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RESEARCH NOTES

Pressure Drop of Non-Newtonian Liquid Flow Through Elbows

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Abstract Experimental data on the pressure drop across different types of elbow for non-Newtonian pseudoplastic liquid flow in laminar condition have been presented. A generalized correlation has been developed for predicting the frictional pressure drop across the elbows in the horizontal plane.

Keywords non-Newtonian liquid, pseudoplastic, elbow

1 INTRODUCTION

The flow through the curved geometry is very complex in nature due to the action of centrifugal forces which cause secondary flow to develop and there are very few experimental data available in literature. Edwards *et al.*[1] and Das *et al.*[2,3] reported experimental studies of non- Newtonian liquid flow through various piping components and empirical correlation were suggested for individual piping components. Das[4] reviewed the flow through various piping components. However, data or equations for pressure losses through piping components for non-Newtonian liquids are meager and the present study is an attempt to generate experimental data on pressure drop through elbows and to develop a generalised correlation for non-Newtonian liquid in laminar flow through elbows.

2 EXPERIMENTAL SET-UP AND TECHNIQUE

The schematic diagram of the experimental apparatus is shown in Fig. 1. The experimental set-up consists of a liquid storage tank (0.45 m^3), a test section, flow rate and pressure measuring devices *etc.* For all the elbows which were tested the upstream portion was identical but the downstream portion and the separator were shifted as necessary.

Four aqueous solutions of sodium salt of carboxymethyl cellulose (SCMC) of concentrations 0.2 to 0.8 kg·m^{-3} were used as the non-Newtonian liquid. Biological degradation was prevented by adding a trace amount of formalin. The contents of the tank were kept at a constant temperature by circulating water through a copper coil.

The test section consisted of a horizontal upstream straight tube 1.2 m long, a elbow and a horizontal downstream straight tube 1.15 m long. The test section was fabricated from a mild steel tube of 0.0127 m internal diameter. The main reason of the long horizontal upstream and downstream portions before and after the elbow is to achieve fully developed flow conditions before the liquid reaches the elbow. The pressure drop across the elbow was obtained from the difference between the static pressure of the upstream, fully developed flow region and the static pressure of the downstream, fully developed flow region across the elbow. The test section was provided with pressure taps (piezometric ring) at different points in the upstream/downstream sections of the pipe. The static pressures at different points were measured by means of U-tube manometers containing mercury beneath water. Three different types of elbows, *i.e.*, 45° to 135°. were used; their radii of curvature and linear length are given in Table 1. The elbows were specially manufactured to ensure uniform internal diameter, constant curvature and roundness.

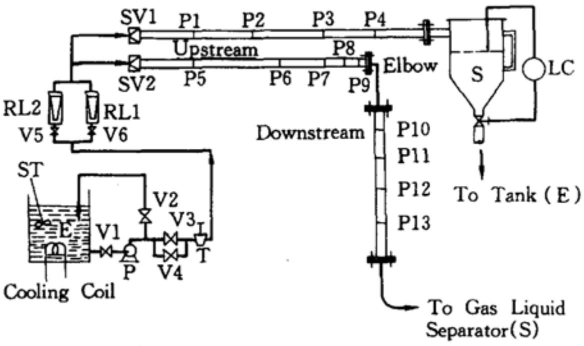

Figure 1 Schematic diagram of the experimental setup

E—tank; P—pump; S—separator; P1—P13—manometer tappings; ST—stirrer; V1—V6—valves; LC—level controller; RL1,RL2—rotameters; SV1,SV2—solenoid valves

Table 1 Dimension of the elbow

Elbow angle θ, (°)	Radius of curvature R_c, m	Linear length of elbow L_{eb}, m
45	0.011	0.014
90	0.022	0.011
135	0.017	0.016

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In the actual experiment, first the liquid entered the horizontal tube of the same diameter and material of construction as the elbow. After taking observations on straight tube, the solenoid valve SV_1 was closed; simultaneously the solenoid valve SV_2 was opened and the same liquid flow was passed through the test section through an entrance length of 1.5 m which was more than 50 pipe diameter needed to ensure fully developed flow in the test section. Under the steady state condition the liquid flow rate was noted from the rotameters; the readings of the manometers attached to the tapping were also noted. The liquid flow rate used in the experiments varies in the range from 3.75×10^{-5}—$21.94 \times 10^{-5}\,\mathrm{m^3{\cdot}s^{-1}}$. In this range of flow rates, only laminar flow conditions were observed. The temperature of the liquid used in the experiments were maintained at (31.0 ± 1.5)℃, *i.e.* ambient temperature.

Rheological and physical properties of the test liquids are given in Table 2. The dilute SCMC solutions display shear-thinning behaviour in the shear rate range 32 to $950\,\mathrm{s^{-1}}$ and follow the Power law model. Calculations are carried out on the basis of the effective viscosity, μ_{eff}, which is given as

$$\mu_{\mathrm{eff}} = K' \left[\frac{8V}{D}\right]^{n'-1} \tag{1}$$

Table 2 Physical properties of the SCMC solutions

Concentration $\mathrm{kg{\cdot}m^{-3}}$	Flow behaviour index n'	Consistency index K', $\mathrm{N{\cdot}s^{n'}{\cdot}m^{-2}}$	Density ρ, $\mathrm{kg{\cdot}m^{-3}}$
0.2	0.9013	0.0142	1001.69
0.4	0.7443	0.1222	1002.13
0.6	0.6605	0.3416	1002.37
0.8	0.6015	0.7112	1003.83

3 RESULTS AND DISCUSSION

3.1 Effect of non-Newtonian characteristics on pressure drop across the elbow

Fig. 2 shows the pressure drop across the 135° elbow as a function of liquid flow rate. It is clear from the graph that as n' decreases pressure drop increases at constant liquid flow rate.

