Hydrology and Earth System Sciences, 4(3), *511–515* (2000) © EGS

Hydrology & Earth System Sciences

RESEARCH NOTE

# On the parameterisation of drainflow response functions

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## Abstract

A procedure for the parameterisation of drain flow hydrographs is proposed. This involves the derivation of empirical linear response functions, which are themselves parameterised. The parameters are the time and height of the peak, and the recession characteristics. The recession limb of the hydrograph can be approximated best by the Youngs (1985) analysis, which requires two parameters. The merit of this method is illustrated by an analysis of data from a drainage experiment at North Wyke, Devon, UK; this shows that the model fits the data very well.

**Keywords:** Drainflow hydrographs; response functions

## Introduction

Hydrological investigations on small catchments frequently generate long runs of discharge data. Among these, drainage experiments often record the outflow from experimental plots over a number of years. For example, the North Wyke drainage experiment described by Armstrong and Garwood (1991) generated records of drain and surface flow every 30 minutes at 18 measurement points over a period of 10 years, a data base of approximately 3 million data points. Although field drainage systems are artificial in construction, they represent the means of discharge for large areas of land (e.g. Robinson and Armstrong, 1988).

This note address the problem of summarising large data sets to determine their main characteristics, such as those obtained from such drainage experiments. Studies of the hydrological balance can integrate the area under the discharge versus time graph, to give total flows over specified time periods. However, other studies may require an objective method of characterising the sequence of drain flow hydrographs, to identify changes in drain system performance both through time and between replicate plots. This note identifies a procedure for deriving characteristics of drain flow data series in an objective and reproducible manner.

## Theory

The main characteristics of a drain flow data series are identified by examining the relationship between drain discharge and rainfall, using linear response functions. Such functions give a generalised relationship between the input and output of any system. They have the simple form

$$O(t) = \int_{0}^{t_m} K(\tau)I(t-\tau)d\tau \qquad [1]$$

in which the Input, $I(t)$, is transformed to the Output $O(t)$, by convolution with the kernel function $K(\tau)$. This kernel (the response function) can then be used to describe the transformation of the input into the output. Applying this analysis to drain flow hydrographs, the inputs are precipitation $P_t$ and the outputs, the discharges, $Q_t$. If effective rather than total precipitation is used, then the kernel function becomes an estimate of Unit hydrograph (Dooge, 1973). However, the empirical response function is an objective way of characterising the rainfall-runoff response without any theoretical pre-suppositions. Indeed, the response function, integrated over time, is a direct measure of the relative proportion of the rainfall that appears as discharge. There are, thus, significant advantages in exploring response functions without any constraints imposed upon them. The response function derived from a strictly empirical analysis can be examined as an objective way of describing the relationships underlying the observations.

## Implementation

A response function can be calculated from any paired set of inputs and outputs.

Usually derived from equi-spaced series of data in time, the discrete version becomes:

$$O(t) = \sum_{k=1}^{\min(m.,t)} I(k)H(t-k+1) \qquad [2]$$

in which the unit response function, $H(k)$ is of length m. An efficient algorithm has been presented by Bruen and Dooge (1984): this permits easy estimation of the form of the response function using a least squares fit. This algorithm has already been used, (Armstrong 1988), to describe the interactions between rainfall and water tables. Here, the same form of analysis is used to examine the interactions between rainfall and drain discharge.

The programme presented by Bruen and Dooge has been implemented to take strings of data from the North Wyke (and similar) drainage experiments, as a sequence of rainfall and discharge measurements. This has then generated a number of response functions, representing variations through time, between replicates at a site and between sites.

## Generalisation of hydrograph shapes

However, in search of even greater generalisation, a simpler parameterisation of the response function was required. Most response functions follow the classic Unit Hydrograph shape, with a rapid rise to a peak, and a gradual decline thereafter. Parameters of the response function were, therefore, generated to reduce the overall analysis of the variation in system behaviour to a small number of parameters. Use of the Bruen and Dooge algorithm to estimate the empirical response function enabled reduction of the description to a set of parameters that describe the overall behaviour of the system in a small number of parameters. The response function is similar in form to the Unit Hydrograph, so descriptors similar to those observed in the Unit Hydrograph are appropriate. In particular, the response function can be split into two sections, a rising limb and a falling limb, separated by a single peak.

The parameters describing the rising limb of the hydrograph are relatively easy to define using the simple time to peak and peak height by identifying the highest point in the response functions. Occasionally, the last point in the response function is the highest, implying that the length of the response function is not sufficient to encompass all the variation in the data series—but for drain flow hydrographs these indicate problems with the data rather than an anomalous response function. The recession limb of the response function proved to be more problematical to characterise. A simple recession could neither be expected nor observed and several parameterisations were possible.

