Experimental Studies of Relative Diffusion in Lake Huron

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ABSTRACT

Experimental data were obtained in Lake Huron on the distribution of mean concentration and mean-square fluctuation about the center of gravity of a diffusing plume of fluorescent dye. Some of the mean concentration profiles showed a skewness attributable to the vertical current shear, while others were approximately Gaussian. The ratio of rms concentration fluctuation to local mean appears to be a quasi-universal distribution, in the sense that the typical amplitude of this ratio depends on turbulence intensity, but otherwise individual distributions are similar, their length-scale being equal to the length-scale of the mean concentration distribution.

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1. Introduction

Experimental data on "relative" diffusion (Batchelor, 1952) in the atmosphere are hard to come by and are usually not very detailed, as is clear from the summary of Pasquill (1962). The reason for this lies in the practical difficulty of obtaining instantaneous concentration measurements at a sufficient number of points to define the concentration distribution in a frame of reference moving bodily with a diffusing cloud (which itself executes large-scale random movements known as "meandering"). Since laboratory data appear to be entirely non-existent, we thus possess very little empirical information on this theoretically interesting and practically important phenomenon.

In natural bodies of water, using fluorescent tracer dyes, instantaneous concentration measurements in a diffusing patch or plume are relatively easily carried out. If several fluorometers are carried on a moderately fast moving boat, nearly simultaneous concentration cross sections of a continuous plume (for example) may be obtained. The use of this technique has already been reported in Csanady (1963) (to be referred to hereafter as I), where some fairly crude data were given on the gross properties of relative diffusion in the surface layers of Lake Huron.

A difficulty in elucidating the more detailed properties of relative diffusion at all reliably lies in the great irregularity of the observed individual determinations of concentration distribution. In the theoretical treatment of Batchelor (1952) the *stochastic mean* concentration field is considered, in a frame of reference moving either with a given marked (diffusing) particle, or with the center of gravity of a cloud. For comparison with dye plume experiments only the latter formulation is useful, although it requires the measurement of a fairly large number of realized concentration distributions under "identical" environmental conditions. In I single realizations were compared with the statistical theory, with the result that only some rather gross properties of the relative diffusion process could be assessed. Further measurements of concentration distributions, involving a suitable number of repeated observations, have been carried out in Lake Huron, at the Baie du Dore research station of the Great Lakes Institute, University of Toronto, during 1965 and, in rather greater detail, 1968. Those results which are thought to be of some fundamental interest are presented here. No similar data on the stochastic properties of relative diffusion appear to have been reported in the literature before, except for a brief preliminary discussion in Csanady (1966) of the 1965 results, which is not widely available.

2. Experimental method

Four G. K. Turner Model 111 continuous flow-door fluorometers were carried on deck of a 10 m catamaran. A 5 m sampling boom was hinged below deck, between the hulls, lengthwise approximately at the middle of the craft, so that when not in use the boom lay against the bottom of the deck, reaching to the bow of the boat. It could be lowered by a cable arrangement so that samples could be drawn in at depths of up to 4 m. Normally, the sampling intakes were located at 1, 2, 3 and 4 m. The stability of the catamaran and its large deck area make it particularly suitable for this kind of experiment.

A dye plume was generated by releasing fluorescent rhodamine B liquid dye from an anchored source, about 2 km from the shore at a water depth of 20 m. The dye was stored on the deck of the source (another catamaran) in two 45-gallon drums interconnected by flexible tubing and syphoned out from one of the drums using a constant head syphon apparatus (similar to the one described in I), which discharged dye solution at a constant rate of approximately 12 cm^3 sec^{-1} into the lake. The dye was released 2 m below the surface from a copper tube mounted vertically to a light aluminum tower section extending clear of the source catamaran to avoid any possible disturbance from the boat itself. Horizontal motion of the source catamaran was reduced by anchoring in three directions. With three-point anchoring, the boat could be easily positioned so that the hulls were perpendicular to the waves. Thus, the point of release of the dye, which was located approximately at the longitudinal center of buoyancy of the boat, experienced the least movement.

Depending on prevailing currents, a good plume normally developed in the wake of the point source in $\sim$2 hr. Anchored marker flags were dropped at the edges of the dye plume, by visual estimation of the perpendicular to the plume center line, and sampling was carried out by criss-crossing the dye plume a number of times between the marker flags. The positions of these marker flags and that of the dye source were established from shore-based transit stations. The average time taken to cross the marker flags was also noted. Equally spaced sections were chosen for sampling by visual estimation of the distance between them.

