Stokes Transport by Gravity Waves for Application to Circulation Models

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ABSTRACT

Stokes mass transport by surface gravity waves is related to the often more interesting Lagrangian transport in a manner that is complicated by the earth's rotation. This paper discusses the conditions under which duration- and fetch-limited gravity wave transport will be important driving mechanisms for circulation models. Curves of duration and fetch-limited Stokes transport are given as functions of dimensionless time and fetch.

1. Introduction

Analytic and numerical models of the wind-driven ocean circulation are usually forced by imposed surface wind stress. This mechanical driving produces mass transport in the surface layer identified as Ekman transport which, in turn, drives other low-frequency motions in the models, including motion at depth. An important additional source of mass transport, Lagrangian transport due to surface gravity waves, exists in nature and has not been accounted for in circulation models. Since this Lagrangian wave transport can be comparable to the Ekman transport in magnitude, it will in some applications be necessary to include it in the driving to obtain realistic results.

Unlike the Ekman transport, unfortunately, Lagrangian transport due to surface gravity waves cannot be imposed on a model *a priori*. Rather, the entire fluid dynamic problem must be addressed. Furthermore, simple analytic forms for the Stokes drift (to which the Lagrangian transport is related) due to duration- or fetch-limited gravity wave fields are not available in the literature. The purpose of this paper is thus twofold; first, to discuss in general the conditions under which surface gravity wave transport will be most important to circulation models; and, second, to calculate the Stokes transport using existing wave spectra for duration- and fetch-limited gravity waves.

Recognition of the importance of Stokes transport for a variety of unsteady oceanic motions has come only recently. Longuet-Higgins (1969) called attention to the difference between the mean current at a fixed point (the Eulerian mean) and the mean current following a fluid particle which passes this point (the Lagrangian mean). Generally the Lagrangian mean has the greater physical significance. The difference in these means represents the Stokes velocity, i.e., Lagrange=Euler+Stokes. As Longuet-Higgins noted, the Stokes current may be comparable to or greater than the Eulerian mean for fluctuating motion where the velocity amplitude exceeds the mean. Unlike the Lagrangian or Eulerian means, the Stokes drift may be computed directly from the wave field that produces the fluctuations.

The Stokes current associated with high-frequency surface gravity waves was first examined by Stokes (1847) for long-crested waves of a single frequency. Kenyon (1969) derived a general expression for the Stokes velocity in terms of the two-dimensional gravity wave spectrum and showed the results of computations using the empirical Pierson-Moskowitz spectrum (Pierson and Moskowitz, 1964) for fully developed seas, i.e., for unlimited time and fetch of the wind. In a later paper (Kenyon, 1970) he showed that under the same conditions for winds in the range 10–20 m s^{-1} the Stokes transport, or vertical integral of the Stokes current, was a significant fraction of the Ekman transport.

The Stokes wave transport, however, need not be equal to the Lagrangian wave transport which is vital to the low-frequency (circulation) motion. In fact, Ursell (1950) showed, using an extention of the Helmholtz circulation theorem, that for an unbounded ocean the Lagrangian transport by an infinite train of waves of infinitely long crest and form unchanging with time must vanish on a rotating earth. Thus, for fully developed seas, the Stokes drift investigated by Kenyon (1969, 1970) produces zero net Lagrangian transport. Hasselmann (1970) interpreted this result and showed that for the steady state the Coriolis force acting on fluid particles in their wave orbits produces an Eulerian mean transport equal and opposite to the Stokes transport, so that the Lagrangian mean wave drift is zero.

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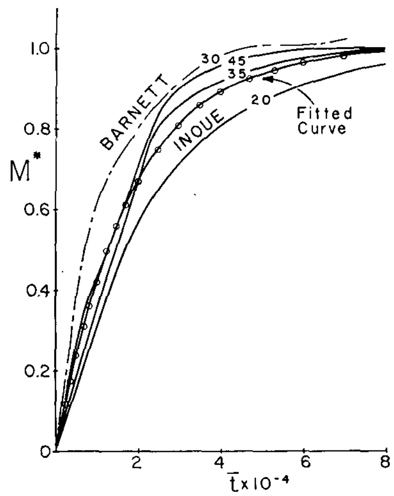

FIG. 1. Normalized Stokes transport as a function of normalized time for duration-limited wind. The "fitted curve" is a representation of the results using the spectra of Barnett (1968) and Inoue (1967). Wind speeds are indicated in knots.

