The Role of Vertical Motion in the Heat Budget of the Upper Northeastern Pacific Ocean

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ABSTRACT

A study of the heat budget of the upper northeastern Pacific Ocean demonstrates the importance of heat changes due to vertical motion. Bathythermograph observations are used to form time series of monthly average thermal structure at ocean weather station November (30°N, 140°W) and various 5° and 2° quadrangles between California and Hawaii. These time series are first used to explore correlations between heat content in a 250 m thick layer and the depth of the 14°C isotherm. High negative correlations between these quantities suggest that vertical motion at 250 m, indicated by the fluctuations of the 14°C isotherm, must play a significant role in altering the heat content of the upper layer. A heat budget equation is derived that includes the heat changes due to vertical motion. At all test locations, correlations between terms show that a stronger relationship exists between surface heat exchange and heat content change when the heat changes due to vertical motion are included. The mean square difference between surface heat exchange and heat content change is computed with and without heat changes due to vertical motion. The ratio of these differences shows about a 50% improvement when heat changes due to vertical motion are included.

1. Introduction

Many studies of the ocean's heat budget have shown that the change in heat content of a given portion of the upper ocean does not equal the net surface heat exchange. This means that heat must be added to the water column or lost from it through processes not occurring at the ocean's surface. Many investigators (Pattullo, 1957; Bryan and Schroeder, 1960; Roden, 1959; Clark, 1967; Bathen, 1970) have tried to explain this difference between the change in heat content ($\partial H/\partial t$) and surface heat exchange (Q) as representing the heat change due to horizontal advection across temperature gradients. The results of these various studies were inconclusive, and the importance of horizontal advection in altering the ocean's heat content is still not clear. Recently, Gill and Niiler (1972) examined the theory of seasonal variability and concluded that horizontal advection might not be very important in the seasonal change of heat content.

Another possible means of changing the heat content of a fixed layer is by vertical motion associated with horizontal convergence or divergence. Using data taken at ocean weather station November (30°N, 140°W), this was explored by comparing a time series of monthly average heat content in the 0 to 250 m layer (H_{250}) with a similar series for the depth of the 14°C isotherm (Fig. 1). The 14°C isotherm was chosen because it never drops below 250 m depth, and it also accurately represents the fluctuations in depth of all isotherms between 150 and 250 m. It was reasoned that vertical motion related to flow divergence would cause fluctuations in the depth of deeper isotherms; in response, the heat content of a given water column should increase as the isotherms moved down and decrease as they moved up. Such a negative correlation between H_{250} and the depth of the 14°C isotherm can be seen in Fig. 1. The correlation coefficient for this comparison is -0.75 and is significant at the 99% level.

This study will further examine the relationship between heat content and vertical motion inferred from temperature fluctuations. Correlations as noted above between heat content and isotherm depth will be computed for various 5° and 2° quadrangles between California and Hawaii. To establish how the heat changes due to divergence help to explain the difference between $\partial H/\partial t$ and Q, a heat budget equation will be derived that includes the heat changes due to vertical motion and divergence. Correlations and a comparison of mean square differences between terms in this equation will show that the agreement between $\partial \mathrm{H}/\partial T$ and Q improves significantly when the heat change due to divergence is included.

2. Data

The temperature data used in this study came from bathythermograph observations taken between California and Hawaii. All temperature values were

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averaged within the month in which the observations were taken and also over various horizontal areas.

The time series of temperature structure at ocean weather station November (Fig. 1) is based on monthly values computed by Ballis (1973). These monthly values were computed from BT observations taken by vessels occupying station November and were restricted to a 1° square around 30°N, 140°W (shown as N in Fig. 2). Since these ships took as many as four BT casts per day, the resulting monthly averages are based on a large number of observations. The series reported by Ballis runs from 1947 to 1971 with some very large gaps. In order to have a quasi-continuous series, this study was restricted to the years 1962–70 in which only a few short gaps exist.

