Wind Stress and Roughness Length over Breaking Waves

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ABSTRACT

The effects of surface wave breaking on the adjacent atmospheric boundary layer are examined. It is argued that the transition from aerodynamically smooth to rough flow in a neutral atmosphere corresponds to the onset of extensive small-scale wave breaking. The association of wave breaking with the generation of turbulence in the boundary layer above leads to the result that the friction velocity is approximately equal to the phase velocity of the breaking waves. It is argued that this approximate relationship holds even when the small-scale breaking waves are riding on a swell. The existence of a minimum phase velocity for surface waves then requires that there be a minimum friction velocity, in the neighborhood of 23 cm s^{-1}, below which rough flow cannot occur. A result of Phillips and Banner (1974) which describes the limiting amplitude of small gravity waves under the action of wind drift and swell is used to derive a relationship between the roughness length and friction velocity which is a generalization of Charnock's (1955) equation. The published field measurements of a number of workers are shown to support these results.

1. Introduction

Since its introduction by Jeffreys (1924, 1925), flow separation above surface waves has from time to time been used to explain momentum transfer across the air-sea interface. However, it was employed without any clear description of the conditions under which it might be expected. This problem has recently been resolved by Banner and Melville (1976) who showed that, in a frame moving with the waves, air-flow separation requires a stagnation point on the interface and that this corresponds to the condition for wave breaking. Thus, as a result of the continuity of the fluid velocity across the interface, wave breaking and flow separation are concomitant. Coincidently, Gent and Taylor (1976b) recently reached the same conclusion.

In Banner and Melville the physical arguments were supported by laboratory measurements which showed that the drag of a breaking wave was an order of magnitude greater than that of a finite-amplitude wave of the same wavelength. Unfortunately the limitations of the laboratory do not permit the correct scaling of the air-sea interaction and the measurements by themselves do not justify the assumption that the onset of wave breaking in the ocean leads to an order-of-magnitude increase in the momentum transfer across the interface. Nevertheless, the measurements do support the suggestion that the onset of wave breaking may be accompanied by a significant increase in the drag. Such an increase would be reflected in the mean wind profiles.

Despite some anomalies most of the mean velocity measurements in the neutral marine atmosphere are well correlated by the logarithmic profile

$$U/u_* = \kappa^{-1} \ln(z/z_0), \tag{1}$$

where $U(z)$ is the mean velocity, u_* the friction velocity, κ von Kármán's constant (0.41) and z_0 the roughness length. The boundary layer is described as aerodynamically smooth, or fully rough, according to whether the roughness Reynolds number, $R_* = u_* z_0/\nu$, is less than 0.13 or greater than 2, respectively.[2] For $R_* \lesssim 0.13$ a viscous sublayer is present between the logarithmic layer and the boundary. As R_* increases, the thickness of this layer decreases until at $R_* \approx 2$ no dynamically significant viscous layer remains. The mean momentum transfer across the interface is usually described by the drag coefficient

$$C_{10} = (u_*/U_{10})^2, \tag{2}$$

where U_{10} is the mean velocity at 10 m above the mean sea level. An increase in C_{10} corresponds to an increase in z_0. While the form of (1) may be derived from very general arguments (Monin and Yaglom, 1971), the magnitudes of u_* and z_0 have not been related con-

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[2] These constants are determined by experiment and may vary slightly.

vincingly to the processes occurring at the interface—especially wave breaking.

The aim of this paper, then, is to examine the response of the atmospheric boundary layer to an underlying surface of breaking waves, and to determine whether the onset of wave breaking in the ocean leads to a significant increase in the momentum transfer across the interface.

In Section 2 it is assumed that the turbulent velocity fluctuations over a train of small-scale breaking waves are dominated by the breaking process. This leads to the result that the friction velocity is approximately equal to the phase velocity of the breaking waves. This relationship is little changed when the breaking waves are riding on a swell. Accordingly, the minimum friction velocity at which we might expect to find extensive small-scale wave breaking is approximated by the minimum phase velocity of capillary-gravity waves. In Section 3 a result of Phillips and Banner (1974) which describes the limiting amplitude of small gravity waves under the action of wind drift and swell is used to derive an expression for the roughness length which is a generalization of Charnock's (1955) relationship. In Section 4 the predictions of Sections 2 and 3 are compared with the published field measurements of a number of workers. While there is evidence to the contrary (see below) a significant body of experimental data, including the more recent observations, supports the prediction that the transition from aerodynamically smooth to rough flow in a neutral marine atmosphere occurs when the friction velocity is in the neighborhood of 23 cm s^{-1} (the minimum phase velocity of surface waves) and is accompanied by a significant increase in the drag coefficient. Furthermore, the expression derived for z_0 is shown to be consistent with the extensive field measurements of Sheppard *et al.* (1972).

