Covariance Functions and Spectra of a Random Wave Field

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ABSTRACT

Covariance functions and spectra of components of fluid particle velocity are obtained, taking into consideration the effects of free surface fluctuations, for a Gaussian, stationary and homogeneous random gravity wave field in deep water, using infinitesimal wave solutions. Approximate representations of the covariance functions and spectra are also derived. It is shown that the covariance functions and spectra presented in this paper differ from those when the effects of free surface fluctuations are ignored, especially at, around and above the equilibrium surface.

1. Introduction

The determination of fluid motion in a random wave field has been the subject of interest in recent years. In their attempts to understand wave dynamics and to interpret field measurements, oceanographers have addressed themselves to the statistical properties of random wave field associated with both surface waves (Shonting, 1967, 1968; Kenyon, 1970; Thornton and Krapohl, 1974) and internal waves (Phillips, 1971; Reid, 1971; Garrett and Munk, 1971). The subject is also of importance to engineers who are responsible for the design of ocean installations since fluid motion is a primary source of forcing function for which the structures must be designed to resist.

In considering the statistical properties of a wave field induced by surface waves, the influence of free surface fluctuations has invariably been ignored in the past. That is, due to fluctuations of the free surface, any point fixed in space, in the vicinity of the equilibrium surface, may rise above or fall below the free surface. At instants when the point under consideration is not submerged, there is no fluid motion. It was shown (Tung, 1975) that by considering the free-surface-fluctuations phenomenon, statistical properties such as probability function of the wave field deviate drastically from those when the phenomenon is ignored especially near or above the equilibrium surface. The statistical properties of an undulating layered medium are similarly affected by the fluctuations of the interfaces as was observed by Phillips (1971), Reid (1971), and Garrett and Munk (1971).

In this paper, taking into account the free-surface-fluctuations phenomenon, expressions of the covariance functions and spectra of the velocity components in a random wave field are derived. For easy reference, a brief account of the description of random sea, the materials of which are contained in Tung (1975), is first given below.

2. Specification of random sea

Consider a rectangular coordinate system with the z axis vertically upward and origin at the equilibrium surface. The free surface displacement, specified by $z=\zeta(\mathbf{x},t)$, and assumed to be Gaussian, stationary in time and homogeneous in horizontal plane, may be represented by (Phillips, 1969)

$$\zeta(\mathbf{x},t)=\int_{\mathbf{k}}\int_{n} dB(\mathbf{k},n)\exp[i(\mathbf{k}\cdot\mathbf{x}-nt)], \quad (2.1)$$

where $\mathbf{x}$ is the horizontal position vector, t is time, and $dB(\mathbf{k},n)$ is a zero mean, complex random function of wavenumber vector $\mathbf{k}$ and frequency n.

Under the assumptions of inviscid, incompressible fluid and irrotational motion, the associated velocity potential $\phi(\mathbf{x},z,t)$, in deep water, to the first order of approximation, is given by

$$\phi(\mathbf{x},z,t)=-i\int_{\mathbf{k}}\int_{n}\frac{n}{|\mathbf{k}|}dB(\mathbf{k},n)\exp(|\mathbf{k}|z) \times\exp[i(\mathbf{k}\cdot\mathbf{x}-nt)],$$

with the frequency

$$n=(g|\mathbf{k}|)^{\frac{1}{2}},$$

where g is gravitational acceleration.

Denoting the unit vector of the z axis by $\mathbf{e}_3$, the velocity vector is

$$\mathbf{u}(\mathbf{x},z,t)=\nabla\phi(\mathbf{x},z,t)$$

$$=-i\int_{\mathbf{k}}\int_{n}(i\mathbf{k}-|\mathbf{k}|\mathbf{e}_3)\frac{n}{|\mathbf{k}|}dB(\mathbf{k},n)\exp(|\mathbf{k}|z)\times\exp[i(\mathbf{k}\cdot\mathbf{x}-nt)]. \quad (2.2)$$

It is noted that (2.2) holds everywhere below the free surface. However, due to fluctuations of the free surface, any point in the vicinity of the equilibrium surface may at certain times not be submerged in which case there is no fluid motion at the point. Stated explicitly, the velocity vector is

$$\bar{\mathbf{u}}(\mathbf{x},z,t)=\mathbf{u}(\mathbf{x},z,t)H[\zeta(\mathbf{x},t)-z], \quad (2.3)$$

where $H(\ )$ is the Heaviside unit function.

