been referred to as elliptic pinions. These lines are obtained by marking the values of the radius r for the angle $k\varphi$ from equation (2) on the angle φ. If the ellipsis which is the basis for drawing an elliptical geometrical figure has the axes 2a and 2b, then its focal distance $c = \sqrt{a^2 - b^2}$, parameter $p = b^2/a$ and the ellipsis eccentricity $e = c/a$. The above-mentioned quantities are the basis for forming k-periodic figures. When $k=2$ is assumed, then the closed figure, a so-called two-period 'ellipsis', is obtained. The point that was the focus of the original ellipsis, after the transformation according to relationship (2), becomes the geometrical centre of the rolling line formed in this way. Two identical pinions, whose rolling lines are the above-described curves, rotated by the angle $\pi/2$ with respect to each other, give an elliptic toothed gear (Figure 2).

A constant distance between the axes of revolution of the pinions is equal to 2a, and is identical as in the elliptic gear formed from the original ellipses, with the axes of revolution in focuses. From the condition of the pinions rolling on each

other without sliding, the gear ratio function has been obtained, namely:

$$i_{1,2} = \frac{d\varphi_1}{d\varphi_2} = \frac{r_2}{r_1} \qquad (3)$$

Substituting $k = 2$ in (2), we obtain:

$$r_1 = \frac{p}{1 + e\cos 2\varphi_1} \qquad (4)$$

Next, if we employ the geometrical properties of the toothed gear used (Figure 2): $2a = r_1(\varphi_1) + r_2(\varphi_1)$, after the transformation of (3), we obtain:

$$\varphi_2 = \int_0^{\varphi_1} \frac{r_1}{2a - r_1} d\varphi_1 = \int_0^{\varphi_1} \frac{1 - e^2}{1 + 2e\cos 2\varphi_1 + e^2} d\varphi_1 \qquad (5)$$

and

$$i_{1,2} = \frac{1 + 2e\cos 2\varphi_1 + e^2}{1 - e^2} \qquad (6)$$

■ Equations of the mechanism motion

In the present paper, the lever mechanism whose schematic view is shown in Figure 1 has been analysed. This is a crank-two-rocker drive of the batten in a loom for manufacturing woven fabrics of very high density. The mechanism has been supplemented with a toothed gear with a variable gear ratio. The crank L_1, fixed rigidly to the passive pinion of the driving elliptic toothed gear, is the active pinion. We obtain two swings of the batten (rocker L_5), whose magnitudes depend on the position of two quadrilaterals with articulated joints that form this mechanism with respect to each other, per one full rotation of the crank with a variable speed. The present investigations are aimed at determining the influence of variable speed of the crank on the angular distance ψ, speed $d\psi/dt$ and acceleration $d^2\psi/dt^2$ of the rocker L_5 versus time,

as well as at analysing their maximum values with respect to the constant speed of the crank.

In the calculations performed, the mechanism was replaced by a closed polygon of vectors, as in Figure 3.

The condition of equilibrium demands that the resultant of these vectors is zero. It is fulfilled if the sums of component vectors are equal to zero. As a result, the equations of equilibrium for the polygons of vectors from Figure 3 give a set of nonlinear algebraic equations (7) in the form:

$F_1 = -L_1\sin(\varphi_2-\pi/2) + L_2\sin\gamma_2 - L_3\cos\gamma_3 + L_7 = 0$,
$F_2 = L_1\cos(\varphi_2-\pi/2) + L_2\cos\gamma_2 - L_3\sin\gamma_3 - L_6 = 0$,
$F_3 = -L_5\sin\psi + L_4\cos\gamma_4 - L_3\cos\gamma_3 + L_7 + L_9 = 0$,
$F_4 = L_5\cos\psi + L_4\sin\gamma_4 - L_3\sin\gamma_3 + L_6 + L_8 = 0$. $\qquad (7)$

From the conditions of cooperation of gear pinions, we obtain:

$$\varphi_2 = \varphi_{02} + \int_0^{\varphi_1} \frac{r_1}{2a - r_1} d\varphi_1 \qquad (8)$$

where: φ_{02} is the angle that determines the position of the crank with respect to pinion 2.