Time series of temperature structure at eight quadrangles of 5° of latitude and longitude were computed along a line between Hawaii and California. The data in this area were largely the result of the XBT program of the National Marine Fisheries Service. In this program XBT sections have been taken twice a month since 1966 between Honolulu and San Francisco. In an effort to increase the density of observations in these quadrangles, other BT observations were added from the National Oceanographic Data Center's mechanical BT file and the Fleet Numerical Weather Central's XBT file. The data were carefully checked to avoid duplication.

This combination of data sets showed the best coverage between the years 1964 and 1973. The data in this 10-year period were sorted into 5° quadrangles and monthly averages at 20 m depth intervals were computed. The eight areas mentioned above were then chosen for study and will be referred to as A through H; their locations are given in Fig. 2.

Similar time series were computed for four 2° quadangles where data coverage was adequate (more than 60% of the months contained data). These four quadrangles will be referred to as 1 through 4, 1 being the westernmost one as shown in Fig. 2.

The surface heat exchange data used in the correlations between heat budget terms were taken from computations made by the Bureau of Commercial Fisheries. The average values of Q were computed for

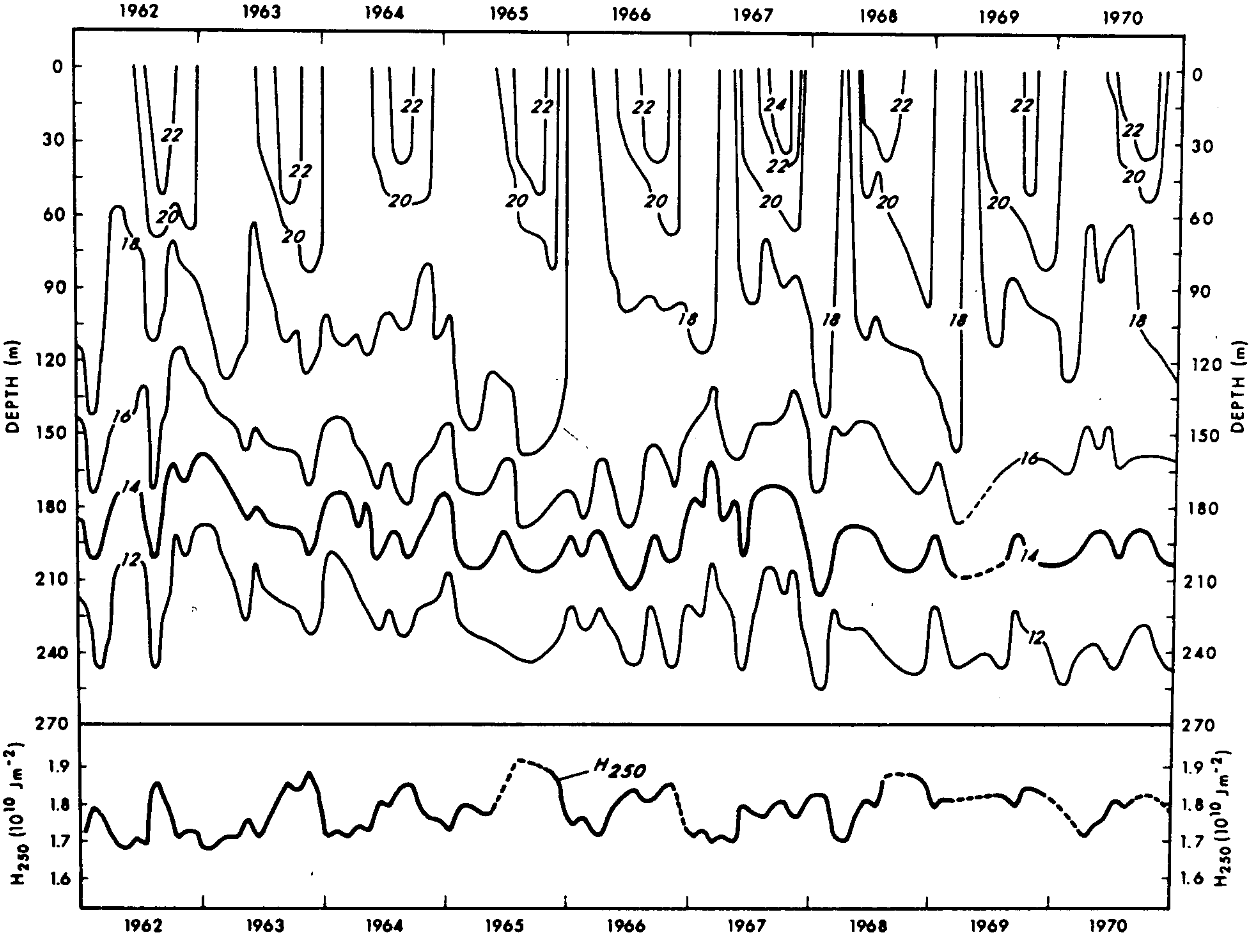

FIG. 1. (Top) Temperature structure (°C) between 0 and 250 m for the years 1962–70 at weather station November. (Bottom) Heat content in the layer 0 to 250 m for the years 1962–70 at weather station November.

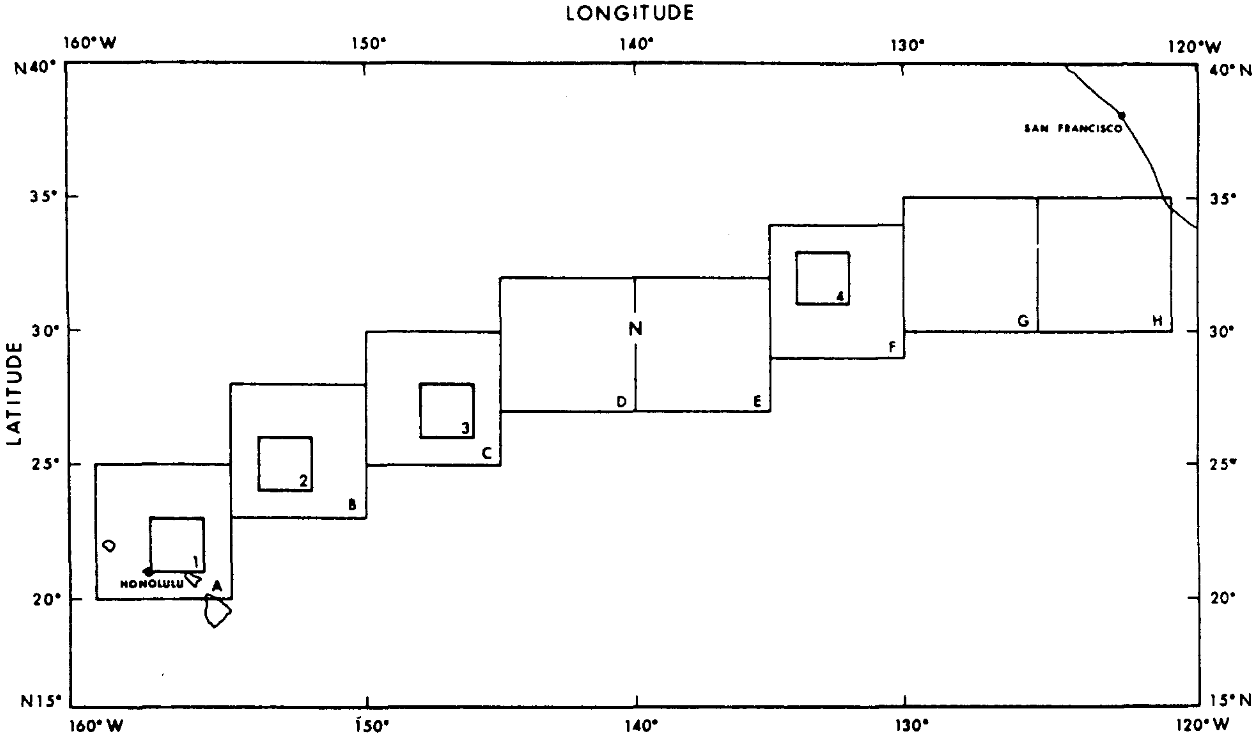

FIG. 2. Locations of the 2° and 5° quadrangles used in this study.