It is immediately clear that while $\mathbf{u}(\mathbf{x},z,t)$ is Gaussian, $\bar{\mathbf{u}}(\mathbf{x},z,t)$, being a nonlinear function of Gaussian processes $\mathbf{u}(\mathbf{x},z,t)$ and $\zeta(\mathbf{x},t)$, is non-Gaussian.

3. Covariance functions and spectra

To determine the covariance functions and spectra of components $\bar{u}_1(\mathbf{x},z,t)$, $\bar{u}_2(\mathbf{x},z,t)$ and $\bar{u}_3(\mathbf{x},z,t)$ of velocity vector $\bar{\mathbf{u}}(\mathbf{x},z,t)$, it suffices to consider the random process

$$\bar{Y}(\mathbf{x},z,t)=Y(\mathbf{x},z,t)H[\zeta(\mathbf{x},t)-z], \quad (3.1)$$

in which $Y(\mathbf{x},z,t)$ is a stationary and homogeneous zero mean random process jointly Gaussian with $\zeta(\mathbf{x},t)$.

The covariance function of $\bar{Y}(\mathbf{x},z,t)$, in general, is the expected value of the product of $\bar{Y}(\mathbf{x},z,t)$ and $\bar{Y}(\mathbf{x}+\mathbf{r},z+r_3,t+\tau)$ in which $\mathbf{r}$ is the horizontal position separation vector, r_3 the vertical position separation variable, and τ the time lag. In this study, to demonstrate the idea underlying the derivation of the convariance function, for convenience, r_3 is set equal to zero.

The covariance function of $\bar{Y}(\mathbf{x},z,t)$, thus defined, and in anticipation that $\bar{Y}(\mathbf{x},z,t)$ is covariance stationary in time and homogeneous in the horizontal plane, is denoted by $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$ and is given by

$$R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)=E[\{\bar{Y}(\mathbf{x},z,t)-E[\bar{Y}(\mathbf{x},z,t)]\}\times\{\bar{Y}(\mathbf{x},\mathbf{r},z,t+\tau)-E[\bar{Y}(\mathbf{x}+\mathbf{r},z,t+\tau)]\}], \quad (3.2)$$

where $E[\ ]$ is the expected value of the quantity enclosed in the bracket.

In (3.2), the quantity $E[\bar{Y}(\mathbf{x},z,t)]$ was obtained previously (Tung 1975) and is

$$E[\bar{Y}(\mathbf{x},z,t)]=r_{Y\zeta}(0,0)\sigma_Y Z(b), \quad (3.3)$$

where

$$Z(\xi)=(2\pi)^{-\frac{1}{2}}\exp(-\tfrac{1}{2}\xi^2),$$

$$b=\frac{z}{\sigma_\zeta},$$

and

$$\sigma_\zeta=\left[\int_{\mathbf{k}}\int_{n}X(\mathbf{k},n)d\mathbf{k}dn\right]^{\frac{1}{2}}$$

is the standard deviation of $\zeta(\mathbf{x},t)$ as can be obtained from (2.1); $X(\mathbf{k},n)$ is the wavenumber-frequency spectrum of $\zeta(\mathbf{x},t)$ (Phillips, 1969).

The quantity σ_Y in (3.3) is the standard deviation of $Y(\mathbf{x},z,t)$. If $Y(\mathbf{x},z,t)$ is identified with the jth component $u_j(\mathbf{x},z,t)$, $j=1, 2, 3$, of $\mathbf{u}(\mathbf{x},z,t)$, it may be verified, from (2.2), that

$$\sigma_Y=\sigma_{uj}=\left[\int_{\mathbf{k}}\int_{n}\frac{k_j^2}{|\mathbf{k}|^2}n^2X(\mathbf{k},n)\exp(2|\mathbf{k}|z)d\mathbf{k}dn\right]^{\frac{1}{2}},\quad j=1,2,$$

where k_j is the jth component of $\mathbf{k}$ and

$$\sigma_Y=\sigma_{u_3}=\left[\int_{\mathbf{k}}\int_{n}n^2X(\mathbf{k},n)\exp(2|\mathbf{k}|z)d\mathbf{k}dn\right]^{\frac{1}{2}}.$$

The correlation function $r_{Y\zeta}(\mathbf{r},\tau)$ is

$$r_{Y\zeta}(\mathbf{r},\tau)=E[Y(\mathbf{x},z,t)\zeta(\mathbf{x}+\mathbf{r},t+\tau)]/\sigma_Y\sigma_\zeta.$$