Samples were drawn at the leading edge of the boom and pumped through the fluorometers on the deck of the boat. The output signal from the fluorometers was stored on strip chart recorders for later processing. A flow rate of approximately 70 cm^3 sec^{-1} was maintained to minimize the lag time in the tubing and to obtain good response characteristics from the sampling system, a typical response time of the system being 10 sec. Because of the time lag between the sample intake and the fluorometer response, sampling was extended well past the visible extent of the dye plume. The extra time so taken up was sufficient for the mean current to displace the disturbed plume section before the next (return) crossing.

At a speed of 1–2 m sec^{-1} the plume was not visibly disturbed by crossings of the boat while a very good resolution on the spatial coordinate of the strip chart recorder was achieved. The average boat speed was determined knowing the distance between the flags from the shore-based surveys and the average time taken to cross them in a number of runs. The time scale of the fluorometer traces was converted to a distance scale knowing the average boat speed and the chart speed used.

3. Treatment of data

Individual concentration distributions showed considerable irregularity. Examples of concentration distributions obtained at a fixed distance of 500 m from the source at different depths from a carefully conducted experiment on 6 June 1968 are shown in Fig. 1. To remove the irregularities, we averaged some 8–26 evaluations of a single dye plume obtained at a particular depth and at different times under what seemed to be identical environmental conditions, lining up the abscissa so that the centers of gravity coincided

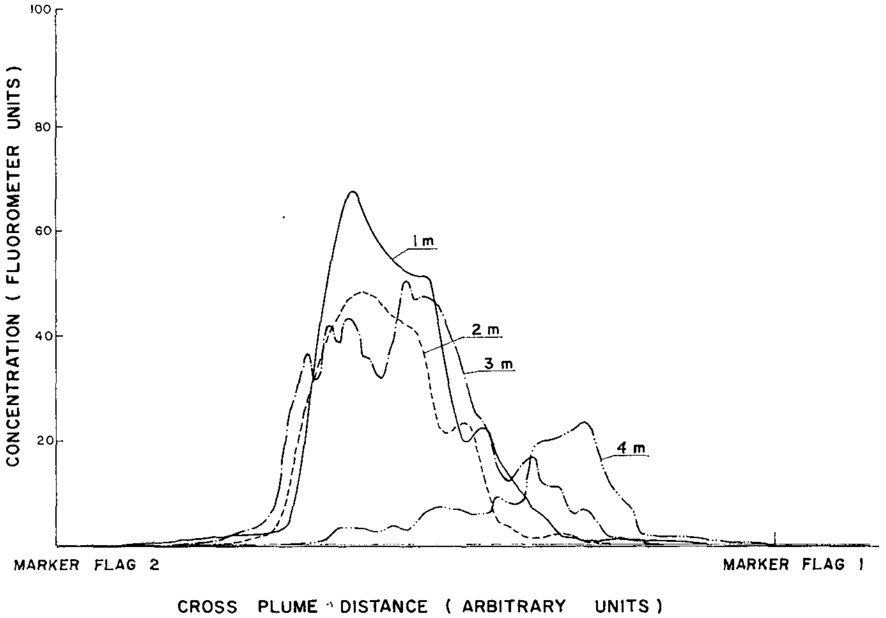

FIG. 1. Example of quasi-instantaneous concentration distributions obtained during a well-documented experiment on 6 June 1968 at the Baie du Dore Research Station. The distance from the source was 500 m.