each month and each 5° quadrangle east of 180°W for the years between 1961 and 1973. The computations of Q were made from observations of surface temperature, air temperature, wet-bulb temperature, and wind and cloud cover as summarized in Johnson *et al.* (1965).

The number of BT observations comprising each monthly average temperature value varied greatly from month to month and area to area. In order to determine if the month-to-month changes between average temperature values were meaningful, the mean standard error was computed for two temperature values adjacent in time, and compared to the temperature change between the same two temperatures. If the mean standard error was less than the computed change in temperature, the change was considered to be statistically significant. Of the approximately 1500 temperature differences used in each series, about 85% were significant on the basis of this test. This percentage is typical of the averages for the 5° and 2° quadrangles, as shown in Emery (1975).

3. Correlations between heat content and temperature

The first evidence of the relationship between vertical motion and heat content was a strong negative correlation between heat content per unit area in the layer of 0 to 250 m depth and the depth of the 14°C isotherm at weather station November. Similar correlations, computed for all 2° and 5° quadrangles (see Table 1) were also strong, further confirming the importance of vertical motion in changing heat content. These strong correlations in both the 2° and 5° quadrangles suggested that the horizontal length of the process responsible for the observed temperature and heat content fluctuations must be greater than 500 km. Such a large process would be adequately sampled by either the 2° or 5° quadrangles as indicated by these correlations.

Correlations were also used to further explore the relationship between temperature and heat content. In order to determine the depth dependence of this relationship, temperatures at three different depth levels were correlated with H_{250}. First, surface temperature (T_0) was used to represent heat changes occurring at the ocean's surface. Similarly, the temperature at 200 m (T_{200}) was taken to represent heat changes at the bottom of the 250 m layer. Then the 200 m temperature was chosen as being representative of temperature changes in the 150 to 250 m layer. The temperature at 60 m (T_{60}) was chosen as an intermediate temperature between the surface (T_0) and bottom (T_{200}) influences.

The correlations were computed for three different sequences— the mean annual, the individual monthly, and the anomaly. The mean annual series is the average of all Januaries, all Februaries, and so forth, over the entire record. (It should be noted that the mean annual cycle is often referred to by other investigators as the mean seasonal variation.) The individual monthly refers to the original time series of monthly averages,

TABLE 1. Correlations between H_{250} and the depth of the 14°C isotherm.

Station November	2° Quadrangles			
	1	2	3	4
−0.75	−0.85	−0.83	−0.72	−0.66

5° Quadrangles							
A	B	C	D	E	F	G	H
−0.72	−0.72	−0.72	−0.66	−0.75	−0.86	−0.71	−0.67

and the anomaly series is the difference between the individual monthly and the mean annual.

The correlations for station November (Table 2) clearly depict the importance of T_0 at the mean annual scale and the importance of T_{200} at the anomaly scale. The mean annual correlation between temperature and H_{250} decreased steadily as deeper temperatures were used, while the anomaly correlation steadily increased. This suggested that on a mean annual scale, heat content is related most strongly to surface processes, while on an anomaly scale, heat content is related more to processes occurring at depth.

The correlations between different temperature records helped to establish the independence or dependence of these temperatures. The lack of significant correlation between T_0 and T_{200}, at all time scales, showed these temperatures to be indicators of very different processes. The higher correlations between T_0 and T_{60} revealed that T_{60} is strongly governed by surface influences.

Similar temperature-heat content correlations were computed for all 2° and 5° quadrangles. As discussed in Emery (1975) these all showed a pattern similar to that at station November. The quadrangles west of station November followed this pattern closely while the quadrangles east of 140°W deviated more and more toward the east.