For the case $Y(\mathbf{x},z,t)=u_j(\mathbf{x},z,t)$, from (2.1) and (2.2),

$$r_{Y\zeta}(\mathbf{r},\tau)=r_{u_j\zeta}(\mathbf{r},\tau)=\left\{\int_{\mathbf{k}}\int_{n}\frac{k_j}{|\mathbf{k}|}X(\mathbf{k},n)\exp(|\mathbf{k}|z)\times\exp[i(\mathbf{k}\cdot r-n\tau)]d\mathbf{k}dn\right\}\Big/(\sigma_{u_j},\sigma_\zeta),\quad j=1,2, \quad (3.4)$$

$$r_{Y\zeta}(\mathbf{r},\tau)=r_{u_3\zeta}(\mathbf{r},\tau)=i\left\{\int_{\mathbf{k}}\int_{n}nX(\mathbf{k},n)\exp(|\mathbf{k}|z)\times\exp[i(\mathbf{k}\cdot\mathbf{r}-n\tau)]d\mathbf{k}dn\right\}\Big/(\sigma_{u_3}\sigma_\zeta). \quad (3.5)$$

It is seen that while the expected value of $\bar{u}_3(\mathbf{x},z,t)$ vanishes, those of $\bar{u}_j(\mathbf{x},z,t)$, $j=1, 2$, remain non-zero. Also, σ_ζ, σ_Y and $r_{Y\zeta}(\mathbf{r},\tau)$ are independent of $\mathbf{x}$ and t and hence so is $E[\bar{Y}(\mathbf{x},z,t)]$. Eq. (3.2) may then be rewritten as

$$R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)=E[\bar{Y}(\mathbf{x},z,t)\bar{Y}(\mathbf{x}+\mathbf{r},z,t+\tau)]-E^2[\bar{Y}(\mathbf{x},z,t)]. \quad (3.6)$$

For convenience in subsequent development, the subscripts 1 and 2 will be used to correspond to quantities evaluated at $(\mathbf{x},z,t)$ and $(\mathbf{x}+\mathbf{r},z,t+\tau)$ respectively. Thus, from (3.1)

$$\left.\begin{aligned}\bar{Y}_1&=Y_1H(\zeta_1-z)\\ \bar{Y}_2&=Y_2H(\zeta_2-z)\end{aligned}\right\},$$

and the first term on the right hand side of (3.6) is

$$\begin{aligned}E[\bar{Y}_1\bar{Y}_2]&=E[Y_1Y_2H(\zeta_1-z)H(\zeta_2-z)]\\ &=E\{H(\zeta_1-z)H(\zeta_2-z)E[Y_1Y_2|\zeta_1\zeta_2]\}\end{aligned}\quad(3.7)$$

(Papoulis, 1965), where $E[Y_1Y_2|\zeta_1\zeta_2]$ is the conditional expected value of Y_1Y_2 given the values of ζ_1 and ζ_2. Since Y_1, Y_2, ζ_1 and ζ_2 are individually zero mean and jointly Gaussian random quantities (Papoulis, 1965),

$$E[Y_1Y_2|\zeta_1\zeta_2]=m_{Y_1|\zeta_1\zeta_2}m_{Y_2|\zeta_1\zeta_2}+C_{Y_1Y_2|\zeta_1\zeta_2},\quad(3.8)$$

where $m_{Y_1|\zeta_1\zeta_2}$ is the conditional expected value of Y_1 given ζ_1 and ζ_2 and is given by (Papoulis, 1965)

$$m_{Y_1|\zeta_1\zeta_2}=a_1\zeta_1+a_2\zeta_2,$$

a linear function of ζ_1 and ζ_2, where

$$\left.\begin{aligned}a_1&=\sigma_Y\sigma_\zeta^3[r_{Y\zeta}(0,0)-r_{\zeta\zeta}(\mathbf{r},\tau)r_{Y\zeta}(\mathbf{r},\tau)]/\Delta\\ a_2&=\sigma_Y\sigma_\zeta^3[r_{Y\zeta}(\mathbf{r},\tau)-r_{\zeta\zeta}(\mathbf{r},\tau)r_{Y\zeta}(0,0)]/\Delta\\ \Delta&=\sigma_\zeta^4[1-r_{Y\zeta}^2(\mathbf{r},\tau)]\end{aligned}\right\},$$

where, from (2.1),

$$\begin{aligned}r_{\zeta\zeta}(\mathbf{r},\tau)&=E[\zeta(\mathbf{x},t)\zeta(\mathbf{x}+\mathbf{r},t+\tau)]/\sigma_\zeta^2\\ &=\left[\int_{\mathbf{k}}\int_n X(\mathbf{k},n)\exp[i(\mathbf{k}\cdot\mathbf{x}-n\tau)]d\mathbf{k}dn\right\}\Big/\sigma_\zeta^2.\end{aligned}\quad(3.9)$$