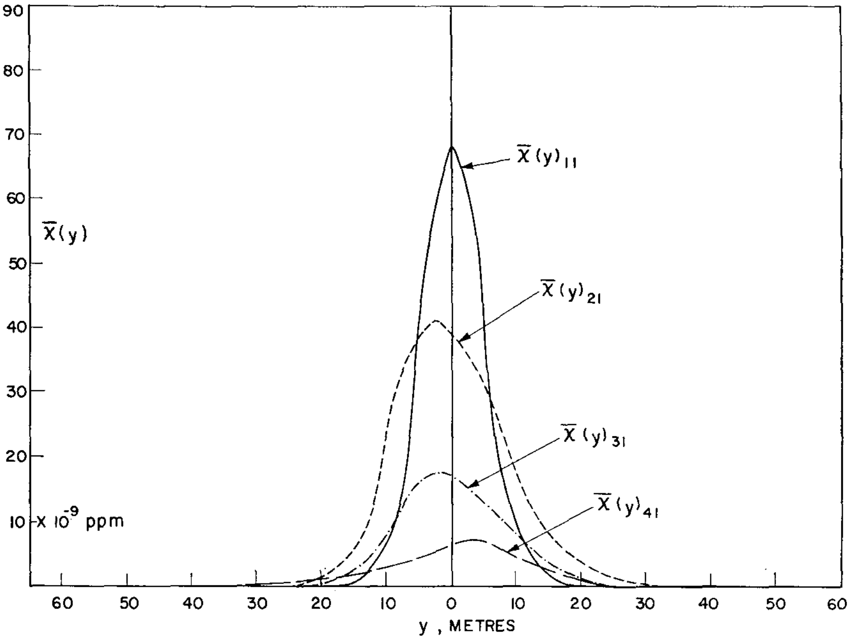

FIG. 2. Ensemble-mean concentration distributions at 1 m depth, at various distances from the source. Subscript code indicates cross-section number (first) and depth of 1 m. Distances of cross sections from source were as follows: 1, 500 m; 2, 1200 m; 3, 2000 m; 4, 2800 m.

(Csanady, 1966). In this manner the rather smoother experimental curves were obtained. The mean concentration distributions corresponding to Fig. 1 are contained in Figs. 2–5.

It should be pointed out here that the above definition of a mean concentration profile $\bar{\chi}(y)$ is not strictly what is considered in Batchelor's (1952) theory, because we have sampled at a fixed distance x and at a fixed depth z, and only "followed the cloud" in the y direction. However, it is reasonable to assume that the mean current does not change during one set of experiments and also that the diffusion along x was less im-

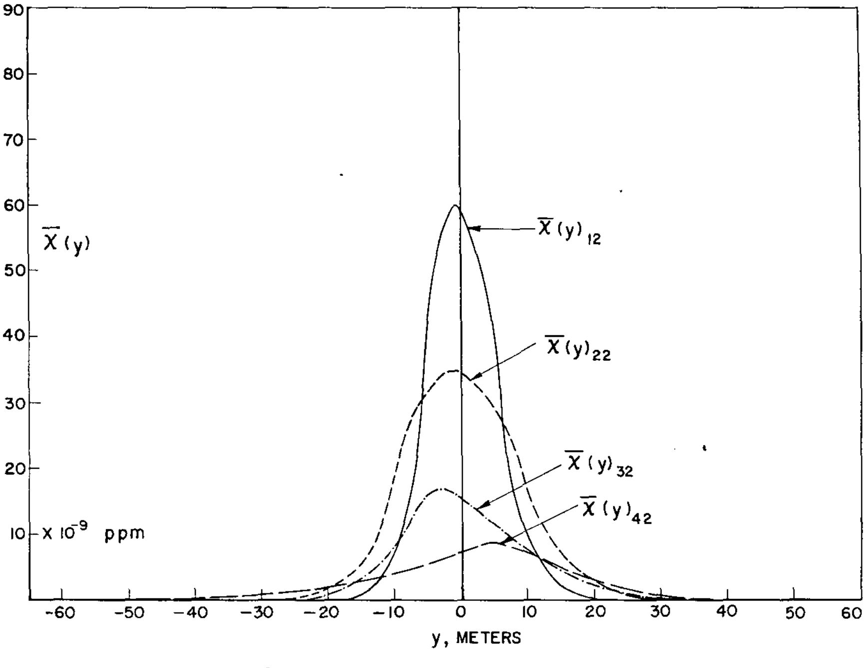

FIG. 3. Same as Fig. 2 except for a 2 m depth.

