Chinese J. Chem. Eng., **10** (3) 358 – 362 (2002)
RESEARCH NOTES

Multi-scale Analysis of Chaotic Characteristics in a Gas-Solid Fluidized Bed*

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Abstract Deterministic chaos theory offers useful quantitative tools to characterize the non-linear dynamic behavior of a fluidized bed and the developed complexity theory presents a new approach to evaluate finite sequences. In this paper, the non-linear, hydrodynamic behavior of the pressure fluctuation signals in a reactor was discussed by chaos parameters and complexity measures. Coherent results were achieved by our multi-scale analysis, which further exposed the behavior in a gas-solid two-phase system.
Keywords gas-solid fluidized bed, pressure fluctuation, chaotic theory, complexity theory

1 INTRODUCTION

The pressure fluctuations in a fluidized bed contain useful information for indexing the quality of fluidization and have been widely applied in industrial practice. Traditionally, in fluidization engineering, time series of pressure fluctuation are analyzed using statistical (*e.g.* averages, standard deviation) or spectral (*e.g.* Fourier transform, power spectrum or autocorrelation function) analysis. Implicitly, these analysis techniques assume that the fluctuations can be described by a linear summation of random variation or by a linear addition of different periodic waves respectively.

In recent years, some new methods based on fractal and deterministic chaos theories have been used to analyze time-dependent fluidized bed data[1]. However, little work has been done on the study of the chaotic behavior in a wide range of operation conditions. Here, the concepts of chaos theory as a tool to characterize the hydrodynamics quantitatively are further investigated. Meanwhile, two complexity measures-fluctuation complexity and algorithm complexity are used to analyze the pressure fluctuation signals.

2 NON-LINEAR PARAMETERS AND ALGORITHM ANALYSIS

The Lyapunov exponents measure the rates at which system processes or destroys information. A system with at least one positive Lyapunov exponent is chaotic. Of several methods available in the literature for estimating the largest Lyaunov exponent, the method of Wolf *et al.*[2] is adopted in this work.

Several fractal dimensions have been proposed in the literature including the capacity dimension, information dimension and correlation dimension. For time series analysis, the correlation dimension is generally used. The most common method, Grassberger and Procaccia(G-P) algorithm[3], to determine the correlation dimension is used in this study.

The degree of chaos is quantified by the Kolmogorov entropy, which is a measure of the state of loss of information in the system (expressed in bits of information per second). The Kolmogorov entropy algorithm adopted in this paper can be referred to Grassberger and Procaccia[4].

Embedding dimension m and delay time τ are two basic parameters in the process of reconstruction of phase space. Theoretically, perfect reconstruction result can be achieved if only m is large enough. However, too large value will cause unnecessary algorithm efficiency loss. Thus the problem is to get the minimum embedding dimension, satisfying the reconstruction condition. Two rules to choose a practical value of embedding dimension in G-P algorithm and Wolf algorithm are discussed as follows.

Generally, $D(m)$ can be considered as the correlation dimension of system attractors when $D(m)$ does not change with m. As illustrated in Fig. 1, $D(m)$ changes with the increase of m and achieves relative saturation when m is 30, which is selected as the embedding dimension in G-P algorithm.

To evaluate the Lyapunov exponent, the dimension of the embedding state space should be large enough to encompass the complete reconstructed attractor. It means that enough degrees of freedom should be taken into account to ensure that the attractor is fully

Received 2001-04-03, accepted 2001-12-25.
* Supported by the National Natural Science Foundation of China (No. 60075003).
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unfolded in the state space without crossing orbits. Takens[5] has shown that embeddings with $m \geqslant 2d+1$ are sufficient to ensure a faithful, complete evolution of the attractor in the reconstructed state space in such a way that the dynamics of the attractor is characterized by the same value of its invariant as the attractor in the true state space. However, in a practical case we never know before hand what the value of d is. Thus in practice we could start with some small values of d (*e.g.* $d = 3$) and increase it subsequently until saturation is observed in the estimated value of the Kolmogorov entropy.