Similarly,

$$m_{Y_2|\zeta_1\zeta_2}=a_2\zeta_1+a_1\zeta_2.$$

The conditional covariance function $C_{Y_1Y_2|\zeta_1\zeta_2}$ is (Papoulis, 1965)

$$C_{Y_1Y_2|\zeta_1\zeta_2}=\sigma_Y^2\{r_{YY}(\mathbf{r},\tau)-\sigma_\zeta^4[2r_{Y\zeta}(0,0)r_{Y\zeta}(\mathbf{r},\tau)-r_{\zeta\zeta}(\mathbf{r},\tau)(r_{Y\zeta}^2(0,0)+r_{Y\zeta}^2(\mathbf{r},\tau))]/\Delta\},$$

where

$$r_{YY}(\mathbf{r},\tau)=E[Y(\mathbf{x},z,t)Y(\mathbf{x}+\mathbf{r},z,t+\tau)]/\sigma_Y^2,$$

and depending on whether $Y(\mathbf{x},z,t)$ corresponds to $u_j(\mathbf{x},z,t)$, $j=1, 2, 3$, it may be obtained from (2.2). That is,

$$r_{u_ju_j}(\mathbf{r},\tau)=\left\{\int_{\mathbf{k}}\int_n\frac{k_j^2}{|\mathbf{k}|^2}n^2X(\mathbf{k},n)\exp(2|\mathbf{k}|z)\times\exp[i(\mathbf{k}\cdot\mathbf{r}-n\tau)]d\mathbf{k}dn\right\}\Big/\sigma_{u_j}^2,\quad j=1, 2,\quad(3.10)$$

$$r_{u_3u_3}(\mathbf{r},\tau)=\left\{\int_{\mathbf{k}}\int_n n^2X(\mathbf{k},n)\exp(2|\mathbf{k}|z)\times\exp[i(\mathbf{k}\cdot\mathbf{r}-n\tau)]d\mathbf{k}dn\right\}\Big/\sigma_{u_3}^2.\quad(3.11)$$

It is noted that $C_{Y_1Y_2|\zeta_1\zeta_2}$ is not a function of ζ_1 and ζ_2.

Inserting $m_{Y_1|\zeta_1\zeta_2}$, $m_{Y_1|\zeta_1\zeta_2}$, $m_{Y_2|\zeta_1\zeta_2}$ and $C_{Y_1Y_2|\zeta_1\zeta_2}$ into (3.8), and (3.8) into (3.7), gives

$$\begin{aligned}E[\bar{Y}_1\bar{Y}_2]=&C_{Y_1Y_2|\zeta_1\zeta_2}E[H(\zeta_1-z)H(\zeta_2-z)]\\ &+2a_1a_2E[\zeta_1^2H(\zeta_1-z)H(\zeta_2-z)]\\ &+(a_1^2+a_2^2)E[\zeta_1\zeta_2H(\zeta_1-z)H(\zeta_2-z)].\end{aligned}$$

Since ζ_1 and ζ_2 are jointly Gaussian, the expected values on the right-hand side of the above equation can all be determined, giving

$$\begin{aligned}E[\bar{Y}_1\bar{Y}_2]=\sigma_Y^2\Bigg\{&r_{YY}(\mathbf{r},\tau)L[b,b,r_{\zeta\zeta}(\mathbf{r},\tau)]\\ &+2r_{Y\zeta}(0,0)r_{Y\zeta}(\mathbf{r},\tau)bZ(b)Q\left[\frac{b[1-r_{\zeta\zeta}(\mathbf{r},\tau)]}{[1-r_{\zeta\zeta}^2(\mathbf{r},\tau)]^{\frac{1}{2}}}\right]\\ &+\frac{1}{\{2\pi[1-r_{\zeta\zeta}^2(\mathbf{r},\tau)]\}^{\frac{1}{2}}}\\ &\times[r_{Y\zeta}^2(0,0)+r_{Y\zeta}^2(\mathbf{r},\tau)-2r_{Y\zeta}(0,0)r_{\zeta\zeta}(\mathbf{r},\tau)r_{Y\zeta}(\mathbf{r},\tau)]\\ &\times Z\left[\frac{2^{\frac{1}{2}}b}{[1+r_{\zeta\zeta}(\mathbf{r},\tau)]^{\frac{1}{2}}}\right]\Bigg\},\end{aligned}\quad(3.12)$$

where

$$Q(\lambda)=\int_0^\lambda Z(\xi)d\xi$$

and (Abramowitz and Segun, 1968)

$$L[b,b,r_{\zeta\zeta}(\mathbf{r},\tau)]=\int_b^\infty Z(\lambda)Q\left[\frac{b-r_{\zeta\zeta}(\mathbf{r},\tau)\lambda}{[1-r_{\zeta\zeta}^2(\mathbf{r},\tau)]^{\frac{1}{2}}}\right]d\lambda.$$

