On Forced, Long Continental Shelf Waves on an f-Plane

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ABSTRACT

Previous studies of forced, long continental shelf waves on an f-plane have considered motion on the shelf and slope which is driven by an alongshore component of the wind stress, essentially through the suction of fluid into the surface layer at the coast. These studies have utilized a boundary condition, which arises consistently in the long-wave nondispersive limit for *free* shelf waves, that at the slope-interior junction the alongshore velocity component $v\approx 0$. This is an extremely useful condition for problems concerning forced motion on the shelf and slope, because it completely uncouples the motion in this region from that in the interior and it allows the shelf-slope problem to be solved independently of the interior problem. It is shown here, however, that this condition is not correct in general for wind-stress-forced f-plane motion. A proper formulation of the f-plane, forced shelf wave problem in the long wave limit is presented. The motion on the shelf and slope, in general, is coupled with and forced by the flow in the interior.

1. Introduction

The generation of motion on the continental shelf and slope by barotropic continental shelf waves which are forced by an alongshore component of the wind stress has been studied by Adams and Buchwald (1969) and by Gill and Schumann (1974). In these studies, an f-plane model with an idealized, exponential shelf and slope bottom topography (Buchwald and Adams, 1968), which is independent of the alongshore coordinate, is utilized. In addition it is assumed that the spatial scale of the wind stress is much larger than that of the shelf-slope width so that the forced shelf waves may be treated in the long-wave, nondispersive limit. The motion is forced by an alongshore component of the wind stress through the suction of fluid into the surface layer at the coast.

The geometry of these models is basically the following. An exponential shelf and slope topography adjoins a flat bottom ocean interior on an f-plane. A Cartesian coordinate system, with velocity components u,v,w in the (x,y,z) directions, is used with the z axis aligned in the vertical direction, the y axis in the alongshore direction and the x axis offshore. The origin $x=0$ is at the coast and the slope-interior junction is at $x=\delta$. The topography does not vary in the y direction. The alongshore scale of the motion is assumed to be greater than the shelf-slope width, i.e.,

$$\delta_y \gg \delta, \tag{1.1}$$

and the time scale is assumed to be larger than an intertial period, i.e.,

$$\delta_t \gg f^{-1}, \tag{1.2}$$

where f is the Coriolis parameter.

In both of the previously mentioned studies an approximate boundary condition at the slope-interior junction,

$$v=0 \quad \text{at} \quad x=\delta, \tag{1.3}$$

is utilized. This is a consistent approximation in the long-wave limit (1.1) for *free* continental shelf waves on an f-plane as shown mathematically and explained physically by Buchwald and Adams (1968). The physical reasoning is basically the following. From the continuity equation the order-of-magnitude estimate $u/v\approx \delta_x/\delta_y$ may be obtained. Over the continental shelf and slope, $\delta_x\approx\delta$ and it follows from (1.1) that $u/v\ll 1$. For the ocean interior it may be expected that the y scale will be similar to the y scale for the slope region and that the x and y scales will be comparable. Consequently, in the interior $u_I\approx v_I$, where interior variables are designated here by a subscript I. At the slope-interior junction the balance of the normal mass flux requires $u\approx u_I$ and the continuity of the pressure requires $v_t\approx v_{It}$ or, essentially, $v\approx v_I$. For consistency with the other order-of-magnitude estimates, this implies that $v(x=\delta)\ll v(0<x<\delta)$ or that $v(x=\delta)\approx 0$ and, hence, gives boundary condition (1.3). This argument was appealed to for the forced problem by Adams and Buchwald (1969) through a reference to Buchwald and Adams (1968) and a similar line of reasoning was given by Gill and Schumann (1974).

We point out, as will be shown in detail in Section 2, that with (1.2), condition (1.3) is equivalent to the assumption that at the slope-interior junction the onshore-offshore flow below the surface frictional layer, of the magnitude of that forced on the shelf and slope by the alongshore component of the wind stress, is geostrophically balanced, i.e., that

$$\rho_0 fu \approx -p_y \quad \text{at} \quad x=\delta, \tag{1.4}$$

where p is the pressure and ρ_0 the density.

