A Rossby Wake due to an Island in an Eastward Current

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ABSTRACT

A mathematical barotropic model based upon the conservation of absolute vorticity is used to determine the effect of the spherical shape of the rotating earth [approximated by the beta (β) effect] on a steady uniform eastward current streaming past a cylindrical island in an unbounded ocean of uniform depth. Upstream far-field conditions are introduced that confine the disturbance pattern produced by the island to the region downstream from the island.

For an initially uniform eastward flow of velocity u_0 streaming past a cylindrical island of radius a, the downstream disturbance consists of a trail of meanders and eddies. The amplitude of these features depends upon the magnitude of the Island number [$\text{Is}=(\beta a^2/u_0)^{\frac{1}{2}}$] and the radial wavenumber equals $(\beta/u_0)^{\frac{1}{2}}$, which is the Rossby wavenumber for stationary planetary waves.

In order to confirm the theoretical results of the beta-plane wake for an eastward flow situation, appeal is made to a laboratory model, consisting of a rotating annulus with a sloping bottom to simulate the beta effect. Dynamic similarity is achieved through the nondimensional Island number. The resulting flow pattern reveals a uniform flow field upstream from the island with the formation of a stationary disturbance downstream that agrees qualitatively with the theoretical results.

1. Introduction

Quasi-geostrophic oscillations with retrograde phase velocity (opposite to the direction of rotation) owe their existence to the spherical shape of the rotating earth as has been shown by Hough (1898). Similar quasi-geostrophic waves have been obtained on a Cartesian coordinate system by Rossby *et al.* (1939) through the principle of conservation of absolute vorticity, where the effect of the spherical shape of the rotating earth is approximated by allowing the Coriolis parameter (f) to vary linearly with the north-south coordinate (y). This is customarily called the beta (β) plane approximation, where $\beta=df/dy$. To make these waves appear stationary with respect to the earth, one can (following Rossby) superimpose them upon an eastward flow of speed (u_0) equal to the phase speed of the westward directed oscillation. These stationary waves have a wavenumber equal to $(\beta/u_0)^{\frac{1}{2}}$.

It is well known that stationary quasi-geostrophic waves (Rossby waves) can occur downstream from a meridional ridge in a uniform eastward flow. It should also be possible to produce such disturbances in the lee of an island in a uniform eastward flow. In this case, however, the disturbance is confined in latitudinal range. We will refer to such a disturbance as a Rossby wake in analogy to, but contrasted with, a von Kármán wake (or trail).

In the present paper a steady-state, barotropic, mathematical model of a Rossby wake based upon the conservation of absolute vorticity is investigated. Attention is confined to initially uniform flow past a simple island in the form of a circular cylinder extending from the sea bed to and through the free surface. In order to check the theoretical results, a laboratory model is also investigated in which the β effect is simulated by bottom topography.

A major difficulty in the theoretical aspects of eastward flow past an obstacle on a rotating sphere was pointed out by Greenspan (1968, p. 267). The boundary condition on the obstacle and the condition of finite or vanishing disturbance in the far field is not sufficient to determine a unique solution. Moreover, this inherent indeterminacy exists only for eastward flow and not for westward flow.

A similar indeterminacy also arises in the two-dimensional case of gravity waves produced by a pressure disturbance on a running stream. Lamb (1945, p. 406) discusses two approaches toward removing the indeterminacy of that problem. First, Rayleigh's (1883) method concerns the utilization of a small viscous force (of the form $\mu\mathbf{V}$) which, when added to the equations of motion, makes the gravity wave problem determinate. After the solution is obtained then μ is set to zero, which results in a non-viscous solution indicating the absence of waves at some distance upstream from the pressure disturbance. Second, Kelvin's (Thomson, 1886) method ignores viscous forces but assumes at the outset that waves upstream from the pressure disturbance are absent. The desired solution is found by superimposing appropriate non-viscous solutions on the general solu-

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tion such that the waves upstream from the pressure disturbance are nullified.