Eq. (3.12), together with (3.3), determines the covariance function $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$. That $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$ is indeed independent of $\mathbf{x}$ and t is clearly seen, indicating that $\bar{Y}(\mathbf{x},z,t)$ is covariance stationary in time and homogeneous in horizontal space. It can be verified that when the point under consideration is far below the equilibrium surface where the influence of free surface fluctuations is small

$$\lim_{r_{Y\zeta}(\mathbf{r},\tau)\to 0}R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)=\sigma_Y^2r_{YY}(\mathbf{r},\tau)=R_{YY}(\mathbf{r},\tau),$$

where $R_{YY}(\mathbf{r},\tau)=E[Y(\mathbf{x},z,t)Y(\mathbf{x}+r,z,t+\tau)]$ is the covariance function of the zero mean random process $Y(\mathbf{x},z,t)$. Far above the equilibrium surface, it may similarly be verified that

$$\lim_{z\to\infty}R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)=0$$

whereas $R_{YY}(\mathbf{r},\tau)$ grows indefinitely.

The spectrum of $\bar{Y}(\mathbf{x},z,t)$, denoted $X_{\bar{Y}\bar{Y}}(\mathbf{k},n)$, may be obtained by taking the Fourier transform of $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$. That is

$$X_{\bar{Y}\bar{Y}}(\mathbf{k},n)=\frac{1}{(2\pi)^3}\int_{\mathbf{r}}\int_\tau R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)\times\exp[-i(\mathbf{k}\cdot r-n\tau)]\,d\mathbf{r}d\tau.\quad(3.13)$$

The above integrals, however, cannot be carried out in closed form and must be performed numerically.

4. Approximate representation

To facilitate computation of $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$ and especially $X_{\bar{Y}\bar{Y}}(\mathbf{k},n)$, it is desirable to simplify the expression of $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$ so that the Fourier transform may be carried out more easily. It is noted that the absolute values of $r_{\xi\xi}(\mathbf{r},\tau)$, $r_{Y\xi}(\mathbf{r},\tau)$ and $r_{YY}(\mathbf{r},\tau)$ are all smaller than unity. This suggests that $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$, considered as a function of $r_{\xi\xi}(\mathbf{r},\tau)$, $r_{Y\xi}(\mathbf{r},\tau)$ and $r_{YY}(\mathbf{r},\tau)$, may be expanded by Taylor's series around the point $r_{\xi\xi}(\mathbf{r},\tau)=r_{YY}(\mathbf{r},\tau)=r_{Y\xi}(\mathbf{r},\tau)=0$.

Without presenting the detailed operations involved, the approximate representation of $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$ is, by retaining only terms up to the first power of $r_{\xi\xi}(\mathbf{r},\tau)$, $r_{YY}(\mathbf{r},\tau)$ and $r_{Y\xi}(\mathbf{r},\tau)$:

$$R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)=\sigma_Y^2[r^2_{Y\xi}(0,0)b^2Z^2(b)r_{\xi\xi}(\mathbf{r},\tau)+2r_{Y\xi}(0,0)bZ(b)Q(b)r_{Y\xi}(\mathbf{r},\tau)+Q^2(b)r_{YY}(\mathbf{r},\tau)]. \quad (4.1)$$

The corresponding approximate spectrum $X_{\bar{Y}\bar{Y}}(\mathbf{k},n)$ is the Fourier transform of (4.1). The Fourier transforms of the quantities $r_{\xi\xi}(\mathbf{r},\tau)$, $r_{Y\xi}(\mathbf{r},\tau)$ and $r_{YY}(\mathbf{r},\tau)$ on the right-hand side of (4.1) are given by (3.9), (3.4), (3.5) and (3.10), (3.11).

It is of interest to note that according to the approximate representation of $R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)$ [given by (4.1)], at the equilibrium surface $z=0$ we have

$$R_{\bar{Y}\bar{Y}}(\mathbf{r},\tau)=\tfrac{1}{4}R_{YY}(\mathbf{r},\tau),$$

indicating that by considering the free surface fluctuations, the covariance function is only one-fourth of that when the phenomenon is ignored. The same observation may be made of the spectra by virtue of (3.13).

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