2. The friction velocity

a. Wave breaking and turbulence generation

We consider a turbulent boundary layer with mean velocity $U_1(x_3)$ blowing in the x_1 direction over a water surface having a displacement $\eta(x_1,x_2,t)$ from its mean height; η is single valued and independent of x_3, the vertical coordinate. By integrating the instantaneous equation of motion from η to a reference height h wholly within the air, averaging horizontally over an area having a length scale much larger than that of the wavy surface, and by assuming the averages are horizontally homogeneous close to the surface, it may be shown (Deardorff, 1967) that the mean shear stress τ_h at the reference level can be related to the surface variables by

$$\tau_h=\overline{p_\eta\frac{\partial\eta}{\partial x_1}}+\mu_a\frac{\partial U_1}{\partial x_3}-\mu_a\overline{\left(\frac{\partial u_1}{\partial x_\alpha}\right)_\eta\frac{\partial\eta}{\partial x_\alpha}},\quad \alpha=1,\,2,\qquad (3)$$

where u_i is the instantaneous velocity vector, μ_a is the viscosity of air, and the subscripts h and η refer to the value of the variables at the reference height and surface, respectively. The first term on the right-hand side of (3) represents the momentum flux across the surface due to pressure forces, while the remaining two terms represent the viscous transfer.

If h is in the logarithmic layer τ_h is a constant, say, τ_0, which defines the friction velocity u_*:

$$\tau_0=\rho_a u_*^2.\qquad (4)$$

This stress may be decomposed into three components with τ_0 given by

$$\tau_0=\tau_w+\tau_t+\tau_\nu=-\rho_a\left(U_1U_3+\overline{u_1'u_3'}+\nu\frac{\partial U_1}{\partial x_3}\right),\qquad (5)$$

where U_i and u_i' are the wave-coherent turbulent velocities which result in the wave-coherent turbulent stresses τ_w and τ_t, respectively, and τ_ν is the viscous stress.

The viscous stress is sensibly zero in the logarithmic region. The physics of the wave-induced stress τ_w is understood in principle (Lighthill, 1962; Phillips, 1966) but its prediction depends on the description of the turbulent field. Our physical understanding of the generation and maintenance of the turbulence over a wavy surface is poor, but recent numerical models devised by Townsend (1971) and Gent and Taylor (1976a) have had some success in predicting the flow. Nevertheless, these models do not explicitly account for the influence of breaking waves on the turbulence.

The processes which are believed to be the source of turbulent energy and stresses in flat-plate boundary layers have been extensively studied over the last decade by Kline and his co-workers. In a recent paper, Offen and Kline (1975) likened these processes to a local separation convected within the fluid in the neighborhood of the boundary. Associated with this motion is the lifting and straining of vortex lines. This model was said to be consistent with all the relevant boundary layer data. It is most likely that the same basic processes are responsible for the generation of turbulence over smooth gravity waves. However, the sea surface is smooth only in the lightest winds, or when contaminated by surfactants. At most times it is covered by breaking waves extending in length from a few centimeters up to tens of meters, and, as was shown by Banner and Melville, the air flow must separate from each of the breaking waves. Consequently, breaking waves propagating over the interface result in separated regions being convected over the surface. If we draw the analogy with Offen and Kline's model, we might assume that breaking waves accompany much of the turbulence generation in the marine atmosphere.

In the absence of breaking there can be no separation from the surface, and according to this model the local

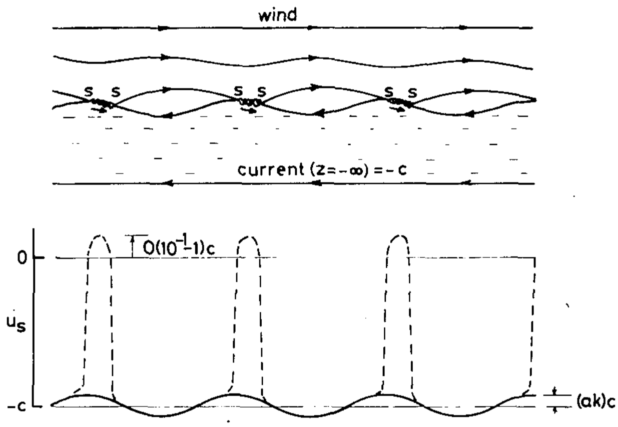

FIG. 1. A sketch of the separating air flow above breaking waves and the corresponding surface velocities. The dashed curve represents the surface velocity u_s for breaking waves, while the solid curve is for unbroken waves. S indicates the stagnation points at the front and rear of each break.

separating regions accompanying the turbulence generation would be advected at a finite distance above the interface. This would permit the establishment of a viscous sublayer contiguous with the surface, and so the atmospheric boundary layer would be smooth, or, at least, not fully rough. With extensive small-scale wave breaking occurring the continued development of the viscous layer will be impeded by a separating air flow. Simple order-of-magnitude estimates indicate that for typical field conditions a horizontal distance on the order of 1 m is required to establish a viscous layer having a thickness corresponding to smooth flow. Accordingly we expect that in a neutral[3] atmosphere the onset of extensive small-scale wave breaking will be accompanied by the transition to a fully rough boundary layer flow. In view of the spatial inhomogeneity and temporal intermittency of the flow it is difficult to quantify the concept of "extensive small-scale wave breaking." Neither the surface nor the flow abruptly changes in the mean from smooth to rough, the transition is more gradual. However, we anticipate that the transition occurs when a fraction of the surface of order unity is covered by small breaking waves.

