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Longitudinal dispersion in natural channels: 3. An aggregated dead zone model applied to the River Severn, U.K.

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Abstract

An Aggregated Dead Zone (ADZ) model is presented for longitudinal dispersion of tracer in river channels, in which the channel cross-section is divided into two parallel regions: the bulk flow and dead zone storage. Tracer particles in the bulk flow are assumed to obey plug-flow advection at the discharge velocity U without any mixing effects. The dispersive properties of the model are completely embodied in the residence time for tracer storage in the dead zone. The model provides an excellent description and prediction of empirical concentration-time distributions, for times $t > x/U$. Its physical realism is demonstrated by using it to describe the evolution of a tracer cloud in the River Severn, U.K., and by comparing it with a more complex model which incorporates the additional effects of shear flow dispersion within the bulk flow. The ADZ model is a potentially useful tool for practical prediction of dispersion in natural channels.

Keywords: Channels; dispersion; dead zones; tracers; River Severn

An Aggregated Dead Zone model of dispersion in natural channels

The relative roles of shear-dispersion and dead zones as mechanisms causing longitudinal dispersion in natural channels were considered by Davis *et al.* (2000). They show that shear-dispersion is unimportant compared to tracer retention in dead zones except during and shortly after an initial period during which cross-sectional mixing is established over the bulk flow region. Indeed, it seemed from their study of empirical data from the River Severn, U.K., that the later evolution of a tracer cloud might be predicted accurately by excluding the shear-dispersion term from the governing partial differential equations. This paper tests this idea by deriving firstly an analytical expression for longitudinal dispersion by dead zone storage processes alone, and then fitting it to the data from the River Severn presented by Atkinson and Davis (2000).

Model formulation

An Aggregated Dead Zone (ADZ) model may be defined by considering the river channel to be divided into two interconnected regions – a central core of moving water, termed the bulk flow region, and dead zones consisting of stationary water around the perimeter of the channel (Davis *et al.*, 2000). Both regions are considered to be internally well mixed at any one cross-section. Tracer particles may be exchanged between the two regions at a rate proportional to the difference in concentrations between them. Thus, the following coupled partial differential equations will apply,

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} - K\frac{\partial^2 C}{\partial x^2} = -\frac{1}{\chi^2}\frac{\partial C_s}{\partial t} \tag{1}$$

for tracer in the bulk flow and

$$\frac{\partial C_s}{\partial t} = \frac{\chi^2}{\tau}(C - C_s) \tag{2}$$

for tracer in the dead zones. Symbols are defined in the list at the end of this paper and in Davis *et al.* (2000). Note that the third term in Eqn. (1) expresses the effects of turbulent shear on dispersion in the bulk flow via the shear-dispersion coefficient K, as originally suggested by Taylor (1954). A pure ADZ model represents the limiting case as K tends to zero, i.e.

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = -\frac{1}{\tau}(C - C_s) \tag{3}$$

Previous work

Equations similar to (3) have been applied to various processes that are conceptually similar to the mass transport problem in natural channels. In chemical engineering such models have been termed 'continuously stirred tank reactor models' or 'cell models' (MacMullin and Weber, 1935; Danckwerts, 1953; Aris and Amundson, 1957; Popovic and Deckwer, 1976). A second application has been in mass transport studies in porous media such as the capacitance models developed by Deans (1963) and Coats and Smith (1964). A third is in modeling of dissolved oxygen and biochemical oxygen demand in rivers, e.g. Young and Beck (1974), Beck and Young (1975). Solutions of (3) for a variety of boundary conditions were studied by Goldstein (1952a, b) and applied to transport of sorbing solutes in porous media by Carnahan and Remer (1984).

The earliest contribution to foreshadow the dead zone concept as the sole mechanism causing longitudinal dispersion in natural channels was by Banks (1974) who derived an analytical solution to a 'cells-in-series' model. Subsequently, Stefan and Demetracopoulos (1981) and Beltaos (1982) have considered discrete-time models with a 'cells-in-series' approach. In these models, the river reach is divided into a number of mixing cells of equal volume, each obeying differential equations of the form (2). The output from an upstream cell then forms the cascaded input for the next cell. Stefan and Demetracopoulos (1981) report that this model "reproduces bulk features of the mass transport (mean travel time and variance) without necessarily particular success in the prediction of instantaneous concentrations", when applied to a 17.2 km reach of the Upper Mississippi River. The most important subsequent contributions are by Beer and Young (1983) and Wallis *et al.* (1989) who formulated a discrete-time, aggregated dead zone model and applied it with considerable success to tracer data from four short reaches (100–150 m) in rivers in north-east England. They used as an initial condition the measured concentrations as a tracer cloud entered the upstream end of a test reach, and fitted the aggregated dead zone model to the cloud leaving the test reach. In all four reaches, a first-order model structure (i.e. a single aggregated dead zone element) was found to be adequate. A more theoretical approach was taken by Smith (1981, 1987) who derived an ADZ model from detailed hydrodynamics with the minor change that the dead zone storage region has a non-zero velocity.

