On the Generation of Baroclinic Rossby Waves in the Ocean by Meteorological Forces¹

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ABSTRACT

The generation of baroclinic Rossby waves in a continuously stratified ocean by fluctuating fields of wind stress, buoyancy flux and atmospheric pressure at the sea surface is studied by means of boundary layer theory. The internal wave field has been represented analytically in terms of the generating meteorological fields and the damping influence of bottom friction. A preliminary application to an example from the eastern Pacific shows that the influence of the atmospheric pressure is negligible compared to that of the other generating agents; on the other hand, fluctuations of the wind stress and the buoyancy flux could be strong enough to generate the waves observed by Emery and Magaard (1976). A more exacting application requires more knowledge about the meteorological fields at the sea surface and has to be left to a later investigation.

1. Introduction

The oceanographic literature contains many papers on the generation of baroclinic gravity waves in the ocean. These papers have been summarized by Thorpe (1975).

Much less is known about the generation of baroclinic Rossby waves. The classic paper by Veronis and Stommel (1956) contains the case of Rossby wave generation by wind in a two-layer inviscid ocean excluding the case of resonance. Perhaps there was little incentive for further theoretical studies in subsequent years because observational evidence of the existence of baroclinic Rossby waves in the ocean was lacking.

In this decade, investigations have concentrated on baroclinic eddies in the Atlantic (e.g., Robinson and McWilliams, 1974) and in the Pacific (Bernstein and White, 1974). These eddies have length scales of up to several hundred kilometers (internal Rossby radius of deformation) and time scales according to the corresponding Rossby wave periods. McWilliams and Robinson (1974) have made an attempt to explain observed fluctuations of these scales by Rossby wave dynamics and have had "partial success." The generation of the eddies is mainly attributed to baroclinic instability (Robinson and McWilliams, 1974; Holland and Lin, 1975).

In a recent paper, Emery and Magaard (1976) pointed out the importance of much longer Rossby waves in the Pacific with periods of one to two years and wavelengths between 1200 and 1700 km. Their

findings gave rise to the study of the local generation of such waves by meteorological forces which is described here.

This paper considers the generation of long baroclinic Rossby waves in a continuously stratified viscous ocean by a prescribed fluctuation of wind stress, buoyancy flux and atmospheric pressure at the sea surface. The analysis is done by means of boundary layer theory (viscous top boundary layer in case of prescribed wind stress, diffusive top boundary layer in case of buoyancy flux, no top boundary layer in case of atmospheric pressure; viscous bottom boundary layer in all three cases). Special attention is given to the case of resonance (calculation of equilibrium amplitudes).

In case of generation by atmospheric pressure, we have a vertical velocity at the sea surface to drive the waves in the interior. In case of generation by wind or buoyancy flux the horizontal divergence of the wind-driven circulation or thermohaline circulation, respectively, in the top boundary layer creates a vertical velocity at the lower boundary of the layer which drives the waves in the interior. In boundary layer theory this velocity can formally be attached to the otherwise rigid sea surface. Hence we consider the motion in the interior to be driven by a vertical velocity at the sea surface.

Generation by a prescribed vertical velocity at the sea surface

We assume the motion in the interior to satisfy the equations

$$\frac{\partial u_I}{\partial t} + f v_I = -\frac{1}{\rho_0} \frac{\partial p_I}{\partial x}, \qquad (2.1)$$

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(2.22)

$$\frac{\partial v_I}{\partial t} - f u_I = -\frac{1}{\rho_0} \frac{\partial p_I}{\partial y},\tag{2.2}$$

$$\tilde{b}_I = i \frac{N^2}{\omega} \tilde{w}_I, \tag{2.18}$$

$$b_I = -\frac{1}{\rho_0} \frac{\partial p_I}{\partial z},\tag{2.3}$$

$$\tilde{p}_I = i \frac{\rho_0}{\omega \lambda^2} \frac{d\tilde{w}_I}{dz}.$$
(2.19)

$$\frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial y} + \frac{\partial w_I}{\partial z} = 0, \tag{2.4}$$