b. Magnitude estimates

In addition to these qualitative descriptions the association of wave breaking with turbulence generation also permits us to derive some approximate quantitative results. First, let us consider the flow over an interface which, except for a train of breaking waves, is flat. We shall assume that the separated regions accompanying the waves are associated with the generation of any significant turbulence in the neighborhood of the boundary. That is, the length and velocity scales of the energetic turbulence correspond to those of the breaking waves. In a frame moving with the waves the air flow and the velocity at the surface are as shown in Fig. 1. For unbroken waves of the same phase speed c, the surface velocity fluctuations are $O(ak)c$, where a is the amplitude and k is the wavenumber. For unbroken waves (ak) is typically $O(10^{-2}$–$10^{-1})$. With the onset of breaking the surface velocity has zero crossings at the stagnation points at the front and rear of each break. Modeling by Longuet-Higgins and Turner (1974) predicts that the positive downslope velocity is $O(10^{-1})c$, so for a train of breaking waves the magnitude of the velocity fluctuations at the interface is $O(c)$. As the boundary layer is rough we expect the logarithmic region to extend to the immediate neighborhood of the surface. Now, from local turbulent energy equilibrium

$$\tau_0 \frac{\partial U}{\partial z} = \epsilon, \tag{6}$$

where ϵ is the rate of viscous dissipation, which is well approximated by $\rho_a u^3/l$, u is the turbulent velocity scale and $l = \kappa z$ is the turbulent length scale. But $u \sim O(c)$, so it follows immediately from (3) and (5) that

$$u_* = O(c). \tag{7}$$

It is difficult to improve this approximation; however, estimates of the rms horizontal surface velocity based on the model of Longuet-Higgins and Turner, and the experiments cited by them (see Table 1 of their paper) lead us to expect that

$$u_* \approx c, \tag{8}$$

with an error of not more than 50%.

This result is supported by this alternative argument: If the surface flow is aerodynamically rough (i.e., R_* large) the viscous transfer across the interface is small and Eqs. (3) and (4) may be combined to give

$$\rho_a u_*^2 \approx \overline{p_\eta \frac{\partial \eta}{\partial x}}. \tag{9}$$

The variation in pressure will be dominated by the velocity fluctuations at the surface and p_η will scale on c^2, i.e.,

$$p_\eta \sim \rho_a c^2.$$

The variations in $\partial\eta/\partial x$ associated with a breaking wave may be $O(1)$ (Longuet-Higgins, 1973); so from (9)

$$u_* = O(c). \tag{10}$$

The absolute magnitude of the right-hand side of (9)

[3] In an unstable atmosphere, or in the presence of a temperature decrease between the water and the air, buoyancy forces may be responsible for a significant fraction of the turbulence generation and the flow may be rough in the absence of breaking waves.

will depend on the phase relationship between the pressure and wave slope.

c. Over a modulated surface

This model of the interface, as a surface comprised of only small-scale breaking waves, is likely to be applicable at only the shorter fetches. Small-scale breaking waves in the ocean are normally to be found riding on a spectrum of large unbroken waves. A first step toward understanding this more complex problem is an examination of breaking waves riding on a monochromatic train of large waves which have a phase speed and wavenumber C and K, respectively. Their slope, AK, is much less than unity.

Since the Reynolds number of the smallest significant features of the flow is large, we again neglect the viscous transfer across the interface and use (9). It is physically instructive to decompose the right-hand side of this equation into a contribution from the large-scale pressure field which is coherent with the large unbroken wave and one from the pressure fluctuations associated with the small-scale breaking waves.

Thus, (9) becomes

$$\rho_a u_*^2 = \overline{\tilde{p}\frac{\partial\tilde{\eta}}{\partial x}} + \overline{p'\frac{\partial\eta'}{\partial x}}, \tag{11}$$

where $p=\tilde{p}+p'$ and $\eta=\tilde{\eta}+\eta'$. The tilde denotes the wave coherent part and the prime denotes the turbulent part which we are associating with the small-scale breaking waves.

While the quadratic dependence of the pressure on the velocity field introduces some difficulty in separately estimating the two terms on the right-hand side of (11), it does provide a mechanism for coupling the small-scale breaking waves to the larger underlying swell. The modulation of the train of small-scael breaking waves by the longer waves may result in the fluctuating velocity field associated with the breaking waves contributing to the component of p in phase with $\partial\tilde{\eta}/\partial x$. This is a possible mechanism for the small breaking waves leading to the growth of the larger unbroken waves.

The pressure fluctuations which are coherent with the larger wave will scale on the mean-square velocity averaged over a length scale which is much larger than that of the breaking waves but much smaller than K^{-1}. The surface velocity is comprised of the orbital velocity of the unbroken wave [$O(AKC)$] and the fluctuations [$O(c)$] from the small breaking waves. It follows that

$$\overline{\tilde{p}\frac{\partial\tilde{\eta}}{\partial x}} \sim \rho_a c^2(1+AKC/c)^2 AK. \tag{12}$$

While the underlying swell will lead to a direct modulation of the breaking wave train, we expect that for AK small enough and the ratio of wavelengths large enough,

$$\overline{p'\frac{\partial\eta'}{\partial x}} \sim \rho_a c^2 \tag{13}$$

will still be an appropriate approximation consistent with the accuracy of (8). It follows from (11), (12) and (13) that

$$u_*^2 \sim c^2[1+O(AK)], \tag{14}$$

where we have assumed that AKC/c is at most $O(1)$.