Mathematically and conceptually, the approximation (1.3) is a very significant boundary condition for problems concerning the forced motion on the shelf and slope, because it completely uncouples the problem in this region from that in the interior. Although (1.3) has been used in the above-mentioned forced problems,[1] the mathematical and physical basis for it has really only been established for free shelf wave motion. In fact, for forced f-plane problems boundary condition (1.3) is not correct in general. It is the purpose of this paper to clarify this point and to present a proper formulation of the f-plane, forced shelf wave problem in the limit (1.1).

The major difference between free and forced problems in this regard is that for forced f-plane problems the forced interior motion, in general, will be such that the geostrophic balance (1.4) does not hold (to sufficiently high order). As a result, the motion on the slope is not uncoupled from the motion in the interior solely by condition (1.1). This becomes obvious after a little thought[2] and after the following analysis, and condition (1.3) would definitely be in error for many possible f-plane laboratory experiments.

The original analyses of Adams and Buchwald (1969) and Gill and Schemann (1974) were meant, of course, to apply to shelf and slope regions adjacent to oceanic interiors in which additional constraints, such as the β-effect, might be important. For simplicity, we limit our consideration here only to the case of a flat bottom, f-plane interior region. The relationship of the results of this case with more general problems is discussed briefly in Section 4.

2. Analysis

We consider a model where a homogeneous fluid is situated on an f-plane which effectively rotates with uniform angular velocity $\mathbf{\Omega}=\frac{1}{2}f\mathbf{k}$, where $\mathbf{k}$ is a constant unit vector in the z (vertical) direction in a Cartesian coordinate system. We assume that the motion is governed by the following set of linear equations in dimensionless variables:

$$u_x+v_y+w_z=0 \tag{2.1a}$$

$$u_t-v=-p_x+\tau_z^{(x)} \tag{2.1b}$$

$$v_t+u=-p_y+\tau_z^{(y)} \tag{2.1c}$$

$$0=p_z. \tag{2.1d}$$

Here $\tau^{(x)}$ and $\tau^{(y)}$ are, respectively, horizontal shear stresses in the x and y directions and the subscripts denote partial differentiation. The variables (u,v,w,p,τ) and (x,y,z,t) have been made dimensionless by $(U_0,U_0,H_0L^{-1}U_0,\rho_0U_0fL_0,\tau_0)$ and (L_0,L_0,H_0,f^{-1}), respectively, where L_0 is a characteristic horizontal scale, H_0 a characteristic depth, τ_0 a characteristic value of the surface wind stress and $U_0=\tau_0/(\rho_0 fH_0)$ a characteristic horizontal velocity.

The fluid is contained in a basin with a rigid upper surface at $z=0$ and a variable depth bottom surface at $z=-H(x,y)$. Eqs. (2.1a, b) are integrated over the depth H. It is sufficient, for the points to be established here, to assume $\tau^{(x)}$ and $\tau^{(y)}$ are negligible at the bottom surface. With the boundary conditions

$$w=\begin{cases} 0, & z=0, \quad (2.2\text{a})\\ -uH_x-vH_y, & z=-H(x,y), \quad (2.2\text{b})\end{cases}$$

and the definitions

$$U=\int_{-H}^{0} udz, \quad V=\int_{-H}^{0} vdz, \tag{2.3a, b}$$

the resulting equations are

$$U_x+V_y=0, \tag{2.4a}$$

$$U_t-V=-Hp_x+\bar{\tau}^{(x)}, \tag{2.4b}$$

$$V_t+U=-Hp_y+\bar{\tau}^{(y)}, \tag{2.4c}$$

where

$$\bar{\tau}^{(x,y)}=\tau^{(x,y)}(z=0). \tag{2.5}$$

By defining a transport streamfunction such that

$$U=\psi_y, \quad V=-\psi_x, \tag{2.6a, b}$$

the following single equation for ψ may be derived from (2.4b, c):

$$[\psi_{xx}+\psi_{yy}-(H_x/H)\psi_x-(H_y/H)\psi_y]_t +(H_x/H)(\psi_y-\bar{\tau}^{(y)})-(H_y/H)(\psi_x-\bar{\tau}^{(x)}) =(\bar{\tau}_y^{(x)}-\bar{\tau}_x^{(y)}). \tag{2.7}$$