For the corresponding vertical displacement ζ (represented by isotherm displacements, for example) we have

$$\frac{\partial b_I}{\partial t} - N^2 w_I = 0. (2.5)$$

$$\tilde{\zeta}_I = -\frac{i}{\omega}\tilde{w}_I. \tag{2.20}$$

For symbols not explained in the text see the Appendix,

Let $\lambda_n, \varphi_n(z)$ be the system of eigenvalues and eigenfunctions, respectively, of the boundary value problem

For the motion in the bottom boundary layer we assume

$$\frac{d^2\varphi}{dz^2} + \lambda^2 N^2 \varphi = 0, \qquad (2.21)$$

$$\frac{\partial u_B}{\partial t} + f v_B = \frac{\partial}{\partial z} \left(A \frac{\partial u_B}{\partial z} \right), \tag{2.6}$$

with

$$\varphi=0$$
 at $z=0$ and $z=H$.

$$\frac{\partial v_B}{\partial t} - f u_B = \frac{\partial}{\partial \sigma} \left(A \frac{\partial v_B}{\partial \sigma} \right), \tag{2.7}$$

Let $\varphi_n(z)$ be normalized so that

$$\frac{\partial u_B}{\partial x} + \frac{\partial v_B}{\partial y} + \frac{\partial w_B}{\partial z} = 0. \tag{2.8}$$

$$\int_{0}^{H} N^{2} \varphi^{2} dz = 1. \tag{2.23}$$

The boundary conditions are

According to (2.9) and (2.12) the boundary conditions for the solution of (2.14) are

$$w_I = -w_T$$
 at $z = 0$, (2.9)

$$\tilde{w}_I = -\tilde{w}_T^{(0)}$$
 at $z = 0$, (2.24)

$$u_I + u_B = 0$$
 at $z = H$, (2.10)

$$\tilde{w}_I = -\tilde{w}_B^{(H)}$$
 at $z = H$. (2.25)

$$v_I + v_B = 0$$
 at $z = H$, (2.11)

The solution of (2.14) with (2.24) and (2.25) can be represented in two different ways:

 $w_I + w_B = 0$ at z = H. (2.12)

$$\widetilde{w}_I = \widetilde{w}_0(z) + \sum_{n=1}^{\infty} A_n \varphi_n(z), \qquad (2.26)$$

Assuming

$$\tilde{w}_I = \sum_{n=1}^{\infty} B_n \varphi_n(z), \qquad (2.27)$$

 $\psi(x,y,z,t) = \sum \tilde{\psi}(\kappa,\eta,z,\omega)e^{i(\kappa x + \eta y - \omega t)}$ (2.13)

where

for all unknowns ψ , we can derive the following equations from (2.1)-(2.5) by using the β -plane approximation:

$$\tilde{w}_0(z) = -\tilde{w}_T^{(0)} + \frac{\tilde{w}_T^{(0)} - \tilde{w}_B^{(H)}}{H} z,$$
(2.28)

$$\frac{d^2\tilde{w}_I}{d\sigma^2} + \lambda^2 N^2 \tilde{w}_I = 0, \qquad (2.14)$$

$$A_n = \frac{\lambda^2}{\lambda_n^2 - \lambda^2} \int_0^H N^2 \tilde{w}_0 \varphi_n dz, \qquad (2.29)$$

where

$$B_n = \frac{\lambda_n^2}{\lambda_n^2 - \lambda^2} \int_0^H N^2 \tilde{w}_0 \varphi_n dz. \tag{2.30}$$

$$\lambda^2 = \frac{\kappa \beta}{\omega} - \kappa_h^2, \qquad (2.15)$$

Note that the sum in (2.26) is uniformly convergent, while the sum in (2.27) is not uniformly convergent.