Model solution

Consider an instantaneous, well-mixed injection of tracer into a channel providing pure advection (plug-flow) in the bulk flow region combined with storage and release of tracer from dead zones. For this situation, Eqn. (3) may be solved rapidly by considering the appropriate solution to Eqns. (1) and (2), then setting $K = 0$. Following Davis *et al.* (2000), the solution to Eqns. (1) and (2) for boundary conditions appropriate to an instantaneous injection of mass M of conservative tracer at $x = 0$, $t = 0$ is,

$$C = C_T(x,t)e^{-t/\tau} + e^{-\chi^2 t/\tau} \int_0^t C_T(x,v).e^{\chi^2 v/\tau}.\frac{\chi}{\tau}.e^{-U/\tau} .\frac{\sqrt{v}}{\sqrt{t-v}}.I_1\left[2\frac{\chi}{\tau}\sqrt{v(t-v)}\right].dv \tag{3a}$$

where I_1 is a modified Bessel function of first order and first kind, and

$$C_T(x,t) = \frac{M}{2A(\pi K t)^{1/2}} \exp\left[-\frac{(x-Ut)^2}{4Kt}\right] \tag{3b}$$

To derive the solution for a pure ADZ model from this result, note that as K tends to zero, from (3b):

$$\underset{K\to 0}{Lim}[C_T(x,t)] = \frac{1}{U}\delta(t - x/U) \tag{4}$$

Applying (4) to (3a) together with

$$\int_0^t \delta(v - x/U)f(v).dv = H(t - x/U)f(x/U) \tag{5}$$

then:

$$C(x,t) = \frac{M}{AU}\delta(t - x/U).e^{-t/\tau} + \frac{M}{AU}H(t - x/U).e^{-\frac{\chi^2}{\tau}(t - x/U)}.e^{-\frac{x}{U\tau}}.\frac{\chi}{\tau} .\left(\frac{x/U}{t - x/U}\right)^{1/2}.I_1\left[2\frac{\chi}{\tau}\sqrt{\frac{x}{U}(t - x/U)}\right] \tag{6}$$

Equation (6) describes the evolution of a tracer cloud from an instantaneous well-mixed injection at $x = 0$, $t = 0$, provided that the tracer is subject only to plug-flow along the stream, plus first-order exchange with well-mixed dead zones. An alternative derivation of Eqn. (6) using a Laplace transform approach is given by Davis (1991).

Properties of the ADZ model

The right hand side of Eqn. (6) consists of two terms. The first represents a Dirac delta function that travels downstream at the discharge velocity, U. The mass in this term decays as a function of longitudinal distance and time according to $e^{-t/\tau}$ as tracer is exchanged with the dead zones. The second term describes tracer that has returned to the bulk flow after storage in the dead zones. Dispersion arises from the exponential distribution of residence time for tracer in dead zones, which accords to $e^{-\chi^2 t/\tau}$. This term represents mass that has been 'stripped off' the Dirac delta function as the latter is advected downstream. The first term approaches zero for large times and distances from the point of tracer injection and the second term then becomes the asymptotic solution to the ADZ model.

The Heaviside function $H(t-x/U)$ is a step function that has a value of zero for $t < x/U$ and a value of 1 for $t > x/U$. This is the arrival time at x of the Dirac delta function which is advected downstream with velocity U. Thus, the ADZ model predicts zero concentrations for $t < x/U$ and an indeterminate concentration for $t = x/U$. In the model, no tracer particles can arrive ahead of the $\delta(t - x/U)$ function. Thus, Eqn. (6) is useful in practice only for the later stages of cloud evolution, when almost all of the tracer particles have passed through a dead zone at least once and are, therefore, travelling more slowly than the rate of advection in the bulk flow. The model expresses this situation for times $t >> \tau$, when the coefficient of the first term approaches zero. The fraction of tracer mass accounted for by the second term is given by $(1 - e^{-x/U\tau})$. Figure 1 shows how this fraction increases with the distance that the leading edge of the cloud has been advected downstream, and asymptotically approaches unity, for parameter values of U,χ and τ which are discussed later.

The effect of variations in χ and τ on the ADZ model (strictly speaking the second term only) is illustrated in Fig. 2 for arbitrary values of $U = 1$ m s^{-1} and $A = 10$ m^2. Figure 2a shows the effect of variations in τ from 1000 s to 5000 s with $\chi = 1$. The effect of increasing τ is seen to be:

(i) To attenuate the peak concentration.
(ii) To increase the variance and time base of the distributions.
(iii) To increase the skewness of the distributions.