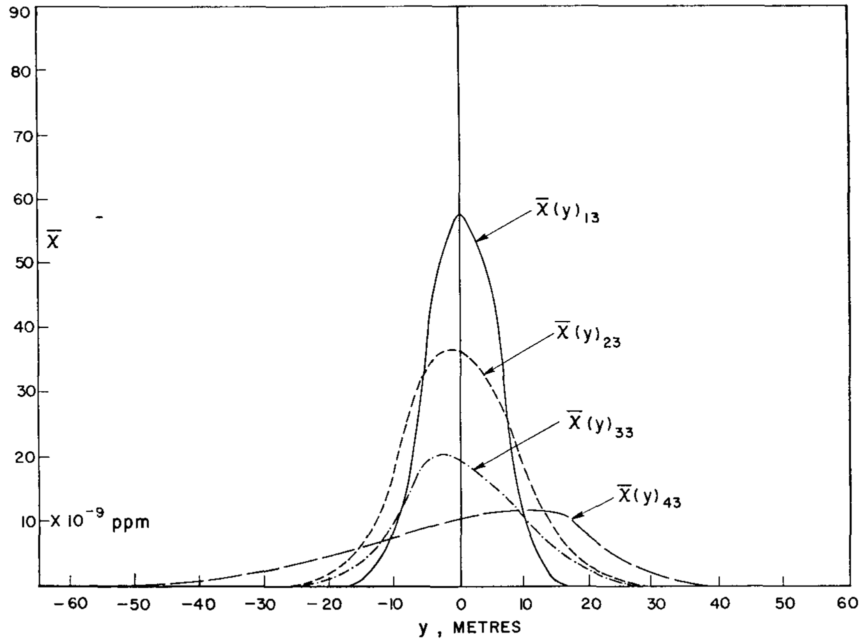

FIG. 4. Same as Fig. 2 except for a 3 m depth.

portant than along y, because the plume was "slender" (the latter being a common assumption when dealing with continuous source plumes). Further, if we assume that the vertical meandering was minimal on the measured occasions, then the experimentally determined mean concentration $\bar{\chi}(y)$ should closely approximate a one-dimensional section of the true stochastic mean concentration field of a diffusing puff.

Having determined a mean distribution, we may calculate the fluctuations $\chi'(y)=\chi(y)-\bar{\chi}(y)$ observed in individual evaluations. A measure of their magnitude (at a given distance from the center of gravity) will be the mean square fluctuations $\overline{\chi'^2}\,(y)$. One may expect the distribution $\overline{\chi'^2}\,(y)$ obtained from a limited number of measurements ($\sim$25) to be rather less regular than $\bar{\chi}(y)$. Typical examples are shown in Fig. 6.

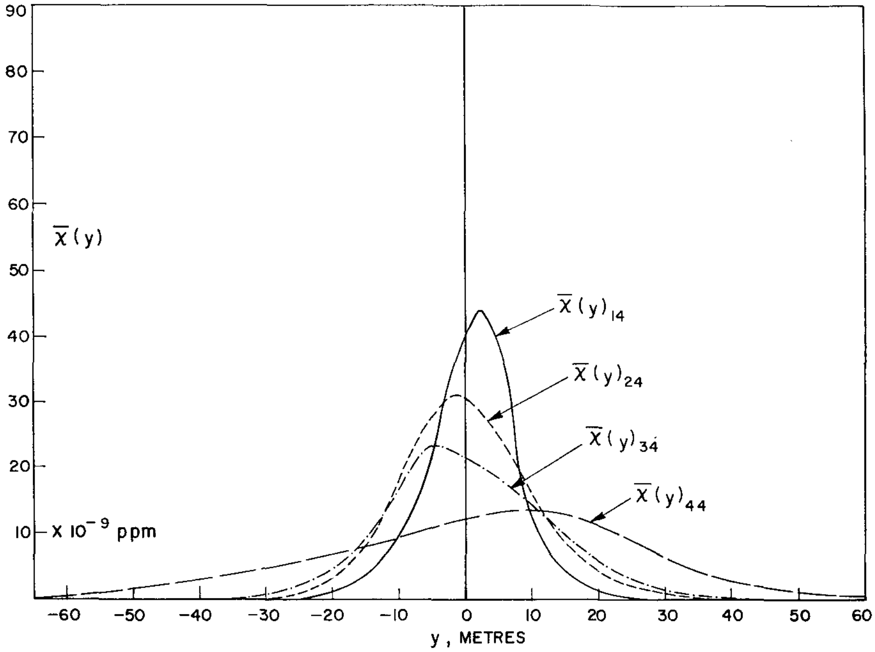

FIG. 5. Same as Fig. 2 except for a 4 m depth.