$$\tilde{u}_I = -\frac{\eta}{\omega f_0 \lambda^2} \frac{d\tilde{w}_I}{dz},\tag{2.16}$$

Solving Eqs. (2.6)–(2.8) for constant A using the boundary conditions (2.10) and (2.11) and again applying the β -plane approximation, we obtain

$$\tilde{v}_I = \frac{\kappa}{\omega f_0 \lambda^2} \frac{d\tilde{w}_I}{dz},\tag{2.17}$$

$$\tilde{w}_{B}^{(H)} = 2^{-\frac{1}{2}} i A^{\frac{1}{2}} f_0^{-\frac{1}{2}} [(\kappa + \eta) \tilde{u}_{I}^{(H)} + (\eta - \kappa) \tilde{v}_{I}^{(H)}]. \quad (2.31)$$

Using (2.16), (2.17) and (2.26) we get

$$\tilde{w}_{B}^{(H)} = \frac{(1+b-aH)h\tilde{w}_{T}^{(0)}}{(1+b)h+H},$$
(2.32)

where

$$h = -\frac{iA^{\frac{1}{2}} \kappa_h^2}{2^{\frac{1}{2}} f_0^3 \omega \lambda^2},$$
 (2.33)

$$a = \sum_{k=1}^{\infty} \frac{\lambda^2}{\lambda_k^2 - \lambda^2} \alpha_k \gamma_k, \tag{2.34}$$

$$b = \sum_{k=1}^{\infty} \frac{\lambda^2}{\lambda_k^2 - \lambda^2} \beta_k \gamma_k, \tag{2.35}$$

$$\alpha_k = \int_0^H N^2 \varphi_k dz, \qquad (2.36)$$

$$\beta_k = \int_0^H N^2 \varphi_k z dz, \qquad (2.37)$$

$$\gamma_k = \frac{d\varphi_k}{dz} \quad \text{at} \quad z = H. \tag{2.38}$$

Using (2.28) and (2.32) we find from (2.29)

$$A_n = a_n \tilde{w}_T^{(0)}, \tag{2.39}$$

where

$$a_n = \frac{\lambda^2 \{\beta_n - \alpha_n H + h \left[a'\beta_n - (1+b')\alpha_n \right] \}}{(\lambda_n^2 - \lambda^2) \left[H + (1+b')h \right] + h\lambda^2 \beta_n \gamma_n}.$$
 (2.40)

The terms a' and b' result from a and b, respectively, by omitting the term with k=n in the sums. In case of resonance (2.40) reduces to

$$a_n^{(r)} = \frac{\beta_n - \alpha_n H + h \left[a' \beta_n - (1 + b') \alpha_n \right]}{h \beta_n \gamma_n}.$$
 (2.41)

Assuming a random field of M uncorrelated resonant baroclinic Rossby wave modes the frequency (horizontal) wavenumber spectrum of the vertical displacement ζ is

$$E_{\xi\xi}(\kappa,\eta,z,\omega) = \sum_{n=1}^{M} \frac{a_n^{(r)} a_n^{(r)*}}{\omega^2} \varphi_n^2(z) E_{ww}(\kappa,\eta,\omega), \quad (2.42)$$

where E_{ww} is the spectrum of $w_T^{(0)}$. In (2.42) κ, η, ω have to satisfy the dispersion relation

$$\left(\kappa - \frac{\beta}{2\omega}\right)^2 + \eta^2 = R_n^2, \tag{2.43}$$

where

$$R_n^2 = \frac{\beta^2}{4\omega^2} - f_0^2 \lambda_n^2.$$
 (2.44)

We will now study $\tilde{w}_{T}^{(0)}$ and E_{ww} as generated by the various meteorological forces.