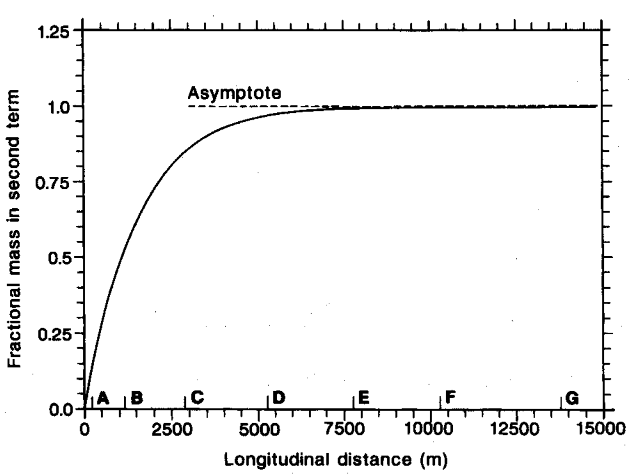

Fig. 1. *The fraction of tracer mass accounted for by the second term of the ADZ model, with appropriate values for the River Severn of* U = *0.66 m.s*$^{-1}$ *and* τ = *2300 s, plotted against longitudinal distance. The locations of the sampling stations A to G are shown.*

(iv) To lessen the time to the peak concentration.

The enhanced retentive effects of an increased residence time of storage (τ/χ^2) account for observations (i), (ii) and (iii). The parameter τ is a characteristic time for exchange of tracer between the bulk flow and the dead zones and hence governs the ratio of the first and second terms. A second effect of an increase in τ is therefore to lengthen the time before tracer particles exchange with the dead zones. With lower values of τ tracer particles are stripped off the $\delta(t - u/U)$ function within a relatively short time and consequently more of the injected mass is accounted for by the second term. With higher values of τ, particles initially spend more time being advected downstream in the bulk flow before entering dead zone storage for the first time. Consequently, the time to the peak concentration is less for higher values of τ, accounting for observation (iv) above.

The asymptotic condition in which the first term has declined to zero is achieved for $\tau = 1000$ s. Figures 2b and 2c show that the influence of τ remains much the same regardless of the value of χ, but that the latter parameter strongly influences the time-to-peak in the asymptotic case.

Figure 2d shows the effects of varying χ from 1 to 5 with τ set to 1000 s. The output concentration distributions are much more sensitive to changes in χ than to τ. The effect of decreasing χ from 5 to 1, equivalent to increasing the effective cross-sectional area of dead zone storage from 4% to 100% of the value for the bulk flow, is seen:

(v) to attenuate rapidly to the peak concentration.
(vi) to increase rapidly the variance and the time-base of the concentration distributions.
(vii) to lengthen the time to the peak concentration.

These features are explained by the increase in residence time in storage for lower values of χ.

Figures 2e and 2f show the same variations of χ for constant values of τ of 3000 s and 5000 s respectively. These solutions are sub-asymptotic, but illustrate the ability of dead zone storage to produce pronounced tailing of the cloud behind the δ-function spike (which is not shown but lies at $t = 12{,}000$ s).

Once the asymptotic condition has been reached, the average particle resides for a period τ in the bulk flow between leaving and re-entering dead zones. Average residence time within a dead zone is τ/χ^2, so each particle spends a proportion $\chi^2/(\chi^2 + 1)$ of time in deadzones with zero streamwise velocity. Thus, the nett migration rate of the cloud centroid will be $U\chi^2/(\chi^2 + 1)$ during the asymptotic period. The effects of χ on retardation can be seen in Fig. 2d in which the time-to-peak varies from about 4% more than the advection time for $\chi = 5$ up to almost twice as much for $\chi = 1$. The overall retardation of the peak will be less than the asymptotic value for so long as the pre-asymptotic period remains a significant fraction of the total time elapsed, i.e. for $t < \sim 12\,\tau$.