In the present problem, a method reminiscent of Kelvin's is employed, i.e., we require that the quasi-geostrophic waves be suppressed at a reasonable distance upstream from the island cylinder. As justification of the assumption of no oscillatory disturbance upstream of the obstacle, we appeal to Frenzen's (1955) laboratory results as well as those described here, where stationary waves were indeed found to be confined to the region downstream from the cylindrical obstacle.

2. The mathematical model

a. Assumptions and constraints

Consider an unbounded homogeneous ocean of uniform depth, in which:

1) The fluid is incompressible and horizontally non-divergent.

2) The motion is steady.

3) The vertical distribution of pressure is given by the hydrostatic relation.

4) All frictional forces are neglected, as are tidal forces.

5) The model ocean is situated on the β plane, where the Coriolis parameter is approximated by $f=f_0+\beta y$.

6) A right circular cylinder of radius a extends from the bottom to above mean water level (see Fig. 1).

7) Attention is confined to the case where the flow in the far-field upstream of the island is uniform.

b. Basic equations

The horizontal equations of motion, using the hydrostatic approximation and other constraints enumerated

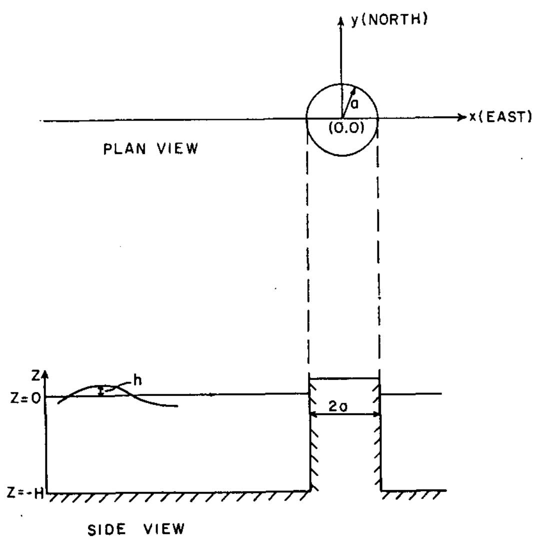


FIG. 1. Schematic of the island model showing coordinates.

above, can be written in the form

$$\left.\begin{aligned} -(f+\zeta)v+\frac{\partial}{\partial x}[(V^2/2)+gh]=0 \\ (f+\zeta)u+\frac{\partial}{\partial y}[(V^2/2)+gh]=0 \end{aligned}\right\}, \qquad (1)$$

where x and y are horizontal coordinates directed eastward and northward, respectively, h is the height anomaly of the sea surface above mean sea level, f the Coriolis parameter, u and v the x and y components of velocity, respectively, V the magnitude of the velocity, and ζ the vertical component of relative vorticity $(\partial v/\partial x-\partial u/\partial y)$.

The kinematic constraint of no horizontal divergence implies that a velocity streamfunction (ψ) exists, such that

$$u=-\frac{\partial\psi}{\partial y},\quad v=\frac{\partial\psi}{\partial x}. \qquad (2)$$

It follows from (1) and (2), for non-trivial conditions ($V\neq 0$), that

$$f(y)+\nabla^2\psi=F(\psi), \qquad (3)$$

$$gh+(1/2)|\nabla\psi|^2=G(\psi), \qquad (4)$$

$$F(\psi)=\frac{dG(\psi)}{d\psi}, \qquad (5)$$

where ∇ is the horizontal gradient operator. Eq. (3) expresses the conservation of absolute vorticity along a streamline while (4) is Bernoulli's relation, the functions F and G being related by (5).

The boundary condition on the island is simply

$$\psi=\psi_0=\text{constant},\quad \text{at } \rho=a, \qquad (6)$$

where ρ is radial distance from the center of the island.