3. Generation by a fluctuating wind field

The motion in the top boundary layer has to satisfy the equations

$$\frac{\partial u_T}{\partial t} + f v_T = \frac{1}{\rho_0} \frac{\partial R_{13}}{\partial z},\tag{3.1}$$

$$\frac{\partial v_T}{\partial t} - f u_T = \frac{1}{\rho_0} \frac{\partial R_{23}}{\partial z}, \tag{3.2}$$

$$\frac{\partial u_T}{\partial x} + \frac{\partial v_T}{\partial y} + \frac{\partial w_T}{\partial z} = 0, \tag{3.3}$$

with boundary conditions

$$R_{13} = -\tau_1$$
 at $z = 0$, (3.4)

$$R_{23} = -\tau_2 \quad \text{at } z = 0. \tag{3.5}$$

The thickness of the boundary layer is

$$D_w = \frac{\tau^{(s)}}{\rho_0 f_0 u^{(s)}}.$$
 (3.6)

Using again the β -plane approximation, we find from (3.1)-(3.5)

$$\tilde{w}_{T}^{(0)} = \frac{i}{\rho_{0} f_{0}} (\eta \tilde{\tau}_{1} - \kappa \tilde{\tau}_{2}). \tag{3.7}$$

For the corresponding spectrum E_{ww} we have

$$E_{ww} = \frac{1}{\rho_0^2 f_0^2} \left[\eta^2 E_{\tau_1 \tau_1} + \kappa^2 E_{\tau_2 \tau_2} - \kappa \eta \left(E_{\tau_1 \tau_2} + E_{\tau_2 \tau_1} \right) \right], \quad (3.8)$$

where $E_{\tau_1\tau_1}$, $E_{\tau_2\tau_2}$, $E_{\tau_2\tau_1}$, $E_{\tau_2\tau_2}$ are the components of the spectrum tensor of the wind stress.

4. Generation by a fluctuating buoyancy flux

This mechanism was studied by Magaard (1973a) for the generation of internal gravity waves. The scaling and the derivation of the boundary layer equations as well as their treatment follows Magaard (1973a,b) and will not be repeated here.

The equations of motion in the top boundary layer at the sea surface alone we obtain are

$$\frac{\partial u_T}{\partial t} + f v_T = -\frac{1}{\rho_0} \frac{\partial p_T}{\partial z},\tag{4.1}$$

$$\frac{\partial v_T}{\partial t} - f u_T = -\frac{1}{\rho_0} \frac{\partial p_T}{\partial y},\tag{4.2}$$

$$b_T = -\frac{1}{\rho_0} \frac{\partial p_T}{\partial z},\tag{4.3}$$

$$\frac{\partial u_T}{\partial x} + \frac{\partial v_T}{\partial y} + \frac{\partial w_T}{\partial z} = 0, \tag{4.4}$$

$$\frac{\partial b_T}{\partial t} = -\frac{\partial F}{\partial z},\tag{4.5}$$

with boundary condition

$$F = F^{(0)}$$
 at $z = 0$. (4.6)

The thickness of the boundary layer is

$$D_b = \frac{F^{(s)}}{\omega h^{(s)}}. (4.7)$$

Introducing an Austausch coefficient K of buoyancy,

$$F = -K \frac{\partial b_T}{\partial z},\tag{4.8}$$

we get

$$D_b = (K/\omega)^{\frac{1}{2}}. (4.9)$$

Again applying the β -plane approximation we obtain from (4.1)–(4.6)

$$\tilde{w}_T^{(0)} = -(1+i)(K/2\omega)^{\frac{1}{2}}\lambda^2 \tilde{F}^{(0)},$$
 (4.10)

$$E_{ww} = -\lambda^4 E_{FF}, \tag{4.11}$$

where E_{FF} is the buoyancy flux spectrum at the sea surface. Moreover, we get

$$\tilde{b}_T(z) = \tilde{b}_T^{(0)} \exp[(i-1)(\omega/2K)^{\frac{1}{2}}z].$$
 (4.12)