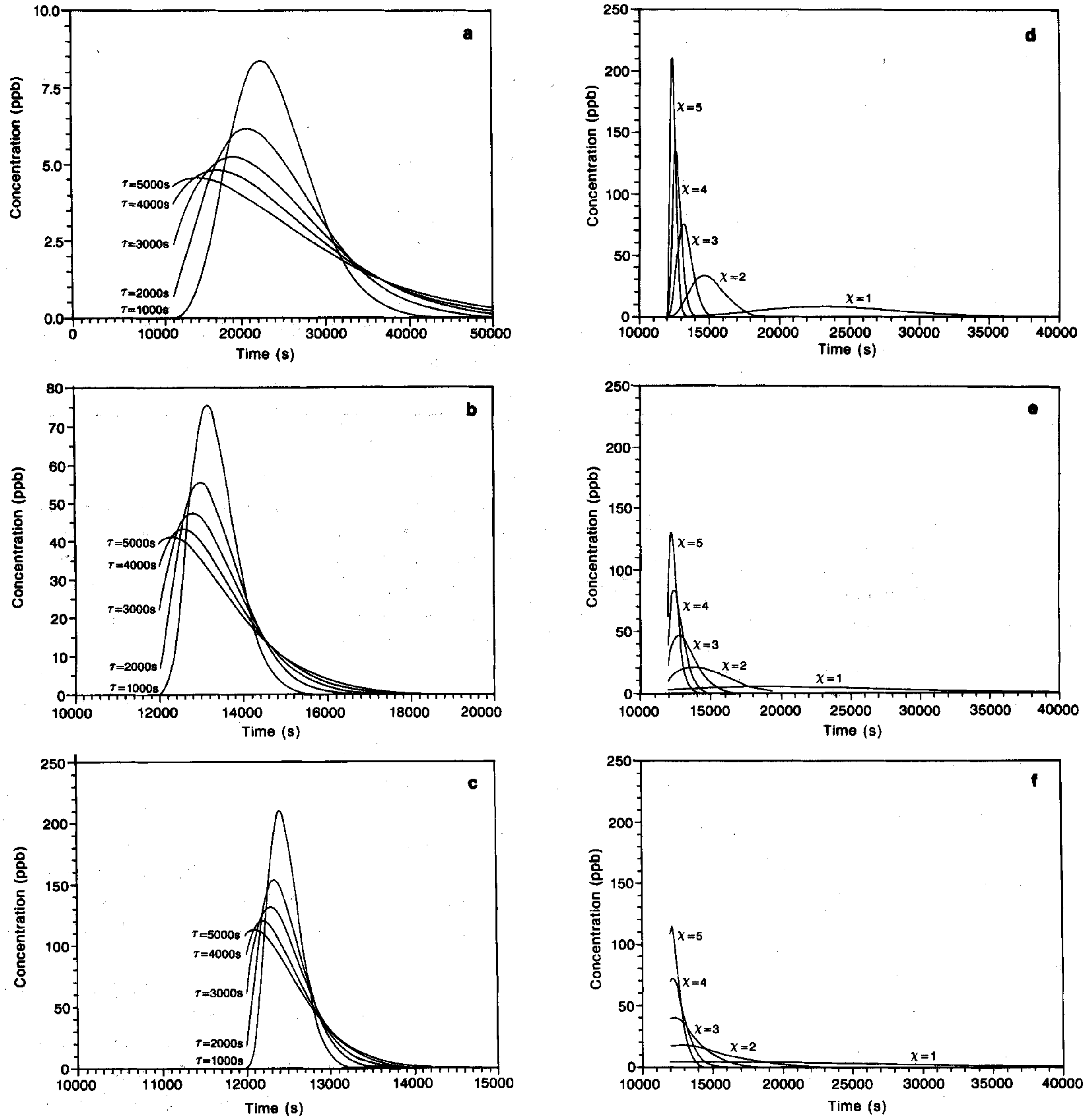

Fig. 2. *Sensitivity of the ADZ model output to different values of parameters χ and τ. For all graphs the value of $U = 0.66\ m.s^{-1}$ and cross sectional area $A = 10\ m^2$. The lines indicating model output begin at $t = 12000$ s which is the translation time of the reach for the chosen values of U and A. The model has no definite solution at this time and gives values of zero for concentration at earlier times. (a) Effect of varying τ from 1000 to 5000 s with $\chi = 1$; (b) Effect of varying τ from 1000 to 5000 s with $\chi = 3$; (c) Effect of varying τ from 1000 to 5000 s with $\chi = 5$; (d) Effect of varying χ from 1 to 5 with $\tau = 1000$ s; (e) Effect of varying χ from 1 to 5 with $\tau = 3000$ s; (f) Effect of varying χ from 1 to 5 with $\tau = 5000$ s.*

Application to the River Severn

The performance of the ADZ model may be tested against data from the River Severn (Atkinson and Davis, 2000) in three ways. Firstly, the model's ability to describe tracer concentration curves at individual stations can be tested. Secondly, the degree of constancy in the fitted parameters can be assessed over the 8500 m of river between stations D and G in which the ADZ model can be expected to apply (see below). Since the channel has essentially constant hydraulic properties in this reach, it might be expected that an accurate model would also show constant parameter values. This is a powerful test of physical realism. Thirdly, the model's ability to predict the form of all the observed tracer clouds from average values of fitted parameters provides a test of its overall usefulness. This test is, of course, only semi-independent of the first two.

The fraction of mass in the second term of the ADZ model is shown as a function of longitudinal distance with $U = 0.66\ \mathrm{m.s^{-1}}$ and $\tau = 2300$ s in Fig. 1. These values of U and τ are appropriate to the River Severn data as analysed by Davis *et al.* (2000) using a dispersion-dead zone model which incorporates the effects of shear-dispersion. The positions of the sampling stations described by Atkinson and Davis (2000) are shown. For downstream distances close to the point of injection only a relatively small proportion of the mass is accounted for in the second term. However, the second term's fractional mass increases rapidly towards its asymptote and, for stations D through G, the second term is an adequate approximation to Eqn. (6).

Optimal solutions and goodness-of-fit

Initially, the ADZ model was calibrated to the seven tracer concentration curves from the River Severn by the parameter fitting method described by Davis *et al.* (2001). The parameters U, M and A were fixed from field measurements, and the sum of squares of residuals $F(\boldsymbol{\alpha})$, $\boldsymbol{\alpha} = (\chi, \tau)$ was minimized. The optimal values χ_o τ_o are given in Table 1 and shown graphically in Fig. 3.