For simplicity in presentation, we assume initially that the fluid is contained in a rectangular basin with boundaries at $x=0$, 1 and $y=0$, L. We also assume that there is an exponential shelf-slope region along the boundary $y=0$, from $x=0$ to $x=\delta$, and that the remainder of the basin is at constant depth, i.e., that

$$H=\begin{cases}\exp[(x-\delta)/\delta_B\delta], & 0\leqslant x\leqslant\delta, \quad (2.8\text{a})\\ 1, & \delta\leqslant x\leqslant 1, \quad (2.8\text{b})\end{cases}$$

where H is independent of y and δ_B is an O(1) constant. The results may be easily generalized to more general basin geometries and onshore-offshore shelf-slope topography.

We will refer to the flat bottom region ($\delta\leqslant x\leqslant 1$, $0\leqslant y\leqslant L$) as the interior and the line $x=\delta$ as the slope-

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[1] Condition (1.3) has also been used by the present author in Allen (1976) and as a co-author in Kundu *et al.* (1975).

[2] Consider, for example, a conceptual problem similar to that in the example of Section 3b.

interior junction. We consider the case where

$$\delta \ll 1, \tag{2.9}$$

i.e., where the scale for the width of the shelf-slope region is small compared with the O(1) x and y scales of the interior. The surface stress components $\bar{\tau}^{(x,y)}$ are assumed to vary on x and y scales which are O(1). The problem will be treated by perturbation methods for the limit

$$\delta \rightarrow 0. \tag{2.10}$$

An initial-value problem is examined where a surface wind stress is imposed at $t=0$ to a fluid at rest. After the initial imposition, the wind stress may vary with time, but we restrict our attention to cases where this variation is on a time scale long compared with f^{-1}. We therefore assume that

$$\bar{\tau}^{(x,y)} = \bar{\tau}^{(x,y)}(x,y,\bar{t}), \quad t>0, \tag{2.11}$$

where we avoid introducing an additional small parameter by assuming that $\bar{\tau}^{(x,y)}$ varies on the time scale

$$\bar{t} = t\delta. \tag{2.12}$$

Since the fluid is at rest initially, ψ satisfies

$$\psi = 0 \quad \text{at} \quad t=0. \tag{2.13}$$

The boundary conditions for ψ are

$$\psi = 0 \quad \text{at} \quad x=0,\,1, \quad y=0,\,L. \tag{2.14a, b}$$

The perturbation problem (2.10) requires different treatment in the interior and shelf-slope regions and the solution will have different expansions in these regions. The discontinuity in bottom slope at $x=\delta$ requires conditions on the continuity of the normal velocity and the pressure along this line. If $\tilde{\psi}$ and $\hat{\psi}$ represent the solution ψ in the interior and in the shelf-slope region, respectively, then the continuity conditions in terms of the streamfunction are

$$\left.\begin{aligned} \tilde{\psi}_y &= \hat{\psi}_y \\ \tilde{\psi}_{xt} &= \hat{\psi}_{xt} \end{aligned}\right\} \text{at } x=\delta. \tag{2.15, 2.16}$$

For the interior we assume that ψ has the expansion

$$\psi = \delta^{-1}[\tilde{\psi}_0(x,y,\bar{t}) + \delta\tilde{\psi}_1 + \cdots], \tag{2.17}$$

where the O(δ^{-1}) magnitude of the lowest order term is dictated by (2.7) and the time scale (2.12). Substituting (2.17) and (2.8b) in (2.7) we obtain the governing equation for $\tilde{\psi}_0$:

$$(\tilde{\psi}_{0xx} + \tilde{\psi}_{0yy})_{\bar{t}} = (\bar{\tau}_y^{(x)} - \bar{\tau}_x^{(y)}). \tag{2.18}$$