3.2 Analysis of data

Initially pressure drop was measured for a straight horizontal tube and the results were found to be in close agreement (within ±5%) with the conventional resistance formula applied for non-Newtonian liquid flow through a straight pipe in laminar flow condition, *i.e.*

$$f_{\mathrm{s}} = 16/Re \tag{2}$$

which signifies the accuracy of the experimental procedure and technique.

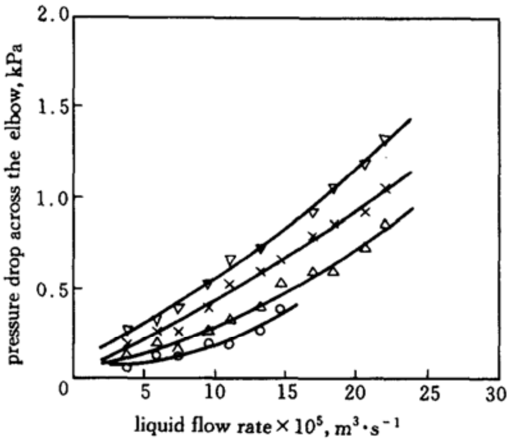

Figure 2 Variation of pressure drop across 135° elbow with liquid flow rate

Concentration of SCMC, $\mathrm{kg{\cdot}m^{-3}}$: ○ 0.2; △ 0.4; × 0.6; ▽ 0.8

A dimensional analysis of the non-Newtonian liquid flow through elbows suggests the following functional relation

$$f_{\mathrm{eb}} = F(Re, R_{\mathrm{c}}/R_{\mathrm{t}}) \tag{3}$$

or

$$f_{\mathrm{eb}} = F(De) \tag{4}$$

Where, De, Dean number really plays the same role as the Reynolds number does in straight pipe. In order to extend the applicability of Eq. (4) to all the different elbows in the horizontal tube, an angle factor has been introduced in functional relationship as follows

$$f_{\mathrm{eb}} = F(De, \alpha/135) \tag{5}$$

In the limiting case when $R_{\mathrm{c}} \to \infty$, *i.e.*, when the elbow becomes straight, the friction factor f_{eb} given by Eq. (5) should be the friction factor f_{s} in a straight pipe. To incorporate this limiting condition Eq. (5) has been modified as follows

$$\frac{f_{\mathrm{eb}}}{f_{\mathrm{s}}} - 1 = F(De, \alpha/135) \tag{6}$$

The functional relationship developed using multivariable linear regression analysis as follows

$$\frac{f_{\mathrm{eb}}}{f_{\mathrm{s}}} - 1 = 7.94 \times 10^{-2} De^{0.718\pm0.082}\left(\frac{\alpha}{135}\right)^{-0.520\pm0.160} \tag{7}$$

for 40< Re <2000; 30< De <2150 and 45° $< \alpha <$ 135°.

The values of $(f_{\mathrm{eb}}/f_{\mathrm{s}} - 1)$ predicted by Eq. (7) have been plotted against the experimental values as shown in Fig. 3. The correlation coefficient and the variance of estimate are 0.8845 and 0.146 respectively. The value of t is 1.98 obtained from the statistical table[5] for 108 degrees of freedom, 0.05 probability level and 95% confidence range.

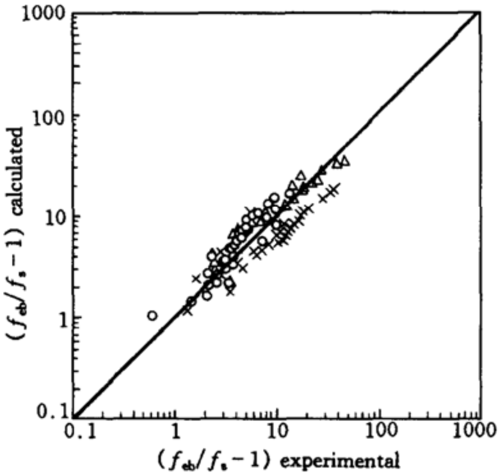

Figure 3 Correlation plot
Elbow angle: △ 45°; × 90°; ○ 135°

3.3 Streamwise pressure loss due to elbow

Observations of pressure were made in the long upstream and long downstream portion of the elbow in order to obtain the overall pressure drop across the elbow. The static pressure starts to deviate from the steady value within 5 pipe diameter of the inlet of the elbow and the pressure recovery lengths were also found to be within 5 pipe diameter.

NOMENCLATURE

D	diameter, m
De	Dean number, $De = V\rho D/\mu_{\text{eff}}(R_t/R_c)^{0.5}$
F	function
f	friction factor, $f = \Delta P_{eb}/(2V^2\rho L_{eb})$
K'	consistency index, $\text{N}\cdot\text{s}^{n'}\cdot\text{m}^{-2}$
L	length, m
n'	flow behaviour index
Δp	pressure drop, $\text{N}\cdot\text{m}^{-2}$
R	radius, m
Re	Reynolds number, $Re = V_\rho D/\mu_{\text{eff}}$
V	velocity, $\text{m}\cdot\text{s}^{-1}$
α	angle, (°)
μ	viscosity, $\text{N}\cdot\text{s}\cdot\text{m}^{-2}$
ρ	density, $\text{kg}\cdot\text{m}^{-3}$

Subscripts

c	curvature
s	straight pipe
t	tube
eb	elbow
eff	effective

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