The fractions of mass in the first terms for stations A, B and C, are 0.428, 0.423 and 0.262 respectively. This explains the high values of the objective function (i.e. poor fits) given in Table 1 for these stations, since the values include the predicted concentrations of zero for $t \leq x/U$. The more complex Dispersion-Dead Zone (D-DZ) model presented by Davis *et al.* (2000) demonstrated that dispersion at these stations does not result solely from the effects of tracer storage and it would be physically inappropriate to interpret the tracer cloud as if they did. Therefore, the fits of the ADZ model to stations A, B and C are ignored.

For stations D, E, F and G, the fractions of mass in the second term of the ADZ model are 0.997, 0.989, 0.999 and 0.999 respectively. The asymptotic form of the model is, thus, appropriate for the whole tracer curve at each of these stations. Because they are not given an explicit treatment, any contributions made by shear flow dispersion are included in the fitted residence times of the ADZ model, but for stations D to G this mechanism is known to be relatively unimportant (Davis *et al.*, 2000).

Figure 3 shows clearly that the ADZ model provides an accurate description of the measured tracer distributions for stations D, E, F and G. The predicted mean residence times of tracer in dead zone storage (τ/χ^2 from Table 1), are 279 s, 475 s, 449 s and 460 s respectively, which are comparable with the values found from the more complex D-DZ model (Davis *et al.*, 2000).

Table 1. Optimal solutions ($\boldsymbol{\alpha}_o$) of the ADZ model with corresponding sums of squares of residuals $F(\boldsymbol{\alpha}_o)$ for the River Severn.

Station	χ_0	$\tau_0(s)$	$F(\chi_0,\tau_0)^\dagger$
A	2.736	359	0.5336
B	3.046	1870	0.7357
C	2.720	3019	0.5434
D	2.123	1257	0.0025
E	2.364	2654	0.0098
F	2.178	2131	0.0125
G	2.267	2362	0.0087

† The values of $F(\boldsymbol{\alpha}_o)$ include the zero concentrations predicted for $t \leq x/\mathrm{u}$.

Sensitivity analysis and downstream constancy of parameters

A sensitivity analysis was undertaken at stations D, E, F and G in the manner described by Davis *et al.* (2000). The optimal solutions and sensitivities of the objective function to χ and τ are shown in Figs. 4a and 4b respectively, together with the values and sensitivities obtained for the D-DZ model for comparison. There is no significant downstream variation in the values of χ for the ADZ model. This shows that the effective cross-sectional areas of dead zones are not a function of downstream distance, suggesting a uniform storage over the entire reach length from station D through to station G. The weighted mean value of χ is 2.23, which as expected is approximately the same as the value of 2.26 from the D-DZ model of Davis *et al.* (2000).

Although the ADZ model's sensitivity to τ is not as pronounced as for χ, there is no significant downstream trend. Figure 4b shows that there is less uncertainty associated with τ in the ADZ than in the D-DZ model, with the former producing slightly lower values of τ. Together with its slightly higher values of χ (Fig. 4a), this gives the ADZ model a more rapid exchange of tracer between bulk flow and dead zones and a shorter dead zone residence time. The weighted mean value of τ from the ADZM is 1877 s compared to 2220 s for the D-DZ model, while the mean residence times in dead zones are 377 s and 435 s respectively.

Constant parameter predictions of cloud evolution

It is now possible to assess the ADZ model's ability to describe the evolution of the dispersing cloud, which is a much more rigorous test than cloud shape at a single station.

The model treats the whole reach as possessing a single longitudinally distributed dead zone. If this structure is an adequate description of the dispersing mechanisms actually operating in the river, then the values of χ and τ should be

