Spectra of Reciprocal Transmission: Generalization to Saturated Regions

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ABSTRACT

The results of Munk *et al.* (1981), relating spectra of ocean sound-speed and water-particle velocity to observable acoustic spectra are applicable only in the geometrical optics region. It is shown that analogous formulas that are valid in the saturated regions can be written if expressions involving the phase spectra are replaced with appropriate expressions involving coherence functions of the acoustic wave function.

The use of reciprocal-transmission experiments to measure covariances of water-particle velocity and cross-covariances between vertical displacement of internal waves and the associated water particle velocities can be an important step toward understanding fluxes of energy and momentum in the ocean. Munk *et al.* (1981, hereafter MWZ) have shown that measurements of the phases $\phi_{\pm}(t)$ and log-intensities can be related to spectra of internal-wave displacement and particle velocity. The phases, ϕ_+ and ϕ_-, refer to travel times multiplied by acoustic center frequency, with the subscripts denoting travel in the positive or negative x direction. Their results are valid only in the geometrical-optics region, as explained in Flatté *et al.* (1979). Only in the geometrical optic region is received phase related to travel time in a direct way.

All of the results of MWZ with regard to phase spectra can be expressed compactly in terms of the phase-structure function described in Flatté *et al.* (1979), and in more detail in Esswein and Flatté (1980)

$$D(2,1) \equiv \left\langle \left(q \int_2 \mu ds - q \int_1 \mu ds \right)^2 \right\rangle, \quad (1)$$

where the integrals are taken over two different paths. These paths might differ in time, space, frequency, or any other property. In the case of reciprocal transmission, the two paths differ also in their direction, with the quantity μ differing on the two paths because of water-particle velocity. By use of this general notation, the acoustic phase behavior in the geometrical optics region is related to the phase-structure function by

$$\langle [\phi(2) - \phi(1)]^2 \rangle = D(2,1). \quad (2)$$

If one specifies path (2) to be in the positive x direction at time t, and path (1) to be in the negative x direction at time zero, then all results of MWZ will follow. In fact, MWZ implicitly assumed (2), and did all of their derivations on the phase-structure function defined in (1). For example,

$$\left\langle \left[\phi_+(t) - \phi_-(t) \right]^2 \right\rangle = D(t_+, 0_-)$$

$$= \left\langle \left[q \int \mu_+(t) ds - q \int \mu_-(0) ds \right]^2 \right\rangle, \quad (3)$$

and the latter expression is just the one MWZ manipulate using their Eq. (4).

Suppose the transmission is in the saturated region, where the phase and travel time are not simply related, due to the presence of micromultipath? This case has been extensively discussed in Flatté *et al* (1979) and also Dashen (1979). The approximate result of general validity is

$$\langle \psi^*(2)\psi(1) \rangle = \exp[-\tfrac{1}{2}D(2,1)]. \quad (4)$$

Thus an acoustic-transmission experiment in the saturated region should relate the phase-structure function to the coherence of the complex wave function, ψ, rather than to the phase of the received signal. As an example, consider the paths to be identified as described above. Then (4) can be manipulated to

$$-2 \ln\langle \psi_+^*(t)\psi_-(0) \rangle = D(t_+, 0_-). \quad (5)$$

The spectra of the logarithmic function on the left can then be directly related to the expressions from MWZ. For example, the quad spectrum would bear

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the same relation to the vertical flux of horizontal momentum as the quad spectrum of $\langle \phi_+(t)\phi_-(0)\rangle$ did in the geometrical-optics region.

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