In the far field we require that

$$u\rightarrow u_0=\text{constant} \qquad (7)$$

as $|x|\rightarrow\infty$. This implies that

$$\psi\rightarrow -u_0 y \qquad (8)$$

in the far field (taking $\psi=0$ at $y=0$). Using (8) in (3) allows the function $F(\psi)$ to be evaluated, i.e.,

$$F=f_0-\frac{\beta}{u_0}\psi. \qquad (9)$$

Moreover from (5) we obtain

$$G=G_0+f_0\psi-\frac{\beta}{2u_0}\psi^2. \qquad (10a)$$

If we take $h=0$ at $y=\psi=0$ in the far field, then from (4)

$$G_0=\tfrac{1}{2}u_0^2. \qquad (10b)$$

Finally, combining (3) and (9) yields

$$\nabla^2\psi+\frac{\beta}{u_0}\psi=-\beta y. \tag{11}$$

It is noteworthy that this relation governing ψ is linear even though the basic equations (1) are nonlinear. However, had we taken u_0 as a function of y, then the resulting equation for ψ would be nonlinear.

The problem is now reduced to that of finding solutions of (11) satisfying conditions (6) and (8) plus possible added conditions to be discussed later. Once the field of ψ is known, then the associated height field h can be evaluated directly from (4) and (10), which yield

$$h=\frac{1}{g}\left(\tfrac{1}{2}u_0^2+f_0\psi-\frac{\beta}{2u_0}\psi^2-\tfrac{1}{2}|\nabla\psi|^2\right). \tag{12}$$

c. Formal solutions for ψ

A particular solution of (11) is simply

$$\psi_p=-u_0y, \tag{13}$$

which corresponds to the far-field streamfunction. Let ψ_1 denote the departure from this far-field function; ψ_1 satisfies the homogeneous version of (11), which in polar coordinate form is

$$\frac{\partial^2\psi_1}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\psi_1}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\psi_1}{\partial\theta^2}+\frac{\beta}{u_0}\psi_1=0, \tag{14}$$

the azimuth θ being measured counterclockwise from the x axis (Fig. 2).

A general solution of (14) is

$$\psi_1(\rho,\theta)=\sum_{k=0}^{\infty}\{J_k(\alpha\rho)[A_k\sin k\theta+C_k\cos k\theta] +Y_k(\alpha\rho)[B_k\sin k\theta+D_k\cos k\theta]\}, \tag{15}$$

where A_k, B_k, C_k, D_k are constants; J_k, Y_k are Bessel functions of the first and second kind, respectively; and

$$\alpha\equiv(\beta/u_0)^{\frac{1}{2}}, \tag{16}$$

the Rossby wavenumber. Eq. (15) is the appropriate form of solution for $u_0>0$, i.e., for a basic eastward flow.

In the case of $u_0<0$, then α becomes imaginary and the appropriate form of solution is

$$\psi_1(\rho,\theta)=\sum_{k=0}^{\infty}\{K_k(|\alpha|\rho)[A_k'\sin k\theta+C_k'\cos k\theta] +I_k(|\alpha|\rho)[B_k'\sin k\theta+D_k'\cos k\theta]\}, \tag{17}$$

where A_k' B_k', C_k', D_k' are constants, and K_k and I_k are the modified Bessel functions of the first and second kind. These have a monotonic behavior in contrast with the oscillatory functions J_k and Y_k. Thus, the character

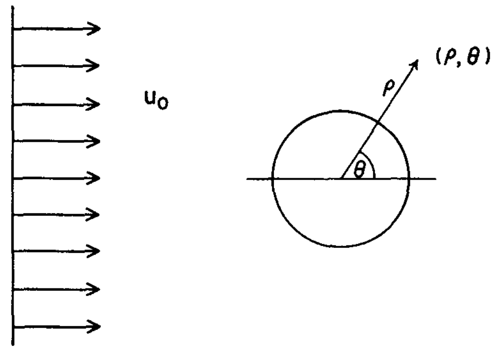


FIG. 2. Schematic of the basic state uniform eastward flow situation.

of the disturbance produced by the island is qualitatively different for eastward and westward flows.