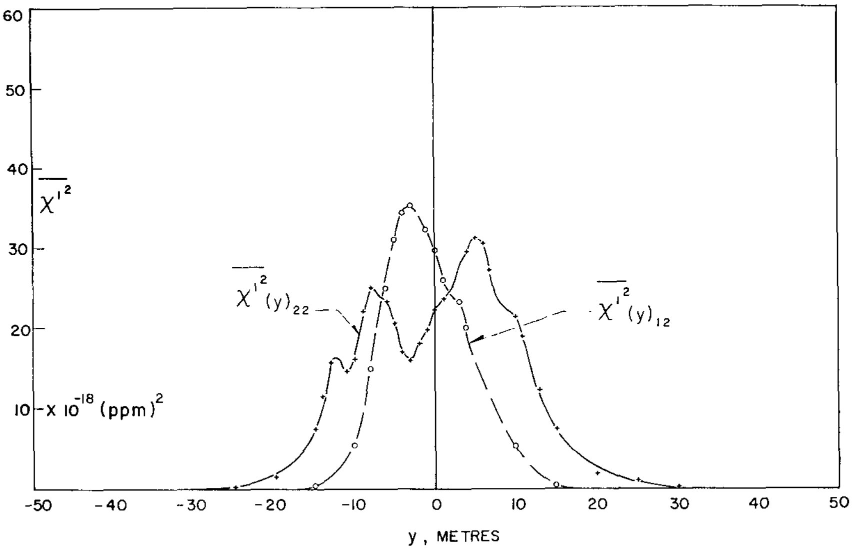

Fig. 6. Example of mean-square concentration fluctuations. Profile subscript code indicates cross-section number (first) and depth of 2 m.

A convenient nondimensional parameter in characterizing fluctuations is the root-mean-square to mean concentration ratio, $s(y)=[\overline{\chi'^2}(y)]^{\frac{1}{2}}/\bar{\chi}(y)$. An example of the distributions of this quantity (corresponding to Fig. 6) is shown in Fig. 7.

4. Mean concentration profiles

Having thus removed much of the random irregularity from our results, we are in a position to look for systematic effects produced by any external variables.

In Csanady (1966) it was shown that the mean concentration distribution approximates a Gaussian distribution. While this is frequently the case, there are marked departures from this form, notably in cross shear. Figs. 2–5 show $\bar{\chi}(y)$ distributions obtained during a particularly well-documented experiment. Some of these exhibit considerable skewness (particularly at the greatest distance from the source in cross section 4) similar to some theoretical profiles obtained by Okubo and Karweit (1969), who calculated numerically (using the classical diffusion equation) concentration

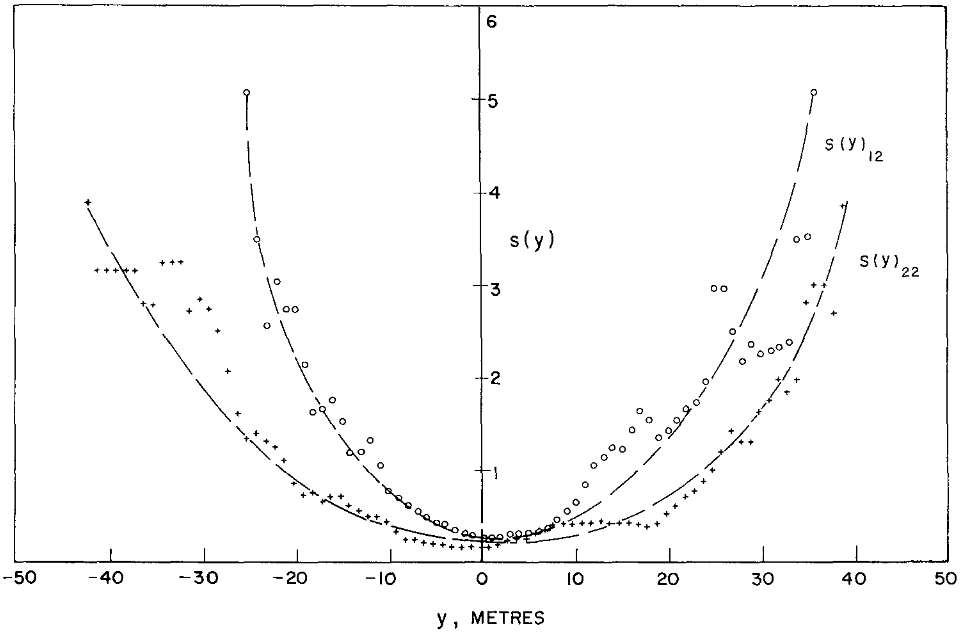

Fig. 7. Ratio of rms to mean concentration for cross sections 1 and 2 at a depth of 2 m.