Initially, a simple exponential curve of the form:

$$Q_t = \alpha e^{-\beta t} \qquad [3]$$

was fitted using linear regression; this does not necessarily pass through the peak and so generally gave a response function with a step in it, corresponding to the intercept at $t = 0$. Further attempts to force the regression through the point of origin did not generate good fits to the form of the response function.

Rather than examine a wide variety of empirical recession curves, Youngs' (1985) theoretical form for the recession of land drainage hydrographs used a transformation of time:

$$T = Cq_0^{\beta} t \qquad [4]$$

where $q_0$ is the rate of flow at the peak, $t$ is time, and $C$ and $\beta$ are constants. The discharge at any time, $q_t$ is then given by Youngs (1985) equation 6:

$$\frac{q_t}{q_0} = (1+T)^{-\frac{1}{\beta}} \qquad [5]$$

These equations cannot be transformed easily into a regression style equation but parameters C and $\beta$ can be estimated by calculating the sum of squared errors and scanning the parameter space for the minimum value, then reducing the step size and repeating the operation around the minimum. This procedure works robustly and without excessive loads on computer time as it converges within a few iterations. (Examination of the 'fitting' surface has shown these are generally well behaved and contain only a single obvious minimum that is easily reached by a simple searching technique). From the sum of squares at the minimum, goodness of fit of the 'best' parameters can be calculated.

This adoption of the Youngs (1985) equation has reduced the recession component of the hydrograph to two parameters, and the whole response curve to four parameters. This seems to be a reasonable degree of simplification to adopt for analysis of the response functions.

The degree to which both the full response function and its parameterised representation fitted the data was explored using the sum of squares (minimised to derive the response function) as a measure of the goodness of fit for the response function model. A correlation coefficient was derived between the observed and predicted data series, as was the Nash Model efficiency criterion (Vanclooster *et al.*, 2000).

## Example

These ideas can be illustrated by examining the drain flow responses for a single discharge point (plot 10) from the North Wyke data set (Armstrong and Garwood 1991). Flow from a mole-drained system was recorded every 30 minutes, and related to rainfall records on the same time base. As this is a drained site, the equation of Youngs (1985) might be expected to be relevant.