The boundary condition (6) requires at $\rho=a$ that

$$-u_0y+\psi_1=\psi_0;$$

and since $y=a\sin\theta$ on the cylinder, then

$$\psi_1(a,\theta)=\psi_0+u_0a\sin\theta. \tag{18}$$

For the case of $u_0>0$, (18) requires that

$$\left.\begin{aligned} A_1J_1(\alpha a)+B_1Y_1(\alpha a)&=u_0a\\ A_kJ_k(\alpha a)+B_kY_k(\alpha a)&=0,\quad k\neq1\\ C_0J_0(\alpha a)+D_0Y_0(\alpha a)&=\psi_0\\ C_kJ_k(\alpha a)+D_kY_k(\alpha a)&=0,\quad k\neq0\end{aligned}\right\}. \tag{19}$$

The far-field condition (8) imposes no further constraint on the coefficients since both J_k and Y_k approach zero in the far field for all k. Thus, for this case, we have twice as many coefficients as equations and the solution is clearly not unique.

On the other hand, for the case of $u_0<0$, the far-field condition (8) demands that B_k' and D_k' be zero for all k since the function I_k approaches infinity for large argument. The Island condition (18) then gives the following values for the remaining coefficients:

$$\left.\begin{aligned} A_1'K_1(|\alpha|a)&=u_0a\\ A_k'&=0,\quad k\neq1\\ C_0'K_0(|\alpha|a)&=\psi_0\\ C_k'&=0,\quad k\neq0\end{aligned}\right\}. \tag{20}$$

The only arbitrary quantity for this case is ψ_0. If one demands symmetry of flow about the y axis, then this would require $\psi_0=0$; the solution in this case is unique. The resulting flow regime near the island for $u_0<0$ looks very much like that of potential flow (corresponding to $\alpha=0$).

d. Added conditions for eastward flow

Clearly, we must add further constraints for the case of $u_0>0$ in order to remove the indeterminacy.

If one demands *symmetry of flow about* $y=0$, as seems physically justifiable for initially uniform flow past a circular cylinder, then ψ_0 and all of the coefficients of the $\cos k\theta$ terms in (15) should vanish (i.e., $C_k = D_k = 0$ for all k). The remaining terms can be combined in the following form:

$$\psi_1(\rho,\theta) = [A_1 J_1(\alpha\rho) + B_1 Y_1(\alpha\rho)] \sin\theta + \sum_{k=2}^{\infty} E_k [Y_k(\alpha a) J_k(\alpha\rho) - J_k(\alpha a) Y_k(\alpha\rho)] \sin k\theta, \quad (21)$$

where A_1 and B_1 satisfy

$$A_1 J_1(\alpha a) + B_1 Y_1(\alpha a) = u_0 a, \quad (22)$$

and the coefficients E_k are thus far arbitrary. Note that there is no contribution to ψ_1 at the island from the terms modified by E_k.

To complete the determination of the Fourier coefficients, we adopt a method similar to that of Kelvin in which the waves upstream are suppressed. The manner in which this is done has certain arbitrary aspects. One possibility would be to minimize (in a least-squares sense) the departure of the streamfunction from a simple potential flow solution in the region upstream from the island. A more simple condition to apply and one which accomplishes the same purpose is to require at $\theta = \pi$ that

$$\rho^{\frac{1}{2}} \frac{\partial \psi_1}{\partial \theta} \rightarrow 0, \quad \text{as } \rho \rightarrow \infty. \quad (23)$$

Indeed, it is found that this is sufficient to make the problem determinate provided that we truncate the series in (21) at $k=2$.

If we make use of the asymptotic property of the Bessel functions for large values of $\alpha\rho$ and set all $E_k = 0$ for $k > 2$, then (23) leads to the condition

$$\{[A_1 - 2E_2 J_2(\alpha a)] \cos(\alpha\rho - \tfrac{3}{4}\pi) + [B_1 - 2E_2 Y_2(\alpha a)] \sin(\alpha\rho - \tfrac{3}{4}\pi)\} = 0. \quad (24)$$

If this is to apply at all $\alpha\rho$ in the far field, then both coefficients in brackets must vanish. This gives

$$\left.\begin{aligned} A_1 &= 2E_2 J_2(\alpha a) \\ B_1 &= 2E_2 Y_2(\alpha a) \end{aligned}\right\}, \quad (25)$$

and substitution into (22) yields

$$E_2 = (u_0 a/2)[J_1(\alpha a) J_2(\alpha a) + Y_1(\alpha a) Y_2(\alpha a)]. \quad (26)$$