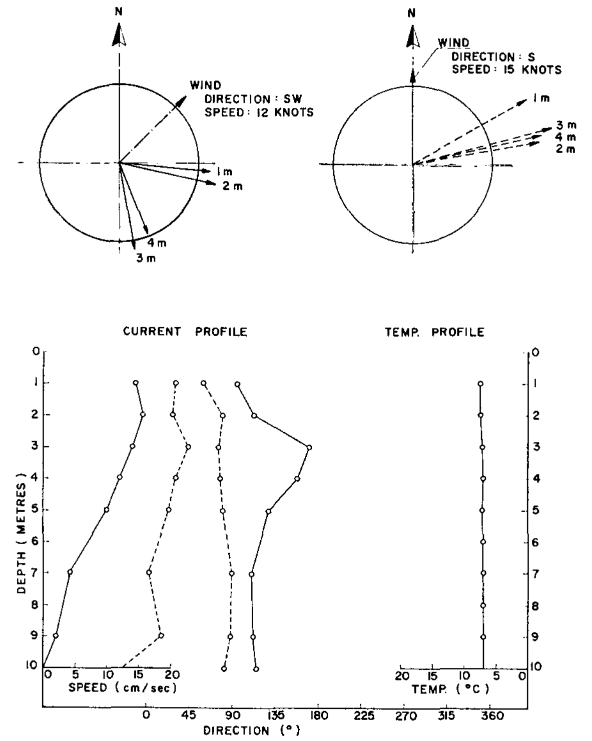

FIG. 8. Current and temperature distributions before and after experiment (water temperature distribution unchanged). Data are shown for 1100 EDT (solid line) and 1635 (dotted line) for 6 June 1968.

distributions in a continuous plume subject to a cross-flow velocity varying linearly with depth.

The current velocity vector distribution measured just before the experiment and during sampling of the two furthest cross sections is shown in Fig. 8. There is considerable difference between the two current profiles, the initial one showing marked shear. The data suggest that most of the current change took place during sampling of the two furthest cross sections, because cross section 4 exhibits rather more widely spread profiles than those of 1–3. The "width" of the distribution may be measured by the standard deviation S_y, defined by

$$S_y^2 = \frac{1}{q}\int y^2\bar{\chi}(y)dy, \qquad (1)$$

in which q is given by (2).

The variation of this quantity with distance is shown in Fig. 9. Up to about 2000 m distance from the source the "final phase" of relative diffusion is seen to hold ($S_y \propto X^{\frac{1}{2}}$) with an effective diffusivity of order $K_y=100$ cm^2 sec^{-1}. This latter value is at the low end of the range of data obtained before. The sudden acceleration of diffusion between cross sections 3 and 4 may be due to the cross-flow shear which seems to have affected this end of the plume only. The strong skewness of the profiles in this cross section is also possibly caused by the cross shear.

The vertical diffusion of dye is best evaluated by comparing the total amount of dye per cross section at different depths, i.e.,

$$q = \int \bar{\chi}(y)dy. \qquad (2)$$

The $q(z)$ profiles corresponding to the previous $\bar{\chi}(y)$ profiles are shown in Fig. 10. Clearly, there is little change of q from the surface down to our maximum measured depth of 4 m even at the two closest cross sections. If we imagine a half-Gaussian profile fitted to this $q(z)$ distribution, the vertical standard deviation may be seen to be of the order $S_z=5$ m at $X=500$ m, yielding an effective vertical diffusivity of order $K_z=50$ cm^2 sec^{-1}, or very much as K_y. This result is in notable contrast to most earlier measurements which showed $K_z \ll K_y$. On this occasion the fairly brisk wind set up a well-defined "window" structure resulting in apparently excellent vertical mixing (and lower than usual horizontal mixing). Visual observations obtained on this occasion are illustrated in Fig. 11.

At the two farthest cross sections the dye distribution with depth is seen to be very different: an increase

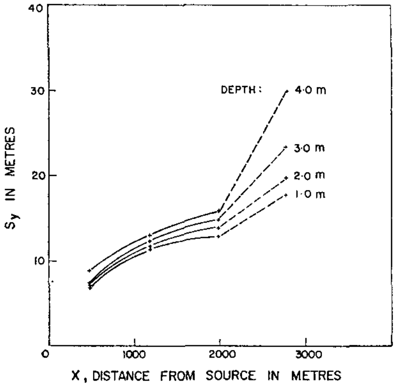

FIG. 9. Growth of plume width.