The set of equations (7) has been treated as the vector function $y = F(x) = 0$ of the variable vector $x = (\gamma_2, \gamma_3, \gamma_4, \psi)$, whose components $(\gamma_2, \gamma_3, \gamma_4, \psi)$ are unknown. Assigning the angle φ_1 as an independent variable and assigning the angle φ_2 from relationship (8), the set of equations (7) has been solved by employing the Newton method. The result of calculations has been improved by means of the Brent
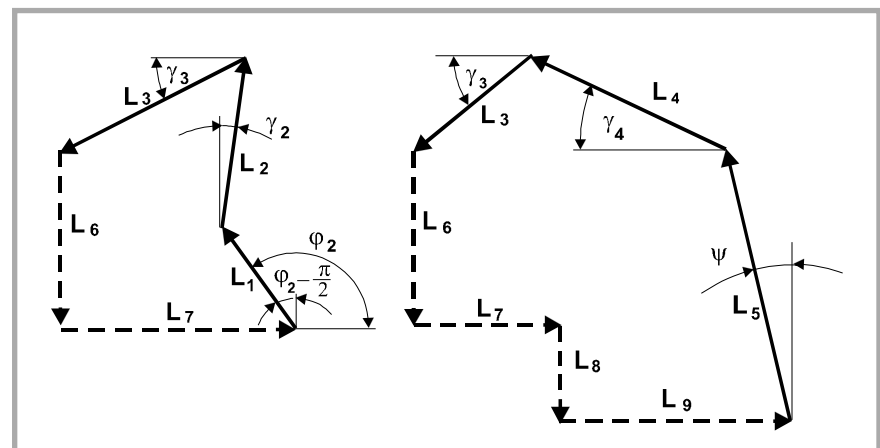

Figure 3. *Polygons of vectors*

algorithm [3]. As a solution, the function $\psi = \psi(\varphi_1)$ sought has been obtained in a discrete form. The discrete function $\psi = \psi(\varphi_1)$ has been replaced by a continuous function in the form of a Lagrange polynomial [4], and the derivatives $d\psi/dt$ and $d^2\psi/dt^2$ have been calculated.

Results of the numerical computations

By inserting an elliptic gear in the rocker driving mechanism of a loom, we obtain a new mechanism. In the mechanism thus obtained, the kinematic functions of the passive pinion 2 are superimposed on the functions of the rocker L_5 (see Figure 2). In the elaboration presented here, a mathematical model in the form of two equations describing the kinematics of the model was developed. The computer solution of the model simulates the kinematics of the real device, and is a tool which can be used for the formation and improving of the parameters characterising the properties of the machine developed by this procedure. The effectiveness of the algorithms presented was proved by the solution of the modelling mechanism, whose geometry was accepted by taking the assumed character of the rocker motion into consideration, and fulfilling the geometrical conditions between the lengths of the links of the crank-and-rocker mechanism. Based on our considerations, the following dimensions have been assumed in our calculations: the link lengths of $L_1 = 0.04$ m, $L_2 = 0.09$ m, $L_3 = 0.08$ m, $L_4 = 0.09$ m, $L_5 = 0.17$ m, $L_6 = 0.1$ m, $L_7 = 0.06$ m, $L_8 = 0.05$ m, $L_9 = 0.11$ m; and the gear parameters $a = 0.05$ m, $e = 0$ and $e = 0.2$, as well as an angular speed of $\omega_1 = 1$ rad/s.

The analysis of the rocker's kinematics was performed by changing the two following parameters:

- the angle φ_{02}, which determines the position the maximum rocker acceleration on the curve, and
- the eccentricity e of the original ellipsis which is decisive for the extreme values of the kinematic parameter.