The final relation for the total streamfunction is

$$\psi = \{-u_0\rho + (u_0 a/Q)[J_{2a} J_1(\alpha\rho) + Y_{2a} Y_1(\alpha\rho)]\} \sin\theta + (u_0 a/Q)[Y_{2a} J_2(\alpha\rho) - J_{2a} Y_2(\alpha\rho)] \sin 2\theta, \quad (27)$$

where

$$\left.\begin{aligned} Q &\equiv J_{1a} J_{2a} + Y_{1a} Y_{2a} \\ J_{ka} &= J_k(\alpha a), \quad Y_{ka} = Y_k(\alpha a) \end{aligned}\right\}. \quad (28)$$

It can be shown from the asymptotic properties of the Bessel functions that

$$Q \rightarrow \begin{cases} 8\pi^{-2}(\alpha a)^{-3}, & \text{as } \alpha a \rightarrow 0 \\ 3\pi^{-1}(\alpha a)^{-2}, & \text{as } \alpha a \rightarrow \infty \end{cases} \quad (29)$$

Moreover, Q remains positive for all αa and hence the solution remains bounded for all finite αa.

Due to the importance of the parameter αa, it is designated the Island number, i.e.,

$$\text{Is} \equiv \alpha a = (\beta a^2/u_0)^{\frac{1}{2}}. \quad (30)$$

An inquiry into the effects of Is upon the near-field streamfunction show that when the Island number is much less than unity, the flow around the island approaches that of simple potential flow around the cylindrical object, *viz.*,

$$\psi \approx \left(\frac{u_0 a^2}{\rho} - u_0\rho\right) \sin\theta + O(u_0 a \text{Is}), \quad (31)$$

for Is≪1. In this case beta clearly has a negligible effect upon the flow pattern.

On the other hand, for values of the Island number greater than unity, a significant disturbance in the streamlines occurs downstream from the island. This is illustrated in Figs. 3a–c which show the streamline patterns for Is equal to 1.0, 2.0 and 3.0, respectively. As a specific illustration we have taken $u_0 = 20$ cm sec^{-1}, and $\beta = 2.0 \times 10^{-13}$ cm^{-1} sec^{-1} common for all three figures so that only the radius varies, these being 100, 200 and 300 km, respectively. For Is = 1.0 there exists a gentle meander pattern. This is much accentuated for Is = 2.0, where also a closed cyclonic eddy exists adjacent to and slightly downstream of the island. For Is = 3.0 the eddy formation has extended meridionally and is accompanied by much more vigorous meandering downstream.

In all of Figs. 3, the disturbance pattern forms a mirror image about the zonal axis of the island, with stagnation points located on this axis at the island boundary, which is the same as in a potential flow situation. The flow pattern is characterized generally by basic state flow upstream from the island and a disturbance pattern in the downstream region. As can be observed, the amplitude of the disturbance pattern increases with increasing Island number; however, the radial wavelength is the same in each of the three figures and is equal to the Rossby wavelength (628 km for this case). The radius a of the island does not affect the wavelength.

In Figs. 3b and 3c, the maximum disturbance produced by the island is found along a radial line that is offset slightly downstream from the meridional axis of the island. This disturbance takes the form of a semi-enclosed cyclonic eddy. In Fig. 3b, only one such eddy has formed, but in Fig. 3c a series of eddies can be seen. These eddies are elliptical in shape with the minor axis in the radial direction and the major axis in the