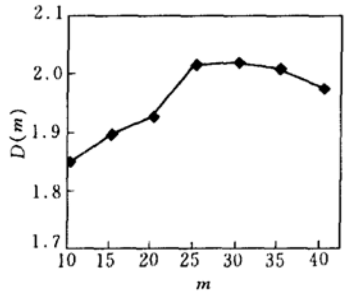

Figure 1 Correlation dimension *vs.* embedding dimension
[polyethylene (PE) particles, $d_p = 280\,\mu$m, $H_s = 630$ mm, $u = 0.412\,\mathrm{m{\cdot}s^{-1}}$, $H_p = 320$ mm]

To constitute a state vector from the data in the time series, we have to choose appropriate delay time between the consecutive elements of the vector. The delay time should not be too long, otherwise successive elements will become uncorrelated. However, it should be large enough so that successive elements are strongly connected. Takens's[5] theorem does not give any clue to what the "best" delay time is in a practical case. In the literature various rules are given to choose a practical value of the delay time. It is suggested to base it upon the first zero crossing or the first minimum of the autocorrelation function or upon the first minimum of the mutual information function[6]. Fraser[7] has suggested that mutual information function be far superior to choosing a zero of the autocorrelation function. As illustrated in Fig. 2, the mutual information function first reaches its minimum when $\tau = 0.075$ s considering most calculation results of our experimental data.

One problem arises from the fact that the reconstruction theory is based on "infinitely" long time series. It is known from the formula of G-P algorithm that $m^* N(N-1)/2$ times calculation must be done in one calculation of the correlation dimension, therefore the number of points must be selected properly to ensure not only a algorithm precision but also a high operation efficiency. In the literature it is generally stated that a number of points of the order of 10^4—10^5 should be taken into account[8]. In our study, 12000 points are selected considering both the algorithm precision and operation efficiency.

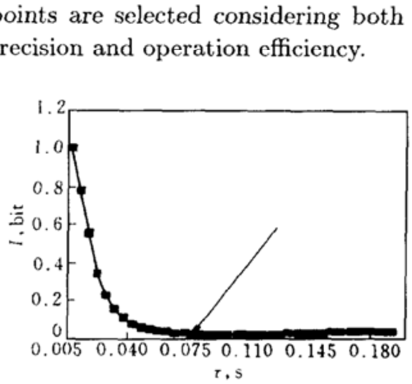

Figure 2 Mutual information function *vs.* delay time
[PE particles, $d_p = 280\,\mu$m, $H_s = 630$ mm, $u = 0.0905\,\mathrm{m{\cdot}s^{-1}}$, $H_p = 320$ mm]

An important way to characterize the complexity of a dynamic system is based on the information gain G_{ij}. It represents the information required to select a state A_j if its preceding state A_i is given.

$$G_{ij} = -\lg P_{i\to j} \tag{1}$$

The mean information gain $<G>$ is defined as

$$\begin{aligned} <G> &= \sum_{i,j=1}^{N} P_{ij} G_{ij} = -\sum_{i,j=1}^{N} P_{ij} \lg P_{i\to j} \\ &= -\sum_{i,j=1}^{N} P_{ij} \lg P_{ij} + \sum_{i=1}^{N} P_i \lg P_i \end{aligned} \tag{2}$$

The mean information loss $<L>$ is defined as the average of information loss L_{ij} over all possible transition $i \to j$. L_{ij} determines the information that a system has lost between a preceding state A_i and the successive state A_j.

$$<L> = \sum_{i,j=1}^{N} P_{ij} L_{i,j} = -\sum_{i,j=1}^{N} P_{ij} \lg P_{i\to j} \tag{3}$$

The net information gain of a system is then expressed by

$$\varGamma_{ij} = G_{ij} - L_{ij} = \lg \frac{P_i}{P_j} \tag{4}$$

During the evolution of a system, $\varGamma_{ij}$ may fluctuate about its mean value and therefore may have a non-vanishing mean-square deviation σ^2, which can be understood as fluctuations in net information gain. It has been introduced as a complexity measure by Bates and Shepared[9].

$$Cf = \sigma^2 = <\varGamma^2> - <\varGamma>^2 = \sum_{i,j=1}^{N} P_{ij} \left(\lg \frac{P_i}{P_j} \right)^2 \tag{5}$$

Fluctuation complexity Cf is a dynamic complexity measure since its definition includes both state probabilities and transition probabilities explicitly and irreducibly.