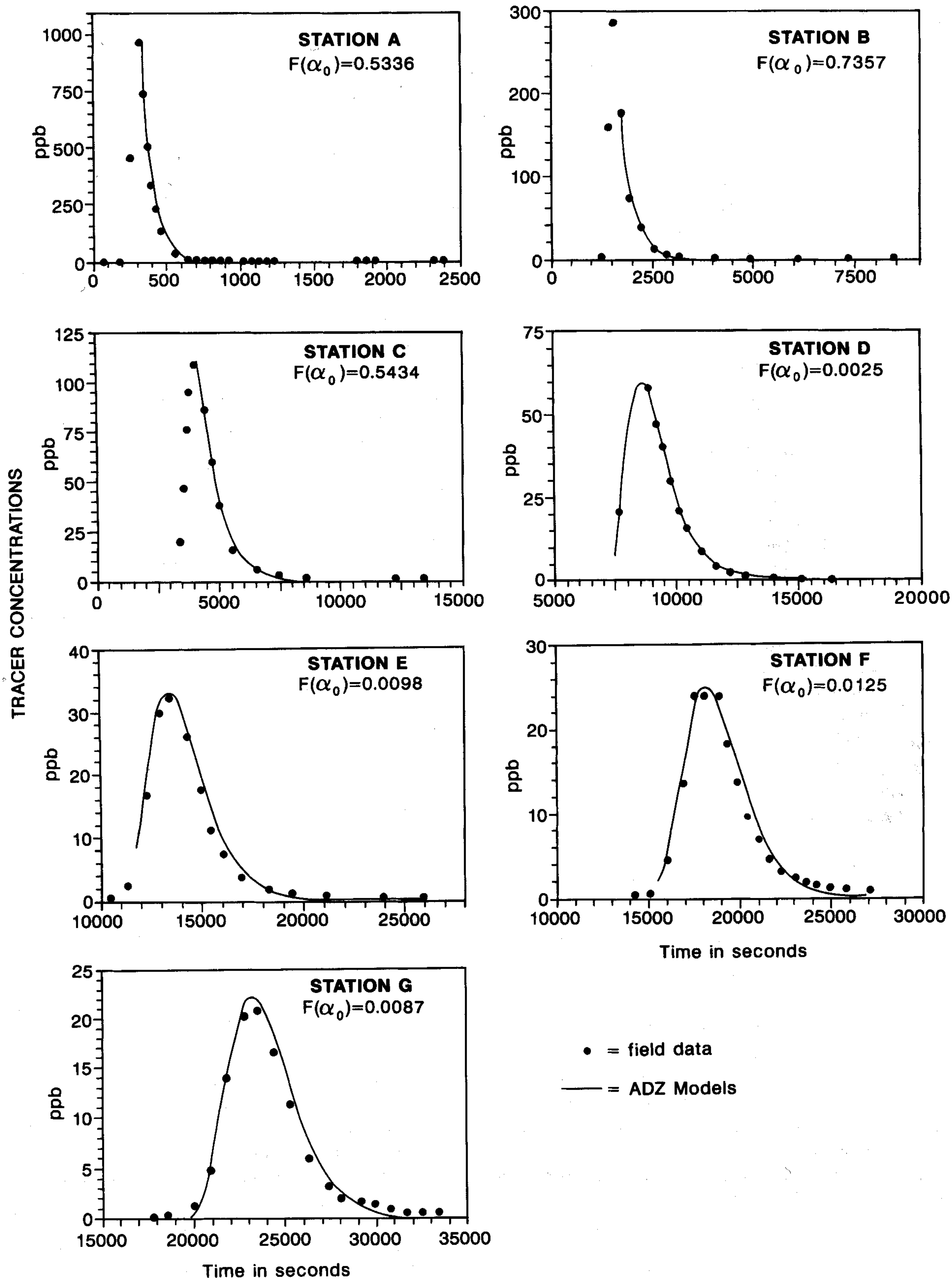

Fig. 3. *Optimal fits of the ADZ model to field data from the River Severn. The continuous lines showing the modelled concentrations commence at their left hand ends at the translation time for the reach.* F(α_o) *is the sum of squares of residuals for the optimum fit of parameters* χ *and* τ.

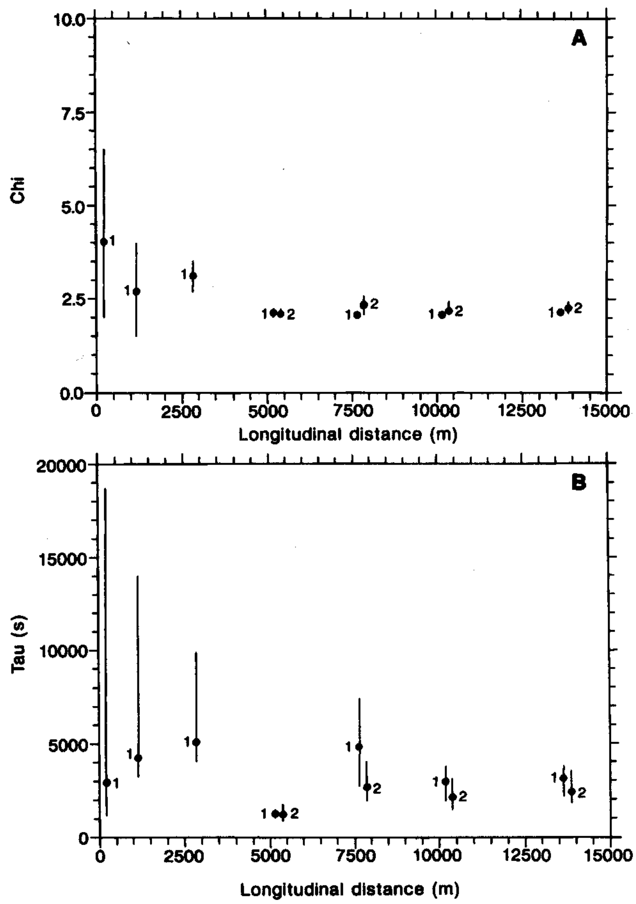

Fig. 4. *Best-fit values of* χ (A) *and* τ (B) *plotted against longitudinal distance. The values for the ADZ model are marked '2'. For comparison, the values for the Dispersion–Dead Zone (D-DZ) model of Davis et al. (2000) are shown marked '1'. The vertical bars indicate the sensitivity ranges of the parameters at each station.*

constant between stations. It is a future goal to relate the dead zone dispersive parameters to the hydraulic properties of the channel reach. At present, it is difficult to suggest any strong physical reasons why χ and τ should be constant over the 8500 m reach from station D through to station G. However, considering the uniformity of the hydraulic parameters over the reach, the dead zone storage regions might have fairly uniform properties also. It is the simplification of the longitudinal mixing process to a single dead zone element that requires χ and τ to be constant if the approximation is to be valid.