The approximations leading to (14) are relatively crude; nevertheless, (14) permits one important conclusion: We have argued that with extensive wave breaking at the surface, the boundary layer flow must be fully rough. Further, we have shown that if the small-scale breaking waves accompany the turbulence generation then the friction velocity u_* will be approximately equal to the phase velocity c of the breaking waves. Now c has a minimum value of ~ 23 cm s^{-1}, so, according to our model, the transition from smooth to rough flow in a neutral marine atmosphere cannot occur below ~ 23 cm s^{-1}. In fact, if, as we anticipate, it is the smallest gravity waves which are the first to break, then the transition will occur in the neighborhood of $u_*=23$ cm s^{-1}.

3. The roughness length

In boundary layers over immobile surfaces the roughness length is usually correlated with the height h of the elements from which the flow is separating. This procedure has also been used to correlate the roughness length of boundary layers over deformable roughness elements such as natural plant cover. Monin and Yaglom (1971) quote values of z_0/h in the range 1/30 (sand grains) to $\frac{1}{5}$ (plant cover). Unlike rigid surfaces, the roughness elements here (i.e., the breaking waves) are generated by the flow. Nevertheless, we express z_0 in terms of the geometry of the surface with no explicit consideration of the dynamics. In defense of this approach it should be stated that there is of course an implicit dependence of the surface geometry on the dynamics.

The small breaking waves, which constitute the roughness elements, are normally found to be intermittent in space and time. This suggests that the breaking is not strong enough to be self-maintaining (see Longuet-Higgins and Turner, 1974) and that the wave amplitude is little changed from its value at the point of breaking. If ζ_m is this maximum amplitude, then following earlier correlations of z_0 with h, we expect that

$$z_0 = M\zeta_m, \tag{15}$$

where $M=O(10^{-1}\text{-}1)$. In a recent paper Phillips and Banner (1974) derived an expression for the limiting amplitude of small gravity waves under the action of

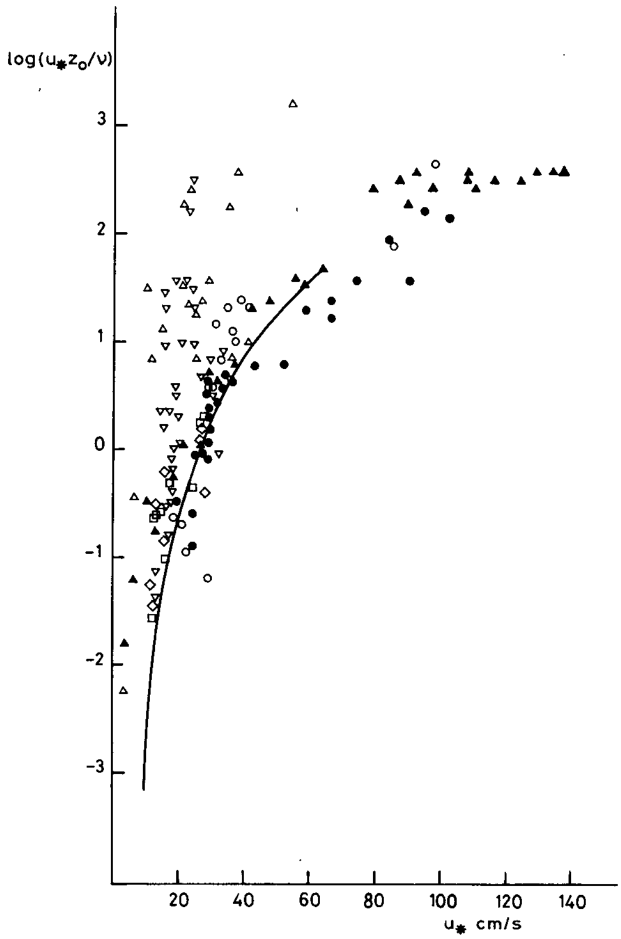

FIG. 2. Published field measurements of the variation of friction velocity u_* with the roughness Reynolds number u_*z_0/ν: Miyake *et al.* (1970), ◇ flux method, □ profile method; Ruggles (1970), △; Seesholtz (1968), ▽; Sheppard *et al.* (1972), solid curve; Smith and Banke (1975), ● thrust anemometer, ○ sonic anemometer; Wu (1969a) ▲.

wind drift and swell:

$$\zeta_m = 0.5(c_c - q_m)^2/g', \tag{16}$$

where $q_m = (C-u_0) - [(C-u_0)^2 - q_0(2C-q_0)]^{\frac{1}{2}}$ is the maximum value of the augmented wind drift, c_c is the phase velocity of the short waves at the long wave crest, g' the apparent local gravity, C the phase speed of the long wave and u_0 its maximum orbital velocity, and q_0 is the wind drift in the absence of swell. Defining the dimensionless height ζ^* by

$$\zeta^* = \zeta_m g c_0^{-2},$$

where c_0 is the phase speed of the short waves in the absence of swell, we have from (16) that

$$\zeta^* = \frac{1}{2\alpha^2}\left\{\alpha - \left[1 - \left(1 - \frac{\gamma(2-\gamma)}{(1-\beta)^2}\right)^{\frac{1}{2}}\right]\right\}^2, \tag{17}$$

where $\alpha = c_0/C$, $\beta = u_0/C$ and $\gamma = q_0/C$. From (14), (15) and (17)

$$z_0 \approx M\zeta^* u_*^2 g^{-1}. \tag{18}$$

Eq. (18) reduces to Charnock's (1955) equation in the absence of wind drift and swell.