Algorithmic complexity is defined as the number of bits of the shortest algorithm (*e.g.*, computer program) which is capable of reproducing a given symbol sequence. A practical realization of this theoretical approach has been proposed by Ziv and Lempel[10]. From the study, for nearly all $x \in [0,1]$, the complexity $c(n)$ tends to be a constant value.

$$\lim_{n\to\infty} c(n) = b(n) = \frac{n}{\log_2 n} \tag{6}$$

And the relative algorithm complexity is given

$$C(n) = \frac{c(n)}{b(n)} = \frac{c(n)\log_2 n}{n} \tag{7}$$

The algorithm complexity of a time series is calculated by Eq. (7). From the definition, it can be seen that $C(n)$ of a white noise series tends to be 1 and $C(n)$ of a periodic series tends to be 0.

3 FACILITIES AND PROCEDURE

A diagram of the experimental facilities is shown in Fig. 3. The fluidized-bed is associated with a bed column, a distributor and a plenum chamber. The bed is 0.250 m in diameter and 5 m in height. The properties of particles we used are listed in Table 1. The fluidizing gas is air. The holes on the distributor are 2 mm in diameter and have a fractional open area of 3%. Pressure probes are installed on the wall of the bed column at four different heights. The outside opening of each pressure probes is connected to one of the two input channels of a differential pressure transducer, which produces an output voltage proportional to the pressure difference between the two channels. The remaining channel is exposed to the atmosphere. The working capacity of the transducer is ±5 kPa, and the relative accuracy error is ±0.5%. The sensitivity of the measuring system is $1\,\mathrm{V{\cdot}kPa^{-1}}$.

Table 1 Particle properties of FCC (fluid catalytic cracking) and PE

	Particle density, $\mathrm{kg{\cdot}m^{-3}}$	d_p, μm	u_{mf}, $\mathrm{m{\cdot}s^{-1}}$
FCC	1480	85	0.0013
PE	962	280	0.02

The range of experimental conditions is listed in Table 2. The sampling frequency is 200 Hz and data length under the same operating condition is 60000 points.

Table 2 Experimental operating conditions

Experimental variables	Test range
ration of static bed height to bed diameter, H_s/D_s	2.0—3.4
distance above distributor, m	0.17—0.77

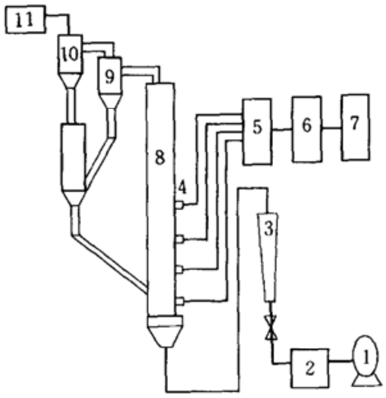

Figure 3 Experimental system
1—fan; 2—cushion pot; 3—rotameter; 4—pressure probes; 5—pressure transducers; 6—A/D board; 7—computer; 8—fluidized bed; 9—first vortex separator; 10—second vortex separator; 11—hop-pocket

4 RESULTS AND DISCUSSION

The flow regime or contacting mode varies widely, depending on the particle size, particle density and particle geometry, gas density, gas viscosity, gas velocity and column configuration. In this paper, the effect of gas velocity is mainly discussed. As illustrated in Fig. 4, with the increase of the gas velocity, the fluidization state of the reactors experiences four flow regimes.