Constancy in model parameters has been demonstrated in Table 1 and Fig. 4. The values of χ and τ are now set to the weighted means for stations D to G of 2.23 and 1877 s respectively. For these values, the calculated fractions of mass in the second term are 0.981, 0.998, 0.999 and 0.999, confirming the applicability of the asymptotic solution. The constant-parameter predictions for tracer concentrations are shown in Fig. 5. The ADZ model describes the overall evolution of the cloud well for these four distal stations. The ratio of the dead zone cross-sectional area to that of the bulk flow region is given by $1/\chi^2$. The chosen value of 2.23 for χ corresponds to dead zones equivalent to 20% of the channel cross-sectional area or approximately 2 m^2.

Relationship between ADZ and D-DZ models

It is evident that for the River Severn at least, the simplification to a single dead zone element is a valid approximation since the ADZ model describes the cloud's evolution over 8500 m of river satisfactorily with constant values of χ and τ (and hence a constant residence time for dead zone storage of 377 s). The properties of the modelled dead zones show no downstream variation. A similar result was obtained for the same data by Davis *et al.* (2000) using the D-DZ model, which includes the effects of dispersion in the bulk flow. Davis *et al.* (2000) showed that the overall evolution of the tracer cloud could be divided into three time periods. In the first, pre-Lagrangian period, full mixing over the cross-section had not occurred and the longitudinal dispersion was characterised by a shear-dispersion mechanism depending upon the contrasts in streamwise velocity between the middle and edges of the bulk flow. In the second, early post-Lagrangian time period, full cross-sectional mixing over the bulk flow had been established but the longitudinal dispersion was effected roughly equally by two mechanisms – shear flow dispersion and dead zone storage. In the third time period, most of the tracer particles have entered a dead zone at least once, as is demonstrated by the dominance of the second term in both the ADZ and the D-DZ models. In this period, the dispersion process is dominated by dead zone storage, although the part of the tracer cloud which is in the bulk flow at any time will also be subject to shear-dispersion.

For the third time period (stations D-G), the ADZ model gives similar values to the parameters χ and τ as the D-DZ model, confirming the relative unimportance of shear-dispersion in the latter. Comparing the values of sums of squares of residuals for the two models (Table 1) it is evident that the ADZ model gives the better fit of the two at stations D–G. Visual comparison of Figs. 3 and 5 of this paper with Figs. 5 and 7 of Davis *et al.* (2000) shows that this superiority extends to constant-parameter predictions of the cloud evolution. The superiority of the ADZ model in this regard again suggests that shear-dispersion may have only a very minor effect compared with dead zones in locations far enough downstream for t/τ to exceed *ca.* 3.9 (station D) or *ca.* 6.4 (station E) during the cloud's passage. Wallis *et al.* (1989) report that a first-order model structure (single dead zone storage element) was adequate to describe the dispersive characteristics of four rivers in north-east England, but their study is limited to the application of a numerical dead zone model to short (100–150m) reach lengths. For the River Severn, it has been shown that a first-