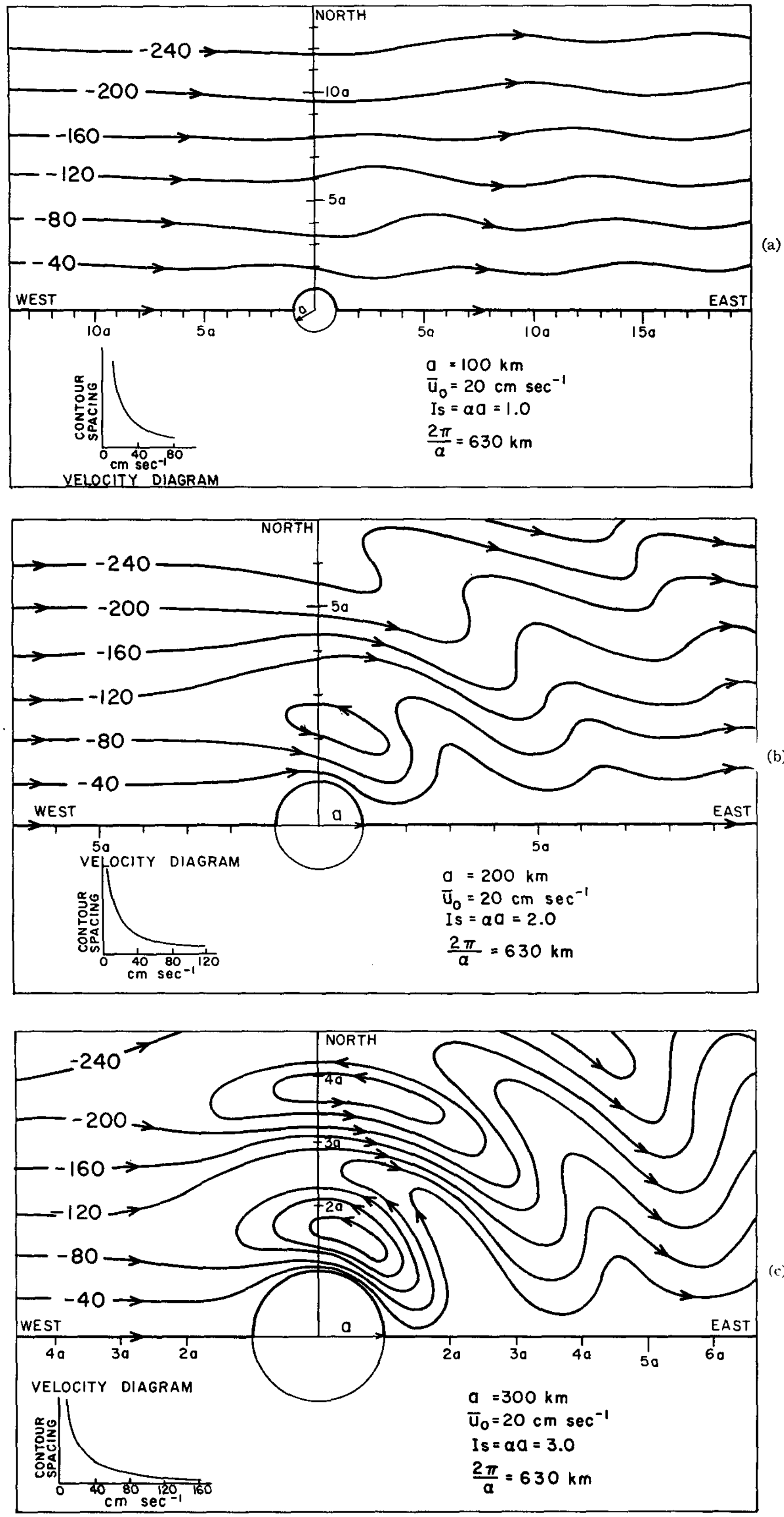


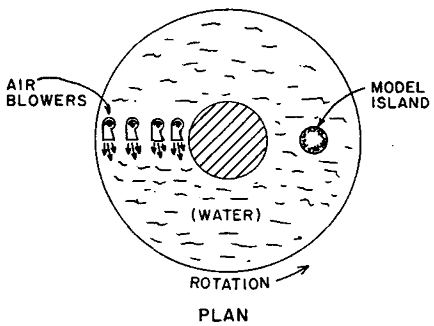
FIG. 3. Field of streamfunction (10^7 cm^2 sec^{-1}) for the uniform eastward flow situation. Is = 1.0, 2.0 and 3.0 for (a), (b) and (c), respectively.

azimuthal direction. In Fig. 3c, this series of eddies attenuates in amplitude with increasing distance from the island and can be expected to vanish in the far field. If friction were taken into account we would expect a greater attenuation of the disturbance away from the island.