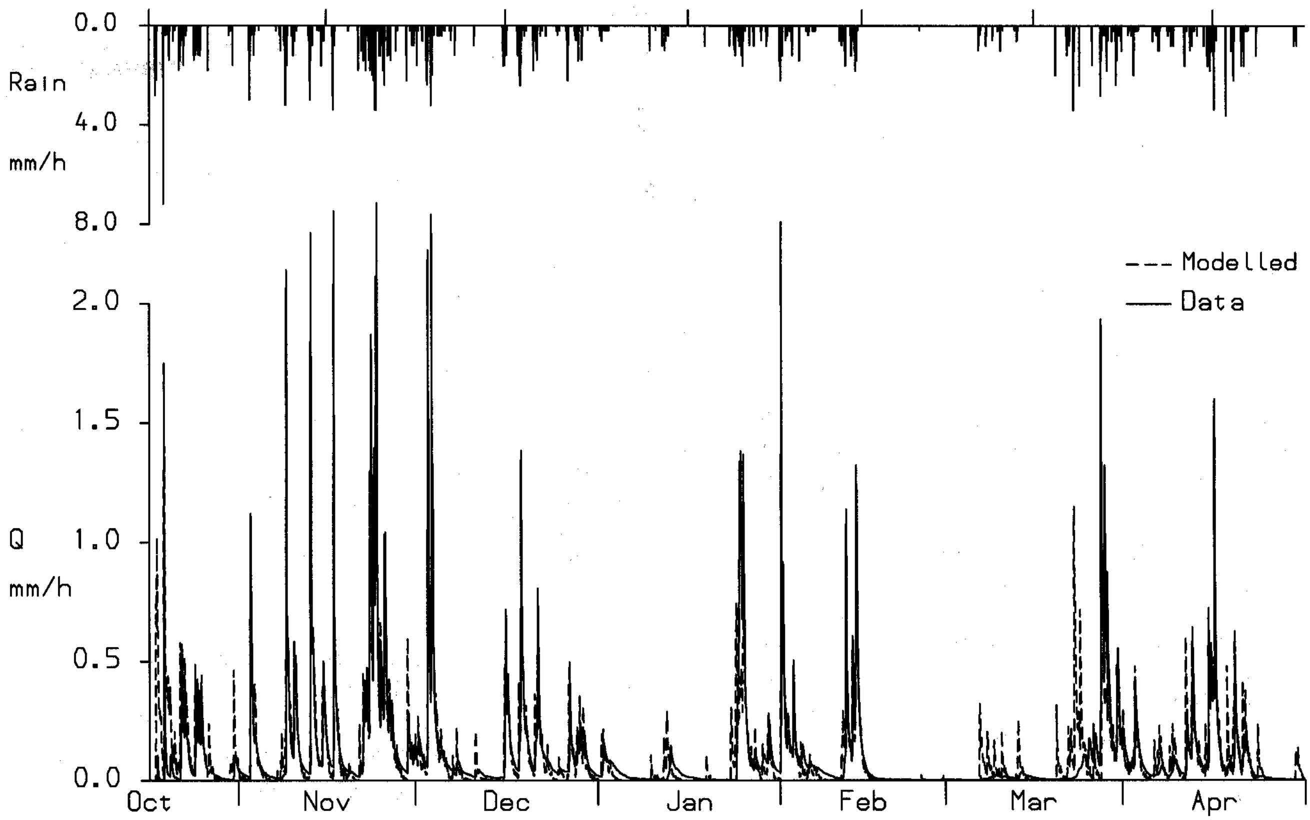


Fig. 1. *Observed and predicted data: Hourly rainfall, and drain discharge from plot 10, North Wyke 1984–85.*

In compiling the input data for the programme, any periods for which there were problems associated with the definition of either variable, perhaps because of instrument failure, were deleted. In addition, summer periods during which there was no drain flow were omitted.

Data were compiled for five hydrological years (running from 1 August to 31 July, so including the whole of the winter drainage season without any break): 1983–84, 1984–85; 1986–87, 1988–89 and 1990–91. Years were omitted where the data quality was insufficient for the analysis. To avoid any effect due to the pattern of evaporation, the effective rainfall was calculated as the actual rainfall—minus the MORECS estimated AET (Thompson *et al.*, 1981). Because ET values are very low during the winter drainage season, the effect of this correction was minimal.

Figure 1 shows the sequence of rainfall and runoff for the year 1984–85. The response function calculated from these data, and the three fitted forms are shown in Fig. 2. The

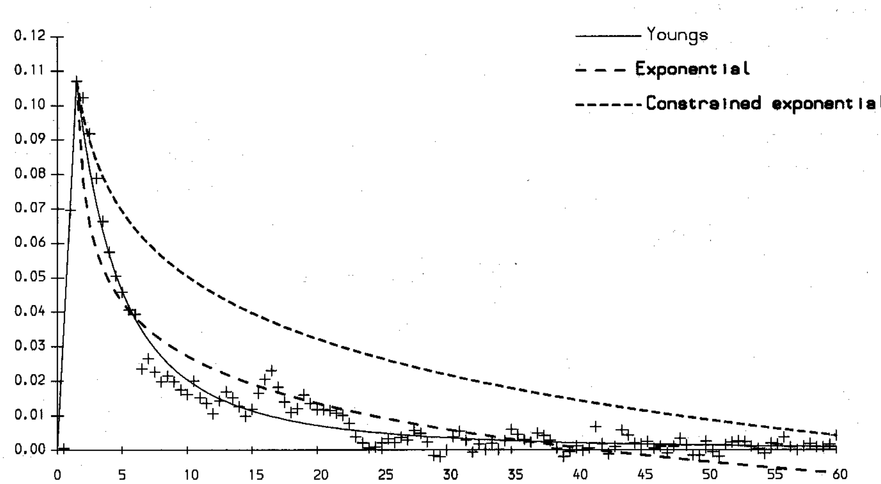


Fig. 2. *Original and modelled response functions, North Wyke 1984–85 Original data points* +.

*Table 1.* Response function parameters: North Wyke 1984–85.

| Model | For different RF models | | | | |
|---|---|---|---|---|---|
| | T to peak | Q peak | | | Fit—r |
| Youngs | 1.5 | 0.107 | C = 0.413 | $\beta = 0.478$ | 0.912 |
| Exponential | 1.5 | 0.107 | $\beta = 0.018$ | $\alpha = 0.042$ | 0.918 |
| Constrained exponential | 1.5 | 0.107 | $\beta = 0.478$ | | |

*Table 2.* Goodness of fit measures North Wyke 1984–5 for different RF models.