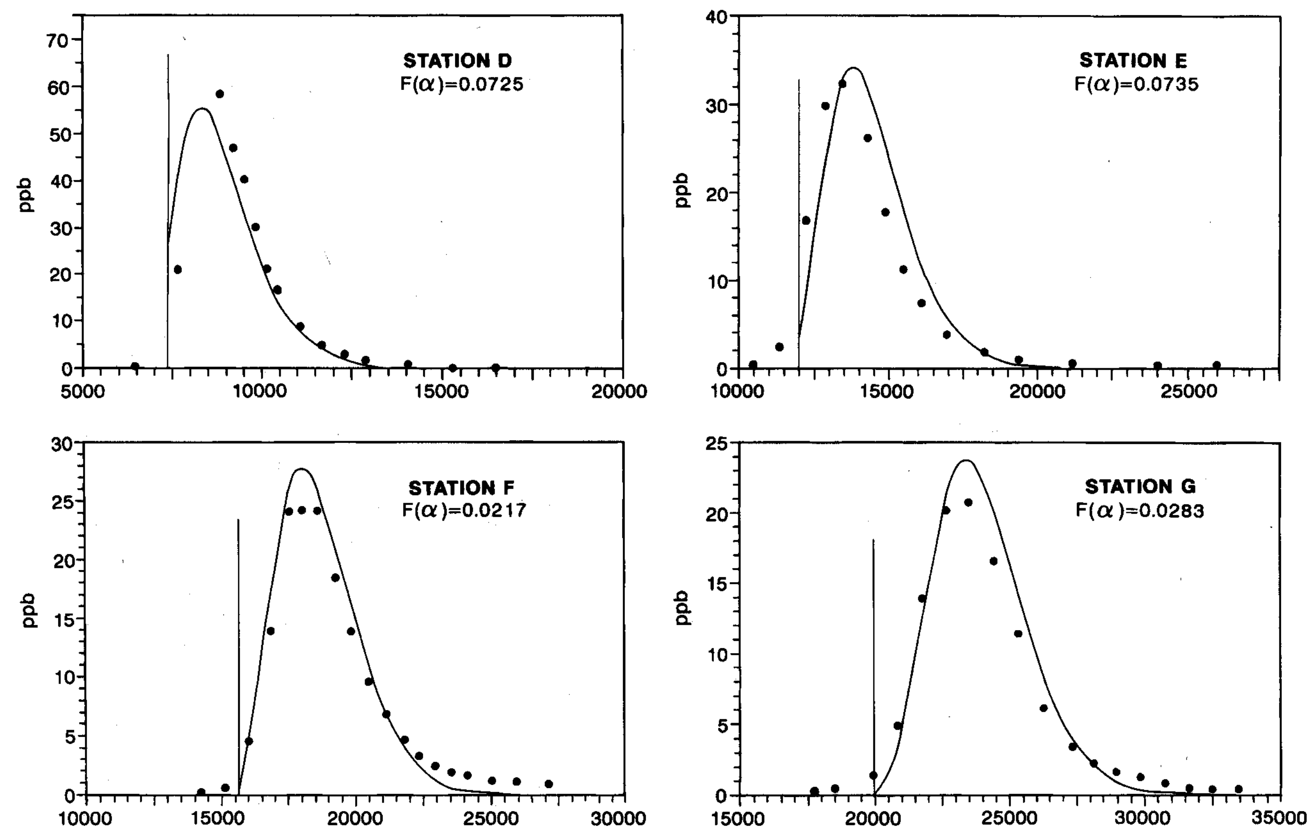

Fig. 5. *Comparison of field data and the ADZ model with parameters χ and τ held constant at $\chi = 2.23$ and $\tau = 1877$ s. The vertical lines mark the translation time for each station, based on measured values of upstream average velocity and cross-sectional area.*

order model structure is capable of describing cloud evolution over the much greater distance from *ca.*5 km to almost 14 km from the injection point, despite the rather idealised assumption that at $t = 0$ the mass is injected in the form of the Dirac delta function.

Conclusions

As shown by Davis *et al.* (2000), tracer dispersion in the River Severn is dominated initially by the effects of shear flow, but the influence of dead zones becomes noticeable after a period in which the cloud velocity slows to the average water velocity in the bulk flow region (the Lagrangian timescale). Thereafter, dead zones and shear flow at first contribute equally to the total amount of dispersion that the tracer cloud undergoes, but the rate due to dead zones exceeds that of shear flow, and dead zones' cumulative effects dominate cloud shape at the more distal stations D-G. In this paper this dominance has been confirmed by demonstrating that cloud evolution at these distal stations can be modelled more accurately by neglecting shear flow dispersion entirely.

This is a significant result since a simplification to a single dead zone storage element is potentially of great practical importance in engineering. It appears that, for practical purposes, it may be better to ignore the shear-dispersion term in the governing partial differential equations for locations at which all but a very small proportion of tracer particles have passed through dead zone storage at least once. Instead, a superior and physically meaningful description of the longitudinal dispersion can be obtained by assuming that the tracer obeys pure translational advection in the bulk flow (sometimes called plug-flow advection); dispersion is produced by the aggregated effect of storage and residence time in dead zones along the whole river reach. The key parameter for deciding the applicability of such an approach is the value of τ which should be such that $e^{-x/U\tau}$ is greater than *ca.* 4–6. At stations whose location x is nearer the injection point than this, shear dispersion will be significant, and the D-DZ model will be more appropriate than the more simplified ADZ model.

List of Symbols

A	cross-sectional area of the bulk flow region of a channel
A_S	cross-sectional area of dead zone region of a channel

C	cross-sectionally averaged concentration of tracer
C_s	tracer concentration in dead zone
$C_T(x,t)$	concentration as defined by Taylor's solution to the Advection-Dispersion Equation
$f(x)$	function of x
$H(x)$	Heaviside step function of x
$I_I(x)$	modified Bessel function of x, of first order and first kind
K	longitudinal dispersion coefficient in the bulk flow region
M	Mass of tracer injected
t	time elapsed since tracer injection
U	average streamwise velocity in the bulk flow region
x	streamwise distance from point of tracer injection
$\boldsymbol{\alpha}$	the parameter set $[\chi, \tau]$
$\boldsymbol{\alpha}_o$	best-fit parameter set $[\chi_o, \tau_o]$
$\delta(x)$	Dirac delta function of x
v	variable of integration
τ	characteristic time scale of tracer exchange between mobile and stationary regions of the channel
χ	dead zone storage parameter, defined by $\chi^2 = A/A_s$

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