For transient conditions where the wind begins to blow abruptly, but uniformly in space, Hasselmann showed that the Lagrangian transport is initially downwind and equal to the Stokes transport, but begins to rotate in inertia circles as the mean Eulerian current develops. The time scale for this rotation is the inertial period $2\pi/f$, where f is the Coriolis parameter. An inertial period is often of the order or greater than the time needed for the wind to bring a wave field to equilibrium. Thus, the Stokes transport for duration-limited seas will be directly useful in circulation studies involving transient wind fields for time scales less than or of the order $2\pi/f$.

By a simple extension of Ursell's circulation argument one can show that a long-term or steady-state Lagrangian wave drift velocity u_L can exist provided that its spatial scale of variation L is less than or of the order u_L/f; in other words, the Rossby number of the Lagrangian motion, $u_L/(fL)$, must be greater than or of order unity. Such a case may occur for spatially variable winds, from the propagation into an area of a finite length gravity wave train, or for coastal areas and embayments.

The above discussion illustrates that an examination of the Stokes wave transport for other than fully developed seas is of importance. The Lagrangian surface wave transport still can only be obtained by vectorially adding the Stokes transport to the Eulerian mean transport. Determining this latter transport, in turn, requires solution of the circulation problem as a whole, including the presence of boundaries. However, the Stokes transport is a necessary starting point for obtaining the mean wind-driven Lagrangian motion.

2. Stokes transport for duration-limited conditions

Kenyon (1969) has derived a general form for the Stokes drift velocity due to random surface gravity waves as a function of depth. The Stokes mass transport M_s is then found by integrating the drift velocity over depth. In the deep water limit this becomes (Kenyon, 1970)

$$M_s = \rho \int_0^\infty \omega S(\omega) d\omega, \qquad (1)$$

where ρ is water density, ω angular frequency, and $S(\omega)$ the one-dimensional spectrum of mean squared height, assumed to be a function of duration or fetch. To be general, an angular spreading factor should be included in the integrand in (1). For the present we assume that all waves at all frequencies travel in the same direction; modifications to this assumption will be discussed briefly later.

Kenyon (1969) notes the limitations in the theory leading to the derivation of (1) arising from the neglect of wave breaking and the assumption of wave field irrotationality, and cautions that his results should be considered only as an estimate of the Stokes drift velocity.

To find the duration- or fetch-limited Stokes transport an appropriate form for the duration- or fetch-limited spectrum is needed in (1). The growth and decay of the wave spectrum is an area of active research. Barnett (1968), Schule *et al.* (1971) and Hasselmann (1972) review experimental and theoretical work. No single generally accepted analytic or graphical form for the spectrum for use in (1) is available.

We have used duration-limited spectra from Inoue (1967) and Barnett (1968). Inoue develops his spectra from a linear differential equation for the change of spectral density. He fits linear and exponential growth terms according to a "modified Miles-Phillips mechanism" to experimental data, then forces the evolving spectrum to approach the Pierson-Moskowitz spectrum for fully aroused seas in the limit of large duration or fetch. He gives graphical results of spectral evolution for six wind speeds for both duration- and fetch-limited conditions. Inoue artificially incorporates dissipation in his model by forcing the evolving spectrum to approach the Pierson-Moskowitz limit; Barnett more directly models nonlinear terms by including expressions for wave breaking and "wave-wave interactions". He gives results for a 30 kt wind for the duration-limited case.