4. The divergent heat budget equation

The correlations in Tables 1 and 2 suggest that vertical motion alters the heat content of the upper layer. In order to evaluate the extent to which this motion changes the heat content a heat budget is derived that includes heat fluxes by vertical motion through the bottom of a fixed upper layer.

Consider a vertical column of water of horizontal cross-sectional area A and fixed depth D below the main thermocline. Neglecting molecular diffusion, radiative heat flux within the column, and turbulent mixing at the boundaries, we can write the heat budget of the column as

$$\frac{\partial}{\partial t}\iint_A \int_0^D \rho C_p T dz d\sigma = \iint_S \rho C_p T \hat{\mathbf{V}}\cdot\hat{\mathbf{n}} d\sigma - QA, \quad (1)$$

where ρ is the density, C_p the specific heat at constant pressure, S the total surface area of the column, $\hat{\mathbf{n}}$ the outward pointing normal vector, T the temperature, z the vertical coordinate (positive downward), $d\sigma$ an element of area, and $\hat{\mathbf{V}}$ the three-dimensional velocity vector; Q is the net heat flux per unit area due to incoming solar radiation and longwave radiation from the atmosphere, less the emitted longwave radiation, latent heat flux, and sensible heat flux to the atmosphere from the sea surface (Johnson *et al.*, 1965).

Using the divergence theorem of Gauss we can write (1) as

$$\frac{\partial}{\partial t}\iint_A \int_0^D \rho C_p T dz d\sigma = \iint_A \int_0^D \rho C_p \nabla\cdot\hat{\mathbf{V}} T dz d\sigma - QA. \quad (2)$$

Approximating ρC_p by a constant and considering the integrands over area A to be nearly uniform, Eq. (2) becomes

$$\frac{\partial H}{\partial t} = \rho C_p \int_0^D \nabla\cdot\hat{\mathbf{V}} T dz - Q, \quad (3)$$

where

$$H = \rho C_p \int_0^D T dz \quad (4)$$

represents the heat content per unit surface area. Now

TABLE 2. Heat content-temperature correlations at ocean weather station November (30°N, 140°W): upper value, mean annual; middle value, individual monthly; lower value, anomaly. (See text for the definition of these terms.)

	T_0	T_{60}	T_{200}
H_{250}	0.83 0.65 0.38	0.79 0.71 0.58	0.58 0.75 0.87
T_0		0.66 0.65 0.48	0.35 0.13 0.04
T_{60}			0.36 0.24 0.28

in order to introduce vertical velocity w the second term in (3) is separated into horizontal and vertical parts whereby (3) becomes

$$\frac{\partial H}{\partial t}=\rho C_p\int_0^D \nabla_h\cdot(\hat{\mathbf{V}}_h T)dz+\rho C_p\int_0^D \frac{\partial}{\partial z}(wT)dz-Q, \quad (5)$$

where h indicates horizontal quantities. Expanding the second term and considering w to be 0 at the surface and W_D at D we can write (5) as

$$\frac{\partial H}{\partial t}=\rho C_p\int_0^D T(\nabla_h\cdot\hat{\mathbf{V}}_h)dz+\rho C_p\int_0^D \mathbf{V}_h\cdot\nabla_h T dz +\rho C_p W_D T_D-Q. \quad (6)$$

Assuming $\hat{\mathbf{V}}_h$ to be the average velocity in the layer 0 to D the continuity equation can be written as

$$\nabla_h\cdot\hat{\mathbf{V}}_h=-\frac{W_D}{D}. \quad (7)$$

With this assumption and continuity (6) becomes

$$\frac{\partial H}{\partial t}-\frac{W_D}{D}(H-\rho C_p D T_D)+\hat{\mathbf{V}}_h\cdot\nabla_h H=Q, \quad (8)$$

which we shall call the divergent heat budget equation.