| RF Model | Fit RF to data: r | Fit RF to data: ME | Fit Modelled RF to data: r | Fit Modelled RF to data: ME |
|---|---|---|---|---|
| Youngs model | 0.832 | 0.692 | 0.827 | 0.683 |
| Exponential | | | 0.810 | 0.653 |
| Constrained exponential | | | 0.759 | −0.593 |

response function was fitted with a length of 120 steps, (i.e. up to a maximum of 60 hours), and the three alternative models were all fitted to this function. The model parameters are given in Table 1, and the goodness of fit for the three forms in Table 2. The originally calculated values of the response function (plotted as data points, +, in Fig. 2) remain 'noisy', and some of the values are negative, a result that is clearly unrealistic in this case. Modelled response functions are 'smooth' whereas the directly calculated response function is 'noisy'. (The Bruen and Dooge algorithm suggests the use of a 'smoothing' parameter to control this noise, but a smoothing parameter value of 0.08 did not affect the form of the directly calculated response function).

In fitting an equation to these data points, the simple unconstrained exponential also becomes unrealistically negative, whereas the exponential constrained to go through the hydrograph peak does not fit the data adequately. The function fitting the observed recession best was that given by the Youngs (1985) equation; it followed the data, did not go negative and so was adopted for further work. Some small reduction in information is entailed; the correlation coefficient is reduced, on average by 1.5%

*Table 3.* Response function parameters: North Wyke.

| Year | T to peak | Q peak | C | $\beta$ | Fit—r |
|---|---|---|---|---|---|
| 1983–84 | 1.0 | 0.173 | 0.375 | 0.384 | 0.788 |
| 1984–85 | 1.5 | 0.108 | 0.409 | 0.477 | 0.912 |
| 1986–87 | 1.0 | 0.159 | 0.137 | 0.258 | 0.912 |
| 1988–89 | 1.00 | 0.104 | 0.660 | 0.465 | 0.660 |
| 1990–91 | 4.0 | 0.043 | 0.508 | 0.519 | 0.668 |

The parameters for all six years analysed are given in Table 3, and the goodness of fit results for the same 6 years shown in Table 4. These show a consistent pattern of behaviour; the height of the hydrograph peak declines through the years, and the recession becomes slower (Fig. 3). The increase in hydrograph peak in 1986 is a consequence of the renewal of the mole drainage in the previous year. Otherwise, the response functions show a continued decline in performance of the drainage system as it ages, which is to be expected for mole drainage systems Although the response function form behaves in a consistent manner, the parameters describing the recession (Table 4) do not. Hence, it is not possible to use these estimated parameters as a means of back-calculating the performance of the drainage system, despite their origins in the geometry of the drainage system as derived by Youngs. Used in this manner, they can be used only as parsimonious descriptors of response function form.

*Table 4.* Model results: Goodness of fit measures: correlation coefficient r, and Model efficiency, ME, for both simple Response Function (RF) and response function fitted to the Youngs' analysis (YRF), North Wyke.

| Year | RF: r | RF: ME | YRF: r | YRF: ME |
|---|---|---|---|---|
| 1983–84 | 0.926 | 0.858 | 0.919 | 0.837 |
| 1984–85 | 0.833 | 0.694 | 0.828 | 0.685 |
| 1986–87 | 0.799 | 0.634 | 0.794 | 0.623 |
| 1988–89 | 0.766 | 0.580 | 0.740 | 0.547 |
| 1990–91 | 0.393 | 0.139 | 0.383 | 0.124 |

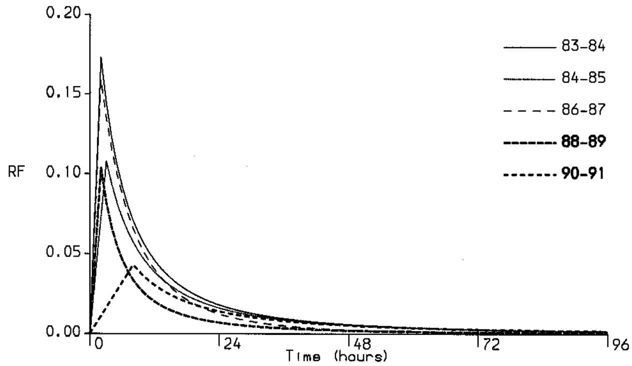


Fig. 3. *Response functions, North Wyke 1993–1990.*

# Conclusions

This note has described the use of linear response functions to the rainfall inputs to characterise the response of drain flow systems. In fitting the response function, the analysis of Youngs (1985) gives a good fit, which is excellent confirmation of his theoretical analysis. The example has shown how the use of this function can achieve a parsimonious description of the hydrograph response of drainage systems. although these descriptors cannot be taken to the point where they can be used to deduce the parameters that underlie the system response function.

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