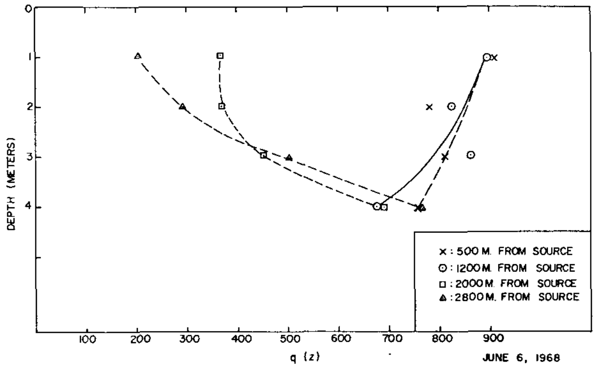

FIG. 10. Integrated dye distribution $q(z)$ vs depth for the four cross sections.

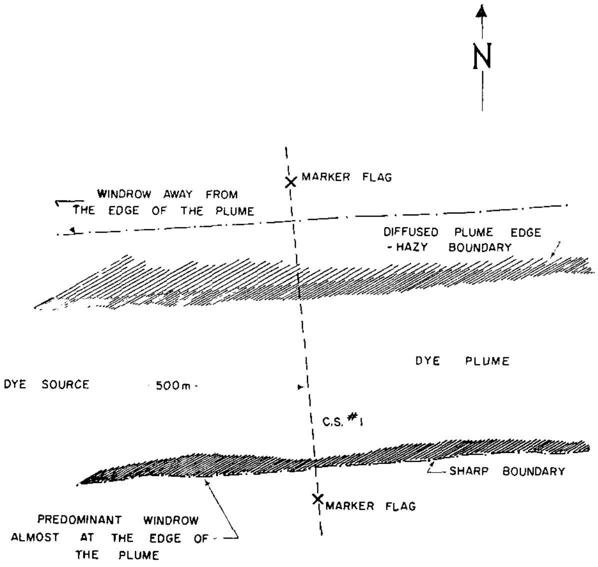

FIG. 11. Visual observations of windrow influence on dye-plume behavior.

of q with depth could be caused by the "windrow" circulation (convergence of surface water and horizontal transport of dye at deeper depths away from the body of an identifiable "plume"). The observed anomaly, no doubt, must be ascribed to some combination of the windrow circulation and the current shift which occurred during the experiment.

5. Concentration fluctuations

The mean-square fluctuation profiles obtained during the 1968 experiment just discussed were rather less regular than the 1965 results reported in Csanady (1966). The $s(y)$ distributions defined in Section 3 show somewhat more easily discernible trends. In spite of considerable scatter, these distributions show very much the same shape, the main difference between individual curves being a change in the horizontal length scale (see Fig. 7).

Particularly significant was thought to be the near constancy of the center value, $s(0)$, for different depths and different sections (excepting perhaps for cross section 4), at about $s(0)=0.3$. This suggests that the $s(y)$ distribution may be "self-preserving," in accordance with some theoretical results and laboratory data discussed by Csanady (1967). According to that study, the horizontal length scale of the mean-square fluctuation should be S_y. A replot in Fig. 12 of all the $s(y)$ profiles of the first three cross sections vs y/S_y as the abscissa indeed yields a more or less well-defined universal curve. Apart from random scatter (not enough measurements per section), the existence of some cross shear on this occasion may also have contributed to the departures from the "universal" profile.

6. Discussion

Most earlier experimental investigations of relative diffusion were oriented toward a determination of the