For the shelf-slope region we assume that ψ has the expansion

$$\psi = \hat{\psi}_0(\xi,y,\bar{t}) + \cdots, \tag{2.19}$$

where ξ is a scaled variable defined by

$$\xi = x/\delta. \tag{2.20}$$

The resulting equation for $\hat{\psi}_0$ is

$$(\delta_B\hat{\psi}_{0\xi\xi} - \hat{\psi}_{0\xi})_{\bar{t}} + \hat{\psi}_{0y} = \bar{\tau}_{(0)}^{(y)}, \tag{2.21}$$

where

$$\bar{\tau}_{(0)}^{(y)} = \bar{\tau}^{(y)}(x=0,\, y,\, \bar{t}). \tag{2.22}$$

It is evident from (2.21) that $\bar{t}$ is a natural time scale for $\hat{\psi}$. From (2.14), $\hat{\psi}_0$ satisfies

$$\hat{\psi}_0 = 0 \quad \text{at} \quad \xi = 0. \tag{2.23}$$

Substituting the expansions (2.17) and (2.19) in the conditions (2.15) and (2.16) and utilizing for the interior variable the Taylor series expansion

$$\tilde{\psi}(x=\delta) = \tilde{\psi}(x=0) + \delta\tilde{\psi}_x(x=0) + \cdots, \tag{2.24}$$

we obtain

$$\tilde{\psi}_{0y}(x=0) = 0, \tag{2.25a}$$

$$\tilde{\psi}_{0x\bar{t}}(x=0) = \hat{\psi}_{0\xi\bar{t}}\ (\xi=1), \tag{2.25b}$$

$$\tilde{\psi}_{1y}(x=0) + \tilde{\psi}_{0yx}(x=0) = \hat{\psi}_{0y}(\xi=1). \tag{2.25c}$$

It follows from (2.25a) that the lowest order motion in the interior is uncoupled from that on the shelf and slope. As a result, (2.18) may be solved for $\tilde{\psi}_0$ with boundary conditions

$$\tilde{\psi}_0 = 0 \quad \text{at} \quad x=0,\,1, \quad y=0,\,L. \tag{2.26}$$

This will determine $\tilde{\psi}_{0x\bar{t}}(x=0)$ which may be used in (2.25b) as a boundary condition for $\hat{\psi}_0$.

To solve the problem for $\hat{\psi}_0$ it is convenient to define

$$\hat{\psi}_0' = \hat{\psi}_0 - \xi\tilde{\psi}_{0x(0)}, \tag{2.27a}$$

where

$$\tilde{\psi}_{0x(0)} = \tilde{\psi}_{0x}(x=0,\, y,\, \bar{t}), \tag{2.27b}$$

and to solve for $\hat{\psi}_0'$. In terms of $\hat{\psi}_0'$, Eq. (2.21) and boundary conditions (2.23) and (2.25b) become

$$(\delta_B\hat{\psi}_{0\xi\xi}' - \hat{\psi}_{0\xi}')_{\bar{t}} + \hat{\psi}_{0y}' = \bar{\tau}_{(0)}^{(y)} + (\tilde{\psi}_{0x\bar{t}(0)} - \xi\tilde{\psi}_{0yx(0)}), \tag{2.28a}$$

$$\hat{\psi}_0' = 0 \quad \text{at} \quad \xi = 0, \tag{2.28b}$$

$$\hat{\psi}_{0\xi\bar{t}}' = 0 \quad \text{at} \quad \xi = 1. \tag{2.28c}$$

The problem for $\hat{\psi}_0'$ is now similar in form to that solved by Adams and Buchwald (1969) and Gill and Schumann (1974) in the sense that the boundary conditions at $\xi=0$, 1 are the same and the forcing function on the right-hand side contains $\bar{\tau}_{(0)}^{(y)}$. Here, however, there are additional forcing terms in (2.28a) which are due to the interior motion $\tilde{\psi}_0$, forced by the curl of the wind stress $(\bar{\tau}_y^{(x)} - \bar{\tau}_x^{(y)})$. We can see that the interior solution drives motion on the shelf and slope through the onshore velocity $\xi\tilde{\psi}_{0yx(0)}$ and through the time rate of change of the alongshore velocity $\tilde{\psi}_{0x\bar{t}(0)}$. Both of these provide a forcing effect of the same magnitude as the direct forcing of the wind stress at coast $\bar{\tau}_{(0)}^{(y)}$. It is evident from (2.28) that the lowest order motion on the shelf and slope is not in general uncoupled from that in the interior.