The fluidization state in the reactor is at the transition state before the bubbling fluidization when the gas velocity is barely above the minimum fluidization velocity. The rotation of particles and expansion of bed floor can be seen from the wall while the obvious bubbles and fluctuation could not be detected. The phenomenon is consistent to the particulate fluidization (Group A powders of the Geldart classification only). When the gas velocity exceeds $0.016\,\mathrm{m{\cdot}s^{-1}}$, a mass of fine bubbles are generated and a small quantity of large bubbles pass through the bed synchronously. It corresponds to the bubbling fluidization, at which state the fluctuation is vehement and bubbling is abundant while the interface between the dense phase and sparse phase is clear. Then when the gas velocity is larger than $0.396\,\mathrm{m{\cdot}s^{-1}}$, the bed reaches the turbulent fluidization (slugging fluidization does not appear in our experiment because it occurs in small vessels only). Now, the surface of bed floor becomes blurred and the break of bubbles quickens. A great deal of particles are caught by the first cyclone segregator. Finally, when the gas is above $0.747\,\mathrm{m{\cdot}s^{-1}}$, the difference between the dense phase region of the lower bed and the sparse phase of the upper bed diminishes or even disappears. Thus the fast fluidization is realized.

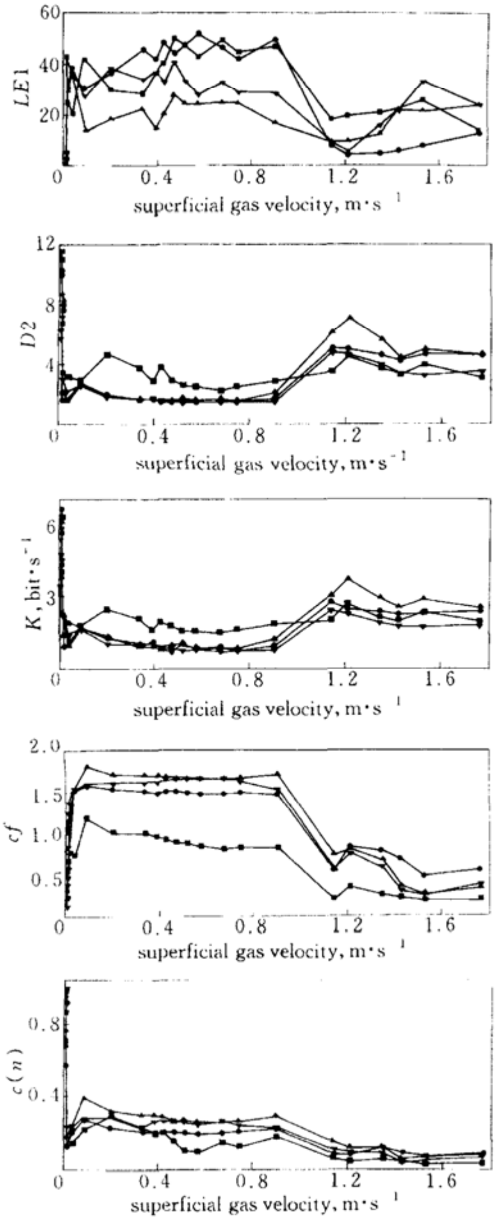

Figure 4 Non-linear invariants *vs.* superficial gas velocity ($m\cdot s^{-1}$)
H_p, mm: ■ 70; ● 170; ▲ 320; ▼ 470

From the trend of non-linear parameters *vs.* gas velocity, we may give explanation as follows.

(1) In the transition region, the Lyapunov exponent fluctuates slightly around zero. At the same time, the correlation dimension and Komogorov entropy are relatively large. Larger values of Komogorov entropy and correlation dimension indicate higher free degree and irregularity. There are probably two reasons, one is that there are no obvious bubbles in the bed and the pressure fluctuation signals are mainly due to the vibration of particles caused by the gas ejection. These signals reflect the whole hydrodynamic behavior, which is close to the random signal. On the other hand, the fluctuation is relatively small and the noise may conceal the useful signals. As shown in Fig. 4, similar to the chaos characteristic parameters, in the region where gas velocity just exceeds the initial fluidization, the tendency of complexity measure indicates that there exists transition region in the fluidized bed where no distinct bubbles are observed. In this region, fluctuation complexity value is relatively lower, which implies the simplicity of motion state while the computational complexity value is close to 1, which is a measure of the complexity of random motion. Therefore, the complexity of motion state in fluidized bed is contrary to each other in terms of the definitions of two complexity measures. In a word, the pressure pulse resulting from the combination of spraying flow and particulate perturbation is close to random signal, which is consistent with speculation obtained by chaos analysis.