This heat budget equation contains the usual terms for the change in heat content $\partial H/\partial t$, horizontal advection $\hat{\mathbf{V}}_h\cdot\nabla_h H$, and surface heat exchange Q. The second term on the left has been neglected in previous heat budget equations and since it represents the heat change associated with divergence it will be referred to as H_{div}. This term accounts for both the heat change due to horizontal divergence $[(W_D/D)H]$ and the heat change due to vertical advection at D $[W_D\rho C_p T_D]$.

In order to evaluate H_{div}, we need to estimate W_D from observations of temperature. As discussed in Krauss (1966, p. 145) this can be done by applying the conservation of heat at depth D under some assumptions to yield

$$W_D=-\frac{\partial T/\partial t}{\partial T_m/\partial z}, \quad (9)$$

where $\partial T/\partial t$ represents monthly temperature changes at D and $\partial T_m/\partial z$ is the mean vertical temperature gradient at D. Since the use of (9) is common practice, a complete discussion of the assumptions necessary to derive (9) will not be given (see Emery, 1975). In brief, however, these assumptions all require that temperature changes at D are due mainly to vertical advection of the mean thermal gradient (at D). It should also be noted that temperature changes at D due to mixing have not been considered since D is below the main thermocline. An estimate of the effect of diffusive flux at D using the diffusion equation with a vertical mixing coefficient of 5 $\mathrm{cm^2\,s^{-1}}$ shows that the diffusive-induced annual signal of temperature at 250 m depth is less than 3% of the surface signal. Shorter period signals are reduced even more.

5. Comparisons between terms in the divergent heat budget equation

To properly evaluate the validity of (8) one should compute all the terms and compare the right- and left-hand sides of the equation, but this was not possible since the advective term could not be calculated. Correlations between the right- and left-hand sides of (8) were therefore used to establish how the known terms were related. Most important is the relationship between Q and $\partial H/\partial t$; these quantities were therefore correlated first. At weather station November this correlation was 0.57, a value which did indicate some sort of connection between the two terms. H_{div} was then calculated and the difference between $\partial H/\partial t$ and H_{div} was found. On correlating this difference with Q, the correlation at station November increased to 0.76 (see Table 3). The 0.19 change is itself significant at the 95% level.

Similar correlations were computed for all 2° and 5° quadrangles (Table 3). An improvement was seen in all areas when H_{div} was included. However, those east of station November showed a smaller increase in correlation when H_{div} was included. These eastern areas also showed a much weaker correlation between Q and $\partial H/\partial t$ itself. These weaker correlations are a sign of the many factors not included in (8) that are present in the transition from the deep ocean region west of station November to the coastal zone off California (Roden, 1971). In the transition region, the California current increases the role of horizontal advection. This em-

TABLE 3. Correlations between heat budget terms (all correlations are between given quantity and $\bar{Q}$).*

	Station November	2° Quadrangles 1	2	3	4
$\frac{\partial H}{\partial t}$	0.57	0.47	0.43	0.41	0.12
$\frac{\partial H}{\partial t}-H_{\mathrm{div}}$	0.76	0.66	0.63	0.61	0.31

5° Quadrangles	A	B	C	D	E	F	G	H
$\frac{\partial H}{\partial t}$	0.58	0.48	0.45	0.62	0.50	0.33	0.11	−0.02
$\frac{\partial H}{\partial t}-H_{\mathrm{div}}$	0.71	0.71	0.60	0.79	0.62	0.45	0.24	0.15

* $\partial H/\partial t$ is the change in heat content, H_{div} the heat change due to divergence (see text).

phasizes the weakness of neglecting horizontal advection which may in some areas play an important role in Eq. (8). The shoaling thermal structure, east of station November, also changes the way temperature fluctuates at D. Both horizontal and vertical mixing become important in the more turbulent coastal zone. Seasonal changes due to upwelling also operate to change the relationship between Q and $\partial H/\partial t$ in the area off California. All of these processes cause heat content changes that are not included in the known terms of (8) nor in the assumptions necessary to use (9).