Since the problem is linear, the solution to (2.28a) for $\hat{\psi}_0'$ may be divided into two parts, the first forced by $\bar{\tau}^y_{(0)}$ and the second forced by the interior motion $(\bar{\psi}_{0xt(0)}-\xi\bar{\psi}_{0yx(0)})$. The solutions presented by Adams and Buchwald (1969) and Gill and Schumann (1974) correspond to the first part of $\hat{\psi}_0'$, which is forced by the wind stress at the coast.

The solution to (2.28) is most easily obtained by the method of Gill and Schumann (1974). This will provide $\hat{\psi}_{0y}(\xi=1)$ which may be used in (2.25c) as a boundary condition for $\bar{\psi}_1$, whose governing equation is

$$\bar{\psi}_{1xx}+\bar{\psi}_{1yy}=0. \tag{2.29}$$

The finite extent of the shelf and slope in the y direction in the present rectangular basin will cause a breakdown of the formulation at $y=0$ (for $f>0$) where incident long shelf waves will be reflected and motion with small y scales will be generated. That part of the problem, however, is not treated here. We point out that if a cylindrical basin is considered, with a uniform shelf-slope region along the perimeter of the basin and if $\delta \ll R_C$, where δ is the width of the shelf-slope region and R_C is the radius of curvature of the basin boundary, the present analysis may be easily generalized and an equation similar to (2.28a) governing the motion on the shelf and slope derived. In that case, the coordinates (ξ,y) here correspond to orthogonal curvilinear coordinates (ξ,η), where ξ varies in the direction of the inward pointing unit normal vector to the boundary $\hat{\mathbf{n}}$ and η varies along the unit tangential vector $\hat{\mathbf{t}}$, oriented so that $\hat{\mathbf{n}}\times\hat{\mathbf{t}}=\hat{\mathbf{k}}$. As before, the boundary is located at $\xi=0$ and the slope-interior junction at $\xi=1$. The forcing terms on the right-hand side are replaced by the appropriate generalizations and (2.28) becomes

$$(\delta_B\hat{\psi}'_{0\xi\xi}-\hat{\psi}'_{0\xi})_t+\hat{\psi}'_{0\eta} = \bar{\boldsymbol{\tau}}_{(0)}\cdot\hat{\mathbf{t}}+[\hat{\mathbf{n}}\cdot\nabla\bar{\psi}_{0t}-\xi\hat{\mathbf{t}}\cdot\nabla(\hat{\mathbf{n}}\cdot\nabla\bar{\psi}_0)]_{(0)}, \tag{2.30a}$$

where

$$\hat{\psi}_0'=\hat{\psi}_0-\xi(\hat{\mathbf{n}}\cdot\nabla\bar{\psi}_0)_{(0)}, \tag{2.30b}$$

and where the subscript (0) denotes evaluation at the boundary. The boundary conditions (2.28b, c) remain unchanged. The interior solution $\bar{\psi}_0$ is obtained, as before, from (2.18) with a boundary condition, analogous to (2.26), of $\bar{\psi}_0=0$ on the boundary. In this problem, there is no breakdown of the formulation such as there is at the corners of the rectangular basin.

The procedure utilized by Adams and Buchwald (1969) and by Gill and Schumann (1974) amounts to solving (2.21) with boundary condition (2.23) and with

$$\hat{\psi}_{0\xi t}=0 \quad \text{at} \quad \xi=1. \tag{2.31}$$

It has been shown that this is incorrect in general, but it is useful to examine the situation for which (2.31) is a proper boundary condition. It turns out, as mentioned in Section 1, that this is the case if the O(1) onshore-offshore flow, i.e., the onshore-offshore flow of the same magnitude as that directly forced on the shelf and slope by the alongshore component of the wind stress, is geostrophically balanced at $x=\delta$. This follows from (2.4c). With expansion (2.17) and with

$$p=\delta^{-1}(\bar{p}_0+\delta\bar{p}_1+\cdots), \tag{2.32}$$

(2.4c) gives, to $O(\delta^{-1})$,

$$\bar{\psi}_{0y}=-H\bar{p}_{0y}, \tag{2.33}$$

and, to O(1),

$$-\bar{\psi}_{0xt}+\bar{\psi}_{1y}=-H\bar{p}_{1y}+\bar{\tau}^{(y)}. \tag{2.34}$$