4. Comparison with published measurements

a. Friction velocity

One of the more comprehensive collations of field measurements of wind stress and roughness at the sea surface is that of Wu (1969a). He correlated the measurements of 30 independent field studies by a number of workers, grouping the data into 1 m s^{-1} bands of U_{10}. No attempt was made to account for atmospheric stability or temperature gradients at the interface, so buoyancy effects are not isolated. He concluded that the transition from smooth to rough flow occurred in the range $3 \leqslant U_{10} \leqslant 7$ m s^{-1}, which corresponded to friction velocities in the range $16 \leqslant u_* \leqslant 36$ cm s^{-1}.

In a recent paper SethuRaman and Raynor (1975) correlated their own field measurements with those of Hsu (1971) and Seesholtz (1968). Of these measurements only those of Seesholtz were made at sea; the others were taken at coastal positions, reportedly close enough to the water for the influence of the land-sea interface to be negligible. Despite these conditions there are no apparent significant differences in the three sets of data. They reported that three drag regimes existed: smooth, moderately rough and fully rough with friction velocities in the range $u_* < 20$ cm s^{-1}, $15 \leqslant u_* \leqslant 25$ cm s^{-1} and $u_* > 20$ cm s^{-1}, respectively.

Sheppard *et al.* (1972) made an extensive set of measurements in near-neutral conditions, which show u_* in the range 18–30 cm s^{-1} for the transition regime $0.13 \leqslant R_* \leqslant 2$.

More recently Smith and Banke (1975) made measurements using both thrust and sonic anemometers mounted on a mast on a sandspit. The sensors were reported to be in a marine boundary layer with a minimum fetch of ~170 km. Their results show a transition to rough flow when u_* is in the range 25–30 cm s^{-1}.

All of these data are consistent with our argument that in a neutral marine atmosphere, the transition from smooth to rough flow may be expected to occur in the neighborhood of $U_* = 23$ cm s^{-1} (±50%). This behavior is shown in Fig. 2 where we have plotted the measurements cited above in the form R_* vs u_*. Note that for $R_* = 2$ (the nominal beginning of the fully rough regime) u_* is in the range 15–30 cm s^{-1}. Also plotted in Fig. 2 is the data of Ruggles (1970) which by displaying u_* as low as 10 cm s^{-1} for $R_* = 33$ is contrary to our result. However, that particular point is based on an average of only three 10 min profile measurements and should be weighed in the light of the other measurements which show significantly larger values of

u_* at this Reynolds number. What is of more interest perhaps is the relatively small change in u_* measured by Ruggles and Seesholtz when compared with that found by other workers. This corresponds to a large variation of the roughness length and hence the drag coefficient.

b. Onset of wave breaking

We know of no field measurements which may be used to accurately relate the onset of wave breaking to the characteristics of the atmospheric boundary layer. However, our argument that the onset of extensive breaking corresponds to rough flow is supported by the Beaufort wind scale and associated sea surface descriptions as published in the *Handbook of Oceanographic Tables* (U. S. Navy Oceanographic Office, 1966). The surface description: "Large wavelets, crests begin to break scattered whitecaps," is given for Beaufort 3, which covers the wind speed range 3.4–5.6 m s^{-1}. This is contained within the transition regime cited by Wu (1969a).

c. The drag coefficient

The drag coefficients measured in the experiments cited above are presented in Fig. 3. Apart from a few scattered points the agreement between the various sets of data is good and shows a significant increase in C_{10} with R_*. In particular, the straight line correlations of SethuRaman and Raynor (1975) show an increase in the gradient by a factor of 5 in going from smooth to rough flow. Over the transition regime C_{10} increases by a factor of 2; and, over the total range of R_* by a factor of 6.

In contrast to the measurements shown in Fig. 3 a number of workers have concluded that C_{10} varies little for moderate wind speeds. In particular Kraus (1972), from a review of some of the earlier evidence, concluded that the drag coefficient was sensibly constant: $(1.3\pm0.3)\times10^{-3}$ for U_{10} in the range 3–16 m s^{-1}. However, he did suggest (and emphasized by repetition: pp. 141, 163) that "If the drag coefficient does increase with wind velocity ... this might be explained by an approach to something like genuinely rough flow; that is, local separations of the boundary layer from the surface, associated with the mechanical disruption of the wave crests and with the formation of spindrift." This is just the mechanism which is at the crux of the arguments developed above. Our departure point is the association of local separation with turbulence generation and our demonstration that the pressure and slope fluctuations associated with the breaking of even the smallest waves may be large enough to transfer essentially all the momentum across the surface. That is, "genuinely rough flow" occurs when there is extensive breaking of even the smallest gravity-capillary waves.