In order to establish a more complete understanding of the relationship between Q and $\partial H/\partial t - H_{\text{div}}$, a mean square difference between these quantities was computed and compared to a similar difference between Q and $\partial H/\partial t$. These differences account for the magnitudes of the respective terms as well as how the terms change and therefore provide a better estimate than correlations of the importance of including H_{div}.

The mean square difference between Q and $\partial H/\partial t$ is

$$\epsilon_1 = \frac{1}{P}\int_0^P \left(\frac{\partial H}{\partial t} - Q\right)^2 dt, \tag{10}$$

where 0 and P are the beginning and end of the time series used. Similarly the mean square difference between Q and $(\partial H/\partial t - H_{\text{div}})$ is

$$\epsilon_2 = \frac{1}{P}\int_0^P \left[\left(\frac{\partial H}{\partial t} - H_{\text{div}}\right) - Q\right]^2 dt. \tag{11}$$

Expanding both terms and forming the ratio between ϵ_2 and ϵ_1, we can write

$$\frac{\epsilon_2}{\epsilon_1} = \frac{\overline{\left(\frac{\partial H}{\partial t} - H_{\text{div}}\right)^2} - 2\overline{\left(\frac{\partial H}{\partial t} - H_{\text{div}}\right)Q} + \overline{Q^2}}{\overline{\frac{\partial H^2}{\partial t}} - 2\overline{\frac{\partial H}{\partial t}Q} + \overline{Q^2}}, \tag{12}$$

where the overbar indicates the time average from 0 to P.

If this ratio is less than 1 the combination of $\partial H/\partial t - H_{\text{div}}$ balances with Q better than does $\partial H/\partial t$ alone. As can be seen in Table 4 this ratio was found to be substantially less than 1 for all test locations. At most locations the ratio was less than 0.5 demonstrating a better than 50% improvement by including H_{div}. As in the correlations between heat budget terms, the areas to the west of station November responded best and had the lowest ratios. The largest ratio occurred in the 5° quadrangle closest to California (area H) which also showed the poorest response to H_{div} in the correlations. The low value in area F was surprising, however, especially when compared to the higher value in the corresponding 2° quadrangle 4. This was also in contrast to the ratios in the other 2° quadrangles that were all less than the ratios in the corresponding 5° quadrangles. The poor correlations in area 4 may be due to the presence of a transition zone in water masses (Roden, 1971) or they may be due to data problems since this quadrangle had the poorest coverage of all the areas. The smaller ratios in the western 2° areas (when compared to the western 5° areas) may be an indication that the former are sensing processes with horizontal length scales <500 km. This difference was not as clear in the correlation studies above.

TABLE 4. Ratio of mean square differences (ϵ_2/ϵ_1).

Station November	2° Quadrangles 1	2	3	4
0.39	0.30	0.34	0.28	0.55

5° Quadrangles A	B	C	D	E	F	G	H
0.43	0.40	0.38	0.33	0.50	0.35	0.46	0.61

6. Summary and conclusions

High correlations between heat content in the upper layer (0–250 m) and the depth of the 14°C isotherm suggest that vertical motion, as inferred from isotherm displacements, acts to change the heat content of the upper northeastern Pacific Ocean. Correlations between terms in a heat budget equation, which include heat changes due to vertical motion and divergence, show that the correlation between surface heat exchange and the change in heat content improves significantly when the heat changes due to vertical motion and divergence are included. A ratio comparing the mean square difference between surface heat exchange and the change in heat content also shows a substantial improvement (~50%) when the heat changes due to vertical motion and divergence are included. These improvements are largest in the region between Hawaii and station November (30°N, 140°W) where the assumptions for the computation of the heat changes due to vertical motion are best fulfilled. Further east between station November and California, the improvements are not as large. Such improvements indicate that heat changes due to vertical motion and divergence improve the balance between surface heat exchange and the change in heat content and help to explain the observed difference between those two terms.

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