The results of Inoue's spectra for 20, 35 and 45 kt winds and for Barnett's 30 kt spectra are shown in Fig. 1. The ordinate is the normalized Stokes transport M^*, obtained by dividing M_s from (1) by the Stokes transport for the fully developed (Pierson-Moskowitz) spectrum as given by Kenyon (1970):

$$M_s \text{ (fully developed)} = 3.06 \times 10^{-3} \rho V^3/g, \qquad (2)$$

where V is the wind speed at 19.5 m. The abscissa is normalized time $\bar{t} \equiv gt/V$. A fitted curve which has the analytic form

$$M^* = 1 - \exp(-0.55 \times 10^{-4}\bar{t} - 0.83 \times 10^{-11}\bar{t}^2), \quad (3)$$

is also plotted in Fig. 1 and labelled "fitted curve." It represents a fit of both Inoue's and Barnett's results which may be used as an estimate of the duration-limited Stokes transport and which differs at most from the actual computed values by about 20% over the range for $\bar{t}$ shown. From (3) the normalized e-folding or response time required for M_s to approach the fully developed value is about 2×10^4. Hence the dimensional response time is $t_r \approx 2 \times 10^4 V/g$. Longer response times are required for higher wind speeds because the fully developed wave spectrum peak then occurs at lower wave frequencies which require a longer excitation time.

3. Fetch-limited conditions

To calculate the Stokes transport for fetch-limited conditions we used Inoue's (1967) spectra as well as those of Hasselmann *et al.* (1973) and Liu (1971). Hasselmann *et al.* fitted a functional form to an extensive series of fetch-limited spectra obtained during the Joint North Sea Wave Project (JONSWAP) by adjusting five parameters to produce an optimal fit for each spectrum. Their function for the spectrum yields the following for Stokes transport after integration in (1):

$$M^* = 0.011\alpha\bar{X}^{0.99}, \quad (4)$$

where α is the Phillips "constant," and $\bar{X} \equiv Xg/V^2$ is the dimensionless fetch. As they noted, their results for α could be fitted by a power law with $\alpha = k\bar{X}^n$. Depending on the particular curve fit desired, they recommended either $n = -0.22$ (for which $k = 0.074$) or $n = -0.40$ (for which $k = 0.30$). Results for M^* from (4) for both of these curve fits are shown in Fig. 2. The JONSWAP data extended in fetch only slightly beyond $\bar{X} = 10^4$. Beyond that the curves in Fig. 2 based on them are not likely to be valid.

Liu (1971) also fitted a functional form to wave spectra data using least-squares analysis. His form when integrated in (1) yields

$$M^* = 0.051\bar{X}^{0.25}, \quad (5)$$

and is plotted also in Fig. 2. Liu's data did not extend significantly beyond $\bar{X} = 10^5$. Because M^* grows as $\bar{X}^{0.25}$ for Liu's spectra, while for the JONSWAP data M^* increases as either $\bar{X}^{0.77}$ (for $n = -0.22$) or $\bar{X}^{0.59}$ ($n = -0.40$), the curve for the former initially increases more rapidly for increasing fetch and then falls below the latter at $\bar{X} \approx 3000$. Even at the largest fetch shown, Liu's results do not reach the level corresponding to saturation, $M^* = 1$.

Normalized mass transport from Inoue's (1967) spectra is also shown in Fig. 2 for wind speeds of 20,

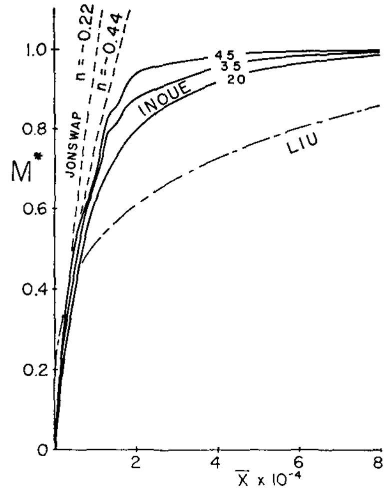

FIG. 2. Normalized Stokes transport as a function of normalized fetch for fetch-limited wind computed form the JONSWAP spectra (Hasselmann *et al.*, 1973) as well as the spectra of Liu (1971) and Inoue (1967). For the latter, wind speed is shown in knots.