In terms of the shelf-slope variables, (2.4c) is

$$-\hat{\psi}_{0\xi t}+\hat{\psi}_{0y}=-H\hat{p}_{0y}+\tau^{(y)}_{(0)}, \tag{2.35}$$

where expansion (2.19) has been used and where

$$p=\hat{p}_0+\cdots. \tag{2.36}$$

We can see from (2.34), (2.35) and (2.25b) that (2.31) is equivalent to the assumption of geostrophic balance of $\hat{\psi}_{0y}$ and $\bar{\psi}_{1y}$ at $x=\delta$, i.e., to the balances

$$\hat{\psi}_{0y}=-H\hat{p}_{0y}+\bar{\tau}^{(y)}_{(0)} \quad \text{at} \quad \xi=1, \tag{2.37a}$$

$$\bar{\psi}_{1y}=-H\bar{p}_{1y}+\bar{\tau}^{(y)} \quad \text{at} \quad x=\delta. \tag{2.37b}$$

3. Examples

It is useful to consider some simple illustrative examples which result in different behavior on the shelf and slope.

a. Rectangular basin $(0\leqslant y\leqslant L)$

1) $\bar{\tau}^{(y)}=T_0(\text{constant}),\ \bar{\tau}^{(x)}=0$

In this problem there is a constant surface stress in the y direction. The solution of (2.18) with boundary condition (2.26) is

$$\bar{\psi}_0=0. \tag{3.1}$$

Therefore, we have $\hat{\psi}_0=\hat{\psi}_0'$ and the problem for $\hat{\psi}_0$ is given by (2.28) with $\bar{\psi}_0=0$ and with $\bar{\tau}^{(y)}_{(0)}=T_0$. The solution for $\hat{\psi}_0$, away from an expanding region about the boundary $y=0$, will be similar to that described in Allen (1976) (for values of y there where $\bar{\tau}^{(y)}_{(0)}\neq 0$).

In this special situation, a boundary condition given by the *a priori* assumption of (2.31) would actually result in a correct formulation. The reason is that in this case the O(1) motion in the interior, below the surface Ekman layer, is geostrophically balanced and (2.37b) holds. This is not true, however, whenever $\bar{\tau}^{(x)}_y-\bar{\tau}^{(y)}_x\neq 0$, for then $\bar{\psi}_0\neq 0$ and the balance in (2.34) is ageostrophic, with all the terms entering. In that case, assumption (2.37) does not hold and the contribution from $\bar{\psi}_0$ has to be taken into account in (2.28) as is illustrated in the next example.

2) $\bar{\tau}^{(y)}=T_0x\sin(\pi y/L)$, $\bar{\tau}^{(x)}=0$

In this case, the solution of (2.18) is

$$\bar{\psi}_0=\bar{t}\,\frac{T_0}{\alpha^2}\left[1-\frac{\cosh\alpha(x-\frac{1}{2})}{\cosh\frac{1}{2}\alpha}\right]\sin\alpha y, \tag{3.2a}$$

where

$$\alpha=\pi/L. \tag{3.2b}$$

The forcing terms in (2.28a) are

$$\bar{\tau}^{(y)}_{(0)}=0, \tag{3.3a}$$

$$\bar{\psi}_{0x\bar{t}(0)}-\xi\bar{\psi}_{0yx(0)} = -T_0\tanh(\tfrac{1}{2}\alpha)[\alpha^{-1}\sin\alpha y-\xi\bar{t}\cos\alpha y]. \tag{3.3b}$$