Expressing $\tilde{F}^{(0)}$ in terms of the density disturbance generated at the sea surface, we have

$$\tilde{w}_T^{(0)} = K \lambda^2 g \tilde{\rho}^{(0)} / \rho_0, \tag{4.13}$$

$$E_{ww} = \frac{g^2 K^2 \lambda^4}{\rho \sigma^2} E_{\rho \rho}, \tag{4.14}$$

where $E_{\rho\rho}$ is the spectrum of the sea surface density. Considering the influence of a temperature disturbance

$$\tilde{w}_T^{(0)} = -g\alpha K \lambda^2 \tilde{T}^{(0)}, \qquad (4.15)$$

$$E_{ww} = g^2 \alpha^2 K^2 \lambda^4 E_{TT}, \qquad (4.16)$$

where E_{TT} is the spectrum of the sea temperature.

fluctuating atmospheric 5. Generation by pressure

The boundary condition at the sea surface is

$$w_I + \frac{1}{g\rho_0} \frac{\partial p_I}{\partial t} = \frac{1}{g\rho_0} \frac{\partial p_a}{\partial t}.$$
 (5.1)

If we restrict ourselves to frequencies and wavenumbers which are sufficiently distant from the dispersion curve of barotropic Rossby waves we get

$$w_I = \frac{1}{g_{00}} \frac{\partial p_a}{\partial t}.$$
 (5.2)

From (2.9) we conclude that

$$\tilde{w}_T^{(0)} = \frac{i\omega}{\tilde{p}_{a}},\tag{5.3}$$

$$E_{ww} = \frac{\omega^2}{g^2 \rho_0^2} E_{pp}, \tag{5.4}$$

where E_{pp} is the spectrum of the atmospheric pressure at the sea surface.

6. Discussion

We will discuss the significance of the various generating agents by utilizing the results of Emery and Magaard (1976) who analyzed temperature fluctuations in the area between the Hawaiian Islands and the position of weather station November (30°N, 140°W). At a wave period of 28 months, 78.4% of the observed fluctuations could be explained by first-order baroclinic Rossby waves propagating along 24.2° west of north with a wavelength of 1390 km and a rms amplitude of 8.2 m (at 500 m depth). Let us consider fluctuations of meteorological fields propagating in the same direction and having the same length and time scales. What amplitudes would these fluctuations need in order to generate the observed waves? Using the eigenvalues λ_n and eigenfunctions $\varphi_n(z)$ as calculated by Emery and Magaard (1976), we find from (2.41)

$$a_1^{(r)} = \frac{(1.44 \times 10^6 \text{ cm}^{\frac{1}{2}} \text{ s}^{-1}) + (3.11 \text{ cm}^{\frac{1}{2}} \text{ s}^{-1})h}{h}, \quad (6.1)$$

where

$$h = -3.08 \times 10^3 i A^{\frac{1}{2}} \text{ [cm]}$$
 (6.2)

and A is given in cm² s⁻¹. Assuming A to be smaller than $100 \text{ cm}^2 \text{ s}^{-1}$ we can neglect the second term in the numerator of $a_1^{(r)}$, when estimating the order of magnitude of $a_1^{(r)}$. Hence with sufficient accuracy

$$a_1^{(r)} = 5 \times 10^2 i A^{-\frac{1}{2}} \left[\text{cm}^{\frac{1}{2}} \text{ s}^{-1} \right].$$
 (6.3)

The corresponding rms magnitude of $\tilde{w}_T^{(0)}$ is 3×10^{-7} $A^{\frac{1}{2}}$ cm s⁻¹. That means that a disturbance of the wind stress (directed perpendicular to the wavenumber vector) would have a rms amplitude of

$$\tau_0 = 4 \times 10^{-4} A^{\frac{1}{2}} [g \text{ cm}^{-1} \text{ s}^{-2}]$$
 (6.4)

in order to generate the observed waves. We will express the necessary magnitude of a buoyancy flux disturbance at the sea surface by considering the corresponding disturbance of the sea surface temperature. The latter would have to have a rms amplitude of

$$T_0 = 10^{-1} A^{\frac{1}{2}} K^{-1} [^{\circ}C].$$
 (6.5)

An atmospheric pressure fluctuation would need a rms amplitude of

$$p_0 = 3A^{\frac{1}{2}} [mb]$$
 (6.6)

to generate the observed waves.