(2) A turning point from the transition region to the bubbling region can be observed clearly in Fig. 4. The point is corresponding to the jump of Lyapunov exponent, and the drop of the correlation dimension and Komogorov entropy. In the bubbling regimes, the fluctuation of bubbles is the dominant source of the pressure signals. The signals of bubbles are much more regular than the rotation of particles, which are some low frequency signals. The jump of Lyapunov exponent foreruns the chaos state of the reactor. Simultaneously, it can be seen that the three chaos parameters behave differently. When the Lyapunov exponent is small, the other two are relatively large. When the Lyapunov exponent becomes large, the other two become small. We can infer from the phenomena that the Lyapunov exponent is consistent with larger scale signal (the bubble phase signals) and the other two parameters correspond to the smaller scale signals, the dense phase signals. It is the different sensitivity in disparate scales between the chaos parameters that cause the phenomena mentioned above. Fluctuation complexity arises and algorithmic complexity plump drastically with the increase of gas velocity at the beginning of obvious bubbling. Meanwhile, the rise of fluctuation complexity and the plump of algorithmic complexity suggest that the gas-solid phase motion is in intermediate state, chaotic state, in-between the order and stochastic. It reconfirms that certain amount of bubbles play important roles in the shift of unitary motion to chaotic state.

(3) With the increase of gas velocity, the fluidization regime becomes turbulent fluidization. However, it can be hardly seen from Fig. 4. It is mainly because that the hydrodynamic characteristics of chaos do not change too much during the transition from bubbling to turbulent fluidization. There is no distinct change with respect to the two complexity parameters either.

(4) Finally, the state of the reactor becomes the fast fluidization. The whole bed is in a sparse phase. The pressure signals are generally caused by the collision of particles suspended in the air and pressure wave formed by gas surge. Due to the lack of bubbles, the signals are smaller scale signals and thus the Lyapunov exponent begins to increase and the other two parameters decline. Both fluctuation complexity and algorithmic complexity slightly slide down when the fluidized bed comes to the state of rapid fluidization.

Various complexity measures are adopted in the research the first time so as to analyze systematically the dynamic complexity under various operating conditions. The results of complexity analysis are compared with that of chaos analysis. Experimental data indicate that these two analytical results coincide well and both satisfy reflections of movement in fluidized beds. However, the coincidence should not be considered as being accidental instead of being certain. As far as the large quantity of experimental data are concerned, the plot of five parameters does not matched so well without essential similarities between the chaos analysis and the complexity analysis. On the other hand, the two complexity theories provide us a new point of view in the time series analysis and the hydrodynamic character analysis in the fluidized bed. Since complexity analysis has simple but effective algorithm with a few changeable parameters, it reduces the uncertainty and subjectivity in the computation so that it is more valuable in implementation. It is shown by our experiments that chaos analysis and complexity analysis provide consistent results in a wide regime scales and the result could be important in the dimensionless scaling of chaotic systems.

NOMENCLATURE

$C(n)$	algorithmic complexity (n the length of symbol series)
Cf	fluctuation complexity
$D(m)$	correlation dimension of system attractors when $D(m)$ does not change with m
D_s	bed diameter, mm
$D2$	correlation dimension
d	limit capacity
d_p	particulate diameter, μm
H_p	pressure probe height, mm
H_s	static bed height, mm
I	the value calculated from the mutual information function of two time series
K	Komogorov entropy (bit·s^{-1})
$LE1$	largest Lyapunov exponent
m	embedding dimension
N	number of data
u	superficial gas velocity, m·s^{-1}
u_{mf}	minimum fluidized velocity, m·s^{-1}
τ	delay time, s

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