It is of interest to note that for $u_*\approx23$ cm s^{-1} and $R_*\approx2$ (the onset of fully rough flow) $C_{10}\approx1.26\times10^{-3}$; in good agreement with the coefficient obtained by Kraus.

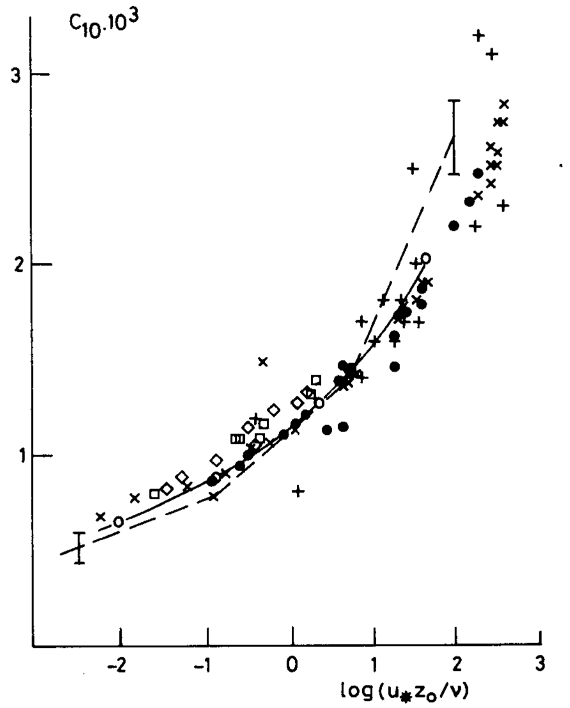

FIG. 3. Published field measurements of the variation of the surface drag coefficient C_{10} with the roughness Reynolds number, u_*z_0/ν: SethuRaman and Raynor (1975), dashed line, three independent sets of data; Wu (1969a), ×, 30 independent sets of data; Ruggles (1970), +; Sheppard *et al.* (1972), ○; Miyake *et al.* (1970), ◇ flux, □ profile; Smith, Banke (1975), ●. (Note that SethuRaman and Raynor published values of C_6, the drag coefficient at 6 m. We have used the conversion to C_{10} recommended by those authors: $C_{10}=0.92C_6$. This will lead to an error of less than ±2%.)

d. The roughness length

With the approximations used to derive (18),

$$\alpha\approx u_*/C, \tag{19a}$$

$$\beta\approx AK. \tag{19b}$$

In the ocean we may take C to correspond to the phase speed C_m of the waves at the spectral peak. (This assumption will break down at high wind speeds when extensive wave breaking may occur over much of the spectrum.) From Kraus (1972) we estimate both u_*/C_m and β to be $O(10^{-2}–10^{-1})$. The ratio $\gamma/\alpha=q_0/u_*$ is more difficult to specify for the ocean. To our knowledge no field measurements exist, but laboratory studies have been made. Measurements by Wu (1975) in a wind-wave facility show q_0/u_* reaching a maximum of ~0.67 at the onset of wave breaking ($u_*\approx40$ cm s^{-1}) and decreasing to what appears to be an asymptotic value of 0.4 at $u_*=100$ cm s^{-1}, the upper limit of his measure-

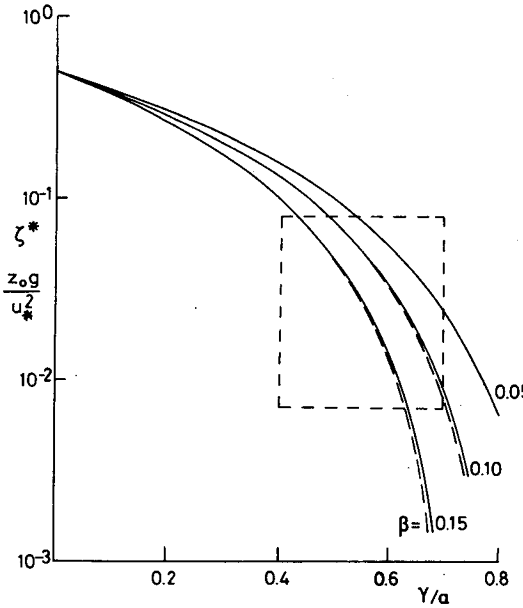

FIG. 4. The limiting dimensionless wave height ζ^* plotted as a function of γ/α, for $\beta=0.05$, 0.1, 0.15 and $\alpha=0.01$ (solid curves); 0.1 (dashed curves). Values of α and β are representative of oceanic conditions. The box is defined by the limits of the field measurements of $z_0 g/u_*^2$ (Charnock's "constant") and the laboratory measurements of γ/α. The intersection of the box by the theoretical curves shows (18) to be consistent with measurements if $M=O(1)$.

ments. These results are supported by Phillips and Banner (1974) whose laboratory study showed q_0/u_* reaching a maximum of 0.69, and decreasing with increasing wind speed.[4] Both sets of measurements are at odds with Wu's (1975) statement that "the wind friction velocity is the sole parameter for correlating this component" (i.e. the wind-drift). If this were so q_0/u_* would be constant. Nevertheless, the range of q_0/u_* found by Wu is consistent with the commonly accepted estimate of wind drift being 3–4% of U_{10}, i.e., with u_* being 4-6% of U_{10}. With these estimates of α, β and γ/α, we can compute ζ^* for the expected range of oceanic conditions.