For $e = 0$, the elliptic gear becomes a circular toothed gear with a constant gear ratio of $i_{1,2} = 1$, which does not alter the nature of the crank motion. In this case, an optimisation cannot be carried out, as the toothed gear inserted is neutral regarding changes to the kinematics. For $e > 0$, the elliptic pinions form the toothed gear which is inserted to the crank driving system. The superposition of the crank-and-rocker mechanism's kinematics with the kinematics of the elliptic passive pinion may create very different functions of the rocker parameters. They depend on the angle φ_{02}, which is decisive for the crank's position in relation to the pinion 2. In our elaboration, the optimisation comes down to searching for such a value of the angle φ_{02} at which the resulting kinematic parameters of the rocker achieve extreme values. As the result of comparing the characteristics for different φ_{02} values in the accepted co-ordination system, it was stated that for $\varphi_{02} = 1.5\pi$, the maximum amplification of the rocker's kinematic parameters occur ??at/in the gear with an established
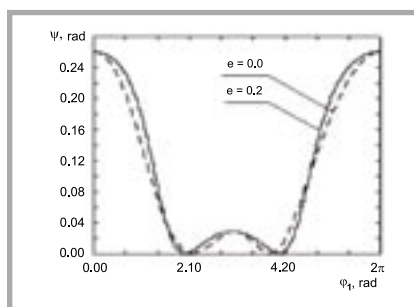

Figure 4. *Plot of the angular distance $\psi = \psi(\varphi_1)$, $e=0 \rightarrow \omega_2 = \omega_1 = const.$, $e=0.2 \rightarrow \omega_2 = \omega_1/i_{1,2}$.*

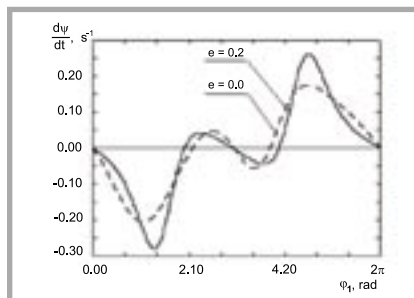**Figure 5.** *Plot of the angular speed $d\psi/dt = f(\varphi_1)$, $e=0 \rightarrow \omega_2 = \omega_1 = const.$, $e=0.2 \rightarrow \omega_2 = \omega_1/i_{1,2}$.*
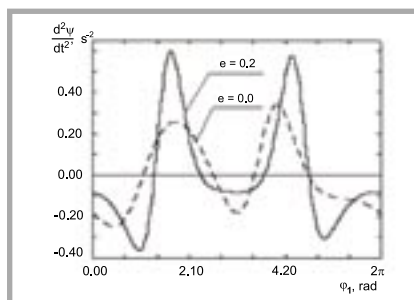

Figure 6. *Plot of the angular acceleration $d\psi^2/dt^2 = f(\varphi_1)$, $e=0 \rightarrow \omega_2 = \omega_1 = const.$, $e=0.2 \rightarrow \omega_2 = \omega_1/i_{1,2}$.*

eccentricity of $e = 0.2$. In the problem discussed in this paper, as optimal functions (runs) are considered as a kinematic function, which as the final effect results in maximum extreme values of the velocity and acceleration of the rocker, thanks to the amplification of the rocker driving mechanism by using an elliptic gear, and by changing the angle φ_{02}. The optimal functions obtained by the kinematic analysis of the rocker motion of the mechanism modelled for the above-discussed case are presented in Figures 4, 5, and 6.

Conclusions

1. The elliptic toothed gear used to drive the lever mechanism crank was enabled at k = 2, to bring the axes of revolution of the pinions to the geometrical centre, and thus to omit the barriers which result from the displacement of the centre of mass with respect to the geometrical centre.
2. The lever mechanism presented, which includes a non-circular toothed gear, allows for the weft beating-up force to be modelled. This is an offer for the designers of looms that are used in the manufacture of heavy woven fabrics.
3. Bearing in mind the achievement of maximum accelerations in the extreme position of the link L_5, the value of the crank angular orientation φ_{02} is established. The kinematic characteristics obtained for the rocker allow us to state that the increment in the rocker angular acceleration maximum is equal to 70% for $\varphi_{02} = 1.5\pi$ and for $e=0.2$. ❑

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