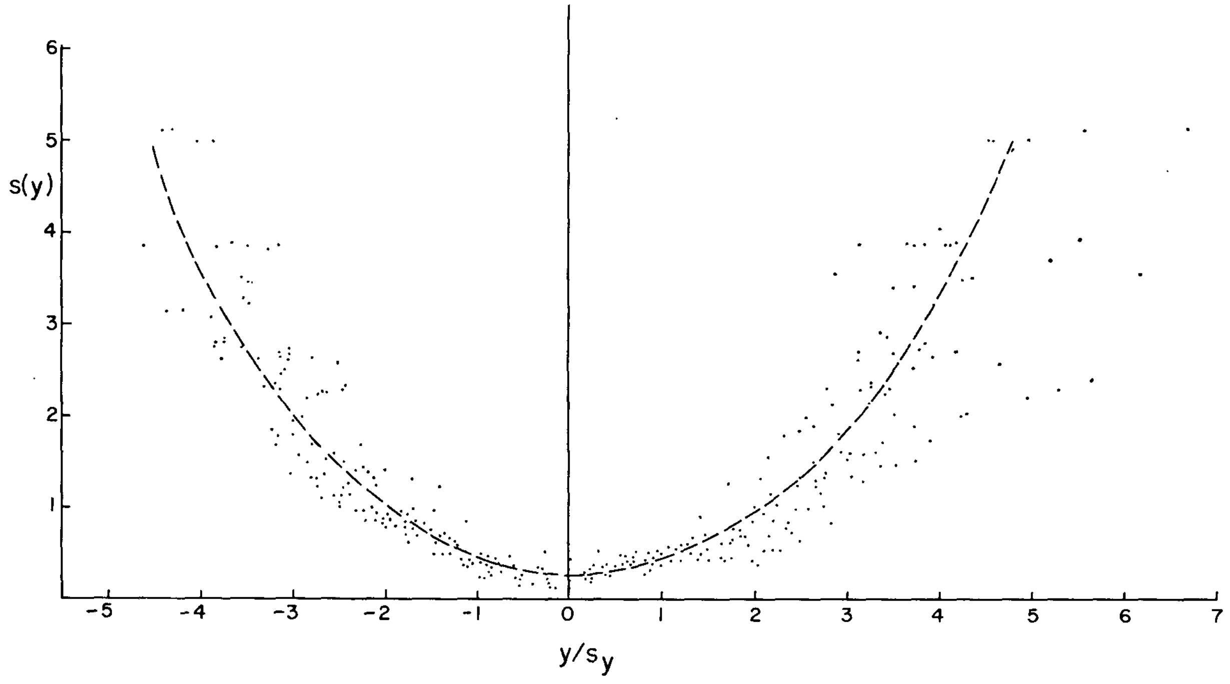

FIG. 12. Universal plot of the ratio of rms to mean concentration for cross sections 1–3, all depths.

growth rate [in our notation this would correspond to the functional relationship $S_y(x)$] or of the eddy diffusivity K_y. A particularly intriguing question was, whether a "$\frac{4}{3}$ power law regime" ($K_y \propto x^{\frac{4}{3}}$) existed in such cases, or, which is equivalent, whether Batchelor's (1952) "quasi-asymptotic" phase ($S_y \propto x^{\frac{2}{3}}$) was practically significant. Some data bearing on this problem were presented in I, showing that such a regime indeed existed but only relatively close to the source. At the distances where the later experiments were carried out, the "final phase" relationships provide a more satisfactory approximation to the observed behavior ($K_y=$constant). To this extent the present data merely confirm earlier results.

Of greater interest seems to be the determination of quite regular stochastic mean concentration profiles $\bar{\chi}(y)$ in a frame of reference moving with the center of gravity of a diffusing cloud. These show a general affinity with a Gaussian distribution. However, it was seen above that cross shear produces considerable skewness in these profiles, much as predicted theoretically by Okubo and Karweit (1969).

From a practical point of view the stochastic mean distribution $\bar{\chi}(y)$ is only a good predictor of observable concentrations (particularly of the maximum concentration) if $s(y)$ is small. From the universal curve in Fig. 12 we conclude in this instance, i.e., near the center [with $s(0)=0.3$], the mean distribution is a fair-to-poor predictor of observable maxima. Near the fringes of a plume it is entirely useless, as $s(y) > 1$.

The universal curve $s(y/S_y)$ also agrees qualitatively with the theoretical results of Csanady (1967). One important conclusion of that theoretical study was that the center value $s(0)$ is a "free" parameter, in the sense that it varies from case to case, apparently being dependent on the turbulence structure. Some atmospheric data suggest that $s(0)$ may be proportional to turbulence intensity. It should be noted during the 1965 experiment (Csanady, 1966) that $s(0)$ was about 0.6, or twice the present value. While no direct turbulence intensity measurements were carried out on the occasion of the diffusion experiments, results from other such measurements showed wide variations of turbulence intensity under apparently similar conditions, so that it is certainly reasonable to attribute the changes in $s(0)$ to this factor. As an aside, we may also add that with $s(0)=0.6$ the mean concentration profile $\bar{\chi}(y)$ would be a poor predictor of observable maxima everywhere, not only on the fringes.

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