35 and 45 kt. An analytic form (not plotted) which represents these results well is

$$M^* = 1 - \exp(-1.1 \times 10^{-4}\bar{X} - 0.85 \times 10^{-9}\bar{X}^2). \quad (6)$$

Because of the assumptions made by Inoue, mass transport for his spectra approaches the Pierson-Moskowitz level, as distinct from the other curves. From (6) the dimensional fetch required to approach the fully developed value of M_s is $X_r \approx 10^4 V^2/g$.

In contrast to the results for duration-limited transport, those for limited fetch are not in substantial agreement across the range of normalized fetch examined. Both the JONSWAP and Inoue spectra give transports in good agreement until the limit of the JONSWAP data ($\bar{X} \approx 10^4$) is approached. For larger fetch the transports from Liu and Inoue are in fair agreement (within about 20%). If a single representative curve is desired for the entire range, then (6), corresponding to the Inoue spectra, is adequate.

4. Concluding remarks

Because the Lagrangian wave transport vanishes for fully developed seas on a rotating earth, direct application of these results to ocean circulation models would be primarily for transient wind forcing where the time scale is less than or of the order of the inertial period, or for spatially limited problems where the Rossby number is greater than or of the order unity. For a wind duration t equal to the inertial period at mid-latitude (6×10^4 s) and wind speed $V = 10$ m s^{-1}, the normalized time would be $\bar{t} = gt/V = 6 \times 10^4$. This corresponds to about three times the response time t_r, so that the Stokes transport would then approximate the fully developed value, or $M^* \approx 1$. The Lagrangian wave transport would also be of this order but would decay subsequently for appreciably larger times as rotation became dominant. Actual wind spectra do contain much energy for periods near the inertial period, so that Lagrangian transport approaching the fully developed level can be expected in nature. We may add further perspective by comparing the fully developed Stokes transport with the steady-state Ekman transport. The latter may be computed from $M_E=\rho_a C_D V_{10}^2/f$ where ρ_a is the air density, V_{10} the wind speed (m s^{-1}) at 10 m height, and C_D the drag coefficient. From Pierson (1964) $V=1.075V_{10}$. Using $\rho_a/\rho=1.25\times10^{-3}$, $f=10^{-4}$ s^{-1} and $C_D=1\times10^{-3}$ (Hasselman *et al.*, 1973), we have for fully developed seas

$$M_s/M_E=0.029V.$$

Thus, for V in the range 10–20 m s^{-1}, the Stokes transport is between 29 and 58% of the steady-state Ekman transport, clearly a significant level.

In applications to spatially limited problems the Rossby number will be of order unity for scales of about 20 km. Then, for a wind speed of 10 m s^{-1} the normalized fetch would be $\tilde{X}=gX/V^2=2000$. Such a fetch correponds only to about $X_r/5$. Fig. 2 shows that $M^*\approx0.25$ then. In general, rotational effects will enter on small enough space scales that the Lagrangian wave transport will not approach the fully developed value for M_s. On these small scales, however, the Ekman transport (more properly, the mass transport due to wind stress) will only be a fraction of M_E, and thus the Lagrangian wave transport may still be relatively important.

We have not presented Stokes drift velocity profiles with depth, but the calculations of Kenyon (1969) also characterize the duration- and fetch-limited results. Motion is always greatest at the surface, but at depth motion increases with time or fetch. For fully developed seas, motion is confined largely to the upper 20 m.

Directional effects on the Stokes transport may be included if desired in particular applications. The JONSWAP data supported a cosine squared directionality factor for the two-dimensional wave spectrum. Using it, our results may be written

$$M^*(\theta)=\begin{cases}(2/\pi\cos^2\theta)M^*, & |\theta|<\pi/2\\ 0\quad, & |\theta|>\pi/2\end{cases}$$

Here θ is the angle between the wind and direction of interest and M^* the normalized transport of Figs. 1 and 2.

The results given in this paper indicate that Stokes transport in surface gravity waves may be estimated with sufficient confidence for application to circulation models. As further wave spectral data become available, these estimates can be further refined.

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