The alongshore component of the wind stress is equal to zero at the coast and therefore does not enter (2.28a) as a forcing function. The term from the interior variables is nonzero, however, and will directly force shelf wave motion in (2.28a). This component of the flow on the shelf would have been completely overlooked, of course, if boundary condition (2.31) had been used.

b. Infinite channel $(-\infty<y<\infty)$

$$\bar{\tau}^{(y)}=T_0 \text{ (constant)},\ \bar{\tau}^{(x)}=0$$

In this problem the surface stress remains the same as in subsection 3a1, but the geometry is changed by removing the boundaries at $y=0$, L so that the fluid is contained in an infinite channel. With $\bar{\tau}^{(y)}$ equal to a constant and with no variation of the geometry in the y direction, there will be no gradients in y of the velocity or pressure. This channel flow represents a problem that might well correspond to a laboratory experiment, i.e., to the axisymmetric motion in a rigidly bounded, narrow-gap annulus.

Since the geometry results in a mathematical problem similar to that in a doubly-connected region, the formal solution of (2.18) for $\bar{\psi}_0$ requires something analogous to a circulation condition and a change in (2.14b). We do not have to pursue that in detail here, however, since the interior solution may be easily found directly from (2.4) and it is

$$\bar{\psi}_{0x}=-T_0\bar{t},\quad \bar{\psi}_{0y}=0,\quad \bar{p}_{0y}=0. \tag{3.4a, b}$$

In this case, the forcing terms in (2.28) vanish identically, which implies

$$\hat{\psi}_0'=0, \tag{3.5}$$

and, therefore, from (2.27)

$$\hat{\psi}_0=\xi\bar{\psi}_{0x(0)}. \tag{3.6}$$

The expression in (3.6) is just the first term of the Taylor series expansion about $x=0$ of $\bar{\psi}_0$ and, in fact, the total solution in both regions is given by (3.4). In this case, of course, the *a priori* application of boundary condition (2.31) is very obviously incorrect.

4. Discussion

It has been shown that for f-plane problems, such as those that might correspond to laboratory experiments, the motion in the shelf-slope region in general is coupled with that in the interior and the use of boundary condition (2.31), for the flow over the shelf-slope region, is not justified solely by condition (2.9). If (2.31) is viewed in terms of the equivalent assumption (2.37) of the geostrophic balance of the onshore-offshore flow at $x=\delta$, we can see that (2.37) breaks down in some f-plane problems because the lowest order forced interior motion causes the O(1) balance at $x=\delta$ to be ageostrophic. Thus, the physical reasoning employed in the order-of-magnitude argument in Section 1 is incorrect and must be altered in this case. This is basically because in the forced problem $v_t\approx v_{It}$ at $x=\delta$. With this condition, rather than $u\approx u_I$, the order-of-magnitude estimate requires for consistency that $u_I(x=\delta)\approx 0$, which applies to the lowest order interior u and is the result expressed in (2.25a).

For oceanic problems, the interior motion will be subject to additional constraints, such as that from the β-effect or that from the bottom slope of the continental rise. If we consider a problem where a continental shelf-slope region adjoins an interior governed by β-plane dynamics, we find that an analysis similar to that given above for the flat bottom f-plane case may be carried through and that an equation similar to (2.28a) governing the motion on the shelf and slope may be derived. The scaling of variables is different in the β-plane case and the problem is more complicated because the interior solution, which enters (2.28a) as a forcing function, has strongly differing behavior on different boundaries. The effects of the coupling with the interior motion will, therefore, vary considerably depending on the boundary location. An investigation of the implications for shelf-slope motion of interior coupling in the β-plane case is in progress and will be reported on separately.

It should be emphasized that, because of the above factors, an assessment of the consistency of the application of boundary condition (2.31) in the oceanic case should not be attempted from the present analysis with a flat bottom f-plane interior, but should await the results from the β-plane case. In addition, regardless of the consistency or inconsistency of the use of (2.31) in idealized models, observations may show that it provides an acceptable "working" approximation for forced shelf motion.

We conclude by noting that the present analysis provides a consistent starting point for the investigation of the important general question of how interior flow may interact with, and force, motion on the continental shelf and slope.

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