Fig. 4 shows curves of ζ^* vs γ/α for typical values of α and β. The dependence on α is very weak in the range $0.01\leqslant\alpha\leqslant 0.1$. The dependence on the slope of the swell (β), and the wind drift (γ/α) is clearly much stronger. For example, with $\beta=0.1$ and $\alpha=0.01$, ζ^* decreases by a factor of 15 from approximately 1.25 $\times 10^{-1}$ to 8×10^{-3}. Also shown in Fig. 4 is the range of $z_0 g/u_*^2$ measured by a number of workers to be 8×10^{-2} to 7×10^{-3} (Kitaigorodskii, 1973; Kraus, 1972). This crude comparison shows that (18) may hold if $M=O(1)$.

A better test of (18) is afforded by comparison with the results of Sheppard *et al.* (1972) and Garratt (1973), for which the values of α and β may be reliably estimated. The measurements were made at Lough Neagh at fetches of 8 and 22 km, with U_{10} in the range 2–16 m s^{-1}. As mentioned above, the frequency of the swell is assumed to correspond to the peak of the wave spectrum, and its amplitude is taken to be the square root of the variance of the surface elevation. With these estimates $\beta\approx 0.056$ and α is in the range $(0.16–0.32)\beta^{\frac{1}{2}}$. Fig. 5a shows a plot of log u_* vs log z_0, taken from Fig. 5 of Sheppard *et al.* (1972). Also shown is (18) with $\gamma/\alpha=0.67$, 0.4 and $M=0.17$. With $\gamma/\alpha=0.4$, the value of M was chosen to fit the data at the largest measured u_*. It should be restated that $\gamma/\alpha=0.4$ was the value found by Wu (1975) for large u_* in his wind-wave tank. With this value of M and $\gamma/\alpha=0.67$, Eq. (18) is shown to intersect the experimental curve at $u_*=21$ cm s^{-1}; very close to the predicted value of u_* at the onset of extensive wave breaking. Wu (1975) found $\gamma/\alpha=0.67$ at the onset of breaking in his laboratory tank. An alternative presentation of the comparison between (18) and the measurements is shown in Fig. 5b. If the range of values of γ/α measured in the laboratory by Wu are applicable to the field measurements then Fig. 5 clearly demonstrates strong support for (18).

5. Discussion

The work reported here points to the importance of small-scale breaking waves in the process of mean momentum transfer across the air-sea interface. Wu (1969b) earlier recognized the importance of the shorter waves, having conjectured that separation occurs from waves having a phase velocity less than the friction

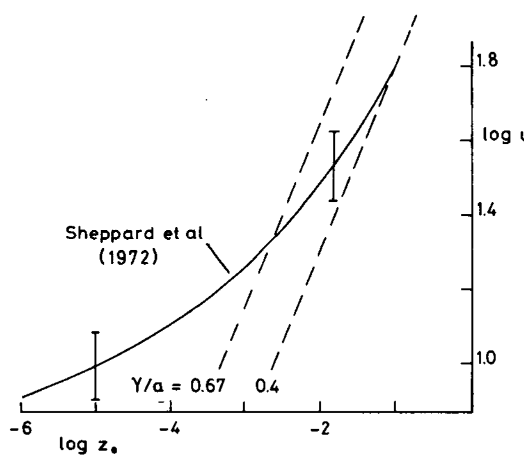

FIG. 5a. Smoothed measurements of Sheppard *et al* (1972) compared with (18) for $M=0.17$ and $\gamma/\alpha=0.4$, 0.67. With $\gamma/\alpha=0.4$, M was chosen to fit the data at the largest u_* measured (63 cm s^{-1}). For $\gamma/\alpha=0.67$, [the value found by Wu (1975) at the onset of breaking] Eq. (18) intersects the data at 21 ± 5 cm s^{-1}, in good agreement with $u_*=23$ cm s^{-1}, the hypothesized value of u_* at the onset of extensive wave breaking.

[4] It is possible that the relative decrease in the wind drift with increasing wind speed is evidence of the shift from viscous to pressure forces transferring the momentum across the interface.

velocity. In some respects Wu's approach is similar to that presented here; however, he was unable to physically account for the concomitance of flow separation and wave breaking, nor did he physically explain the relationship between the friction velocity and the phase velocity of the breaking waves. In a subsequent paper (Wu, 1970),[5] by assuming Charnock's expression and a roughness length proportional to a characteristic wavelength, it was shown that u_* was almost equal to a characteristic phase speed. However, there appears to be some inconsistency in the approximations used; the value of Charnock's "constant" used by Wu (1970) differs by a factor of 1.75 from that found in Wu (1969a), while the data employed in both papers are ostensibly the same. The more important differences between Wu's development and that given here are 1) that instead of conjecturing that there should be a relationship between u_* and the phase speed of the waves from which the flow separates, we have developed plausible physical arguments which lead to that relationship, albeit approximate; and 2) that instead of assuming Charnock's equation we have derived an expression for the roughness length which gives Charnock's result as a special case. Notwithstanding these important differences, Wu's speculation regarding the role of the shorter waves, here assumed to be breaking, is consistent with the physical model we have proposed.