It is seen that the direct influence of atmospheric pressure is negligible because the atmospheric pressure (6.6) would imply a (geostrophic) wind disturbance which is several orders of magnitude larger than that of (6.4). That is to say, the disturbance of the sea level height generated by a fluctuation of the atmospheric pressure is very small compared to the vertical displacement of the lower edge of the top boundary layer generated by the corresponding geostrophic wind.

For a comparison of the wind influence and that of the buoyancy flux, we need a value for K. White and Walker (1974) estimate the rate at which temperature disturbances of time scales from 3 to 6 years penetrate through the top boundary layer to be approximately 100 m per year. According to (4.12) this means K has values between 1 and 2 cm² s⁻¹. Assuming K to be of order 1 cm² s⁻¹, we see that a wind stress disturbance with τ_0 given by (6.4) and a sea surface temperature disturbance with $T_0 = 10^{-1} A^{\frac{1}{2}} [^{\circ}C]$ can likewise generate the observed waves. Assuming A to be in the range between 1 and 10 cm² s⁻¹, the necessary values of τ_0 and T_0 appear to be small. This is particularly true for τ_0 . One must realize, however, that these values are rms values for a sufficiently large number of wave periods. It is planned that spectra of sea surface temperature and wind stress will be calculated. The results should shed more light on the significance of the processes under consideration. An important test will be whether the disturbances show a preference with respect to the direction of propagation of the Rossby waves.

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APPENDIX

List of Symbols Not Explained in Text	
A	Austausch coefficient (vertical) for momentum (horizontal) in the bottom boundary
b_I	buoyancy (associated with the wave field)
$b^{(s)}$	in the interior scale of buoyancy in the top boundary layer buoyancy in the top boundary layer
$egin{array}{c} oldsymbol{b_T} \ oldsymbol{b_T^{(0)}} \end{array}$	b_T at $z=0$
f	Coriolis parameter $[=f_0+\beta y (f_0=6.163 \times 10^{-5} \text{ s}^{-1}, \ \beta=2.075\times 10^{-13} \text{ cm}^{-1} \text{ s}^{-1} \text{ in}$ the numerical example)
F	buoyancy flux in the top boundary layer
$F^{(0)}$	F at $z=0$
$F^{(s)}$	scale of buoyancy flux in the top boundary layer
g	acceleration of gravity (981 cm s ⁻²)
H	depth of water (assumed constant, $H = 4750$ m in the numerical example)
N(z)	Brunt-Väisälä frequency
p_a	atmospheric pressure at the sea surface
Þι	pressure (associated with the wave field) in the interior
p_T	pressure in the top boundary layer
R_{13}, R_{23}	components of the Reynolds stress in the top boundary layer
t	time
$T^{(0)}$	sea surface temperature
u_B, v_B, w_B	x, y, z components of the velocity in the bottom boundary layer
u_I, v_I, w_I	x, y, z components of the velocity in the interior
u_T, v_T, w_T	x, y, z components of the velocity in the top boundary layer
$u^{(s)}$	scale of the horizontal velocity in the top boundary layer
$w_T^{(0)}$	w_T at $z = 0$
$w_B^{(H)}$	w_B at $z = H$
x, y, z	Cartesian coordinates (x westward, y northward, z downward)
α	coefficient of thermal expansion $(\alpha=3\times10^{-4} \text{ (°C)}^{-1})$
κ, η	components of the horizontal wavenumber vector (κ to the west, η to the north)

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components of the wind stress at the sea

constant reference density

 $\kappa^2 + \eta^2$

 $au^{(s)}$

surface

frequency

scale of wind stress

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