Except for our use of a result of Phillips and Banner [1974, Eq. (16)], our treatment is based on simple physical arguments and order-of-magnitude estimates. Eq. (16) has been shown to be strongly dependent on γ/α, or q_0/u_*. The laboratory measurements of q_0/u_* have been shown to be consistent with the field measurements of z_0, but corroborating field measurements of the wind drift are needed to test the validity of (18). Unfortunately, such measurements are likely to be very difficult to make. A recent paper by Wright (1976) describing laboratory measurements of the attenuation of wind-generated waves by mechanically generated swell showed good agreement with the theory of Phillips and Banner (1974), only in the neighborhood of $u_* = 30$ cm s^{-1}. Otherwise, he found that the theory underpredicted the limiting amplitude of the wind waves. On the basis of these measurements it might be argued that our use of (16) is unjustified. However, two aspects of Wright's measurements may be strongly criticized. First, the values of q_0 and u_* were not measured in the same facility: the values of q_0 measured in one laboratory were "associated," in an unspecified manner, with the u_* measured in a secondary laboratory. Given the strong dependence of ζ^* on q_0/u_*, this is a serious deficiency in the experimental technique. Second, in many of the experiments there was little

[5] I wish to thank a referee for drawing my attention to this reference.

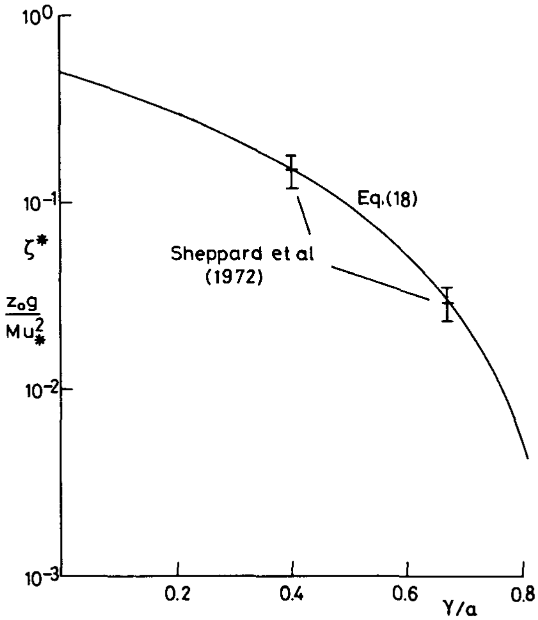

FIG. 5b. An alternative comparison of (18) with the data of Sheppard *et al.* (1972) plotted as ζ^* vs γ/α for $\beta=0.056$, $\alpha=(0.16-0.32)\beta^{\frac{1}{2}}$, the measured values of α and β. Also shown are the measured values of $M^{-1}z_0gu_*^{-2}$ ($M=0.17$) at $u_*=63$ and 23 cm s^{-1}. The former is the largest u_* measured by Sheppard *et al.*, and the latter corresponds to the hypothesized onset of extensive wave breaking. The abscissae are 0.4 and 0.67, respectively, the corresponding values found by Wu (1975) in his laboratory measurements. This comparison shows that if the variation of q_0/u_* in the ocean is the same as that measured in the laboratory, then (18) displays good agreement with the measured change in z_0.

separation of the phase speeds of the wind and mechanically generated waves, with the discrepancy between the theory and experiments increasing as this separation decreased. With the wide separation of scales which exists at the air-sea interface the theory should be applicable over a significant range of conditions, as is implied by the comparisons of Fig. (5).

More recently Kraus (1977) has examined the influence of the Coriolis force on the surface drift velocity, finding that in the steady state the surface velocity is in the direction of the surface geostrophic wind. This result might lead one to question the validity of our use of laboratory data to estimate the wind drift. However, the case Kraus considered is rather special and, as he recognized, his arguments may not be applicable to a wavy surface. Nevertheless, our use of laboratory data to estimate the surface drift is not entirely satisfactory, but in the absence of appropriate field data is probably the best possible course.

If the physical arguments and estimates employed in Sections 2 and 3 are reliable and if the measurements reviewed in Section 4 are representative, then we may conclude that in a neutral marine atmosphere the transition from smooth to rough flow is concomitant with the onset of small-scale wave breaking and occurs in the neighborhood of $u_*=23$ cm s^{-1}. In addition a significant body of experimental data shows an increase of C_{10} with increasing flow roughness (R_*). In particular, the data of Sheppard *et al.* (1972) strongly support our prediction of the roughness length [Eq. (18)].

In contrast, it must be conceded that there is a significant body of data which suggests that the drag coefficient is sensibly constant in the moderate wind speed range. We do not believe that the available evidence is strong enough to categorically resolve the behavior of the drag coefficient: one is always able to criticize some aspect of the experimental technique.

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