Making use of the velocity nomograms in Figs. 3, maximum speeds are encountered next to the perimeter of the island between $\pi/2>\theta>\pi/3$. The magnitude of this maximum depends upon the size of the Island number. For Is=1.0 (Fig. 3a), the maximum speed is approximately $2\bar{u}_0$ and is found slightly downstream from $\theta=\pi/2$, nearly the same as in potential flow around a cylindrical object. For Is=2.0 (Fig. 3b), the maximum speed is approximately $5\bar{u}_0$ and occurs near $\theta=\pi/3$. For Is=3.0 (Fig. 3c), the maximum speed is approximately $8\bar{u}_0$ and is found at $\theta=\pi/3$. This speed intensification is associated with the pronounced eddy formation that lies near the meridional axis of the island.

An important result observed in Fig. 3 is that no perturbation occurs along the zonal axis of the island. This is a direct result of the assumed meridional symmetry of the basic state situation; both beta and the basic state velocity are symmetric (and constant) about the zonal axis of the island. If this symmetry were absent, then a stationary wave could form downstream along the island's zonal axis. This can be seen in the laboratory experiments of Frenzen (1955), where a Rossby wave train occurred downstream along the zonal axis of the cylindrical object. In this latter case both the beta and the basic state velocity changed radically and unsymmetrically across the zonal axis of the cylindrical object.

The field of the sea surface height anomaly h is not presented. It differs very little from the streamfunction field for the case of non-zero f. The only place it will differ very much from that of ψ is for an island centered at or near the equator, where the Coriolis parameter tends to zero and the nonlinear terms in the Bernoulli relation become important.



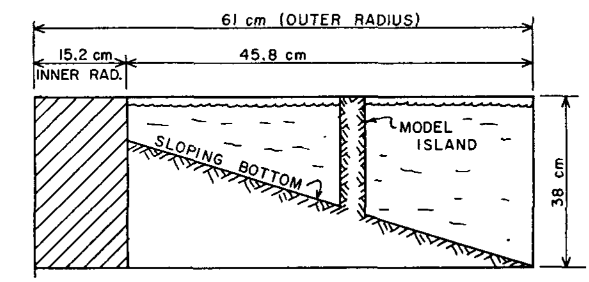


FIG. 4. Schematic plan view and cross section of the laboratory model experiment.

3. Laboratory model

a. Dynamic similarity

The mathematical oceanic island model is based upon the conservation of absolute vorticity. This vorticity balance can also be simulated in a laboratory rotating annulus with a radially sloping bottom. In the rotating annulus, the meridional variation in the Coriolis parameter is simulated by the vorticity changes accompanying the stretching and shrinking of fluid columns that move radially in the presence of rotation (von Arx, 1952). Such a laboratory set-up is here used to observe the circulation pattern associated with a uniform eastward current streaming past a cylindrical model island.

Previously, Frenzen (1955) was able to generate a train of stationary Rossby waves downstream from a cylindrical object on a rotating hemisphere. In his experiment, the object's diameter corresponded to 20° of latitude, a meridional range over which both beta and the ambient flow conditions of the model changed considerably. Therefore, these results are probably not valid for a smaller scale phenomenon, where both the beta parameter and the ambient flow can be considered as constant. In the present experiment, the scale size of the island is of the order of 3° of latitude, over which beta and ambient flow are considered constant.

The experiment was conducted in a Plexiglas rotating tank built by Prof. Takashi Ichiye. The tank is 38 cm deep, is 61 cm in radius, and can be rotated at speeds up to 30 rpm. The annulus is constructed by placing a Plexiglas cylinder (radius of 15.2 cm) at the center of the tank. The sloping bottom is fashioned from sheet aluminum in the shape of a cone that fits into the annulus. The ambient flow speed is produced by a surface wind stress (using four blowers placed opposite to the model island), which, after a sufficient waiting period, produces the required steady state uniform flow field. A schematic diagram of the laboratory set-up is shown in Fig. 4.

For the laboratory model the effective value of β is given by

$$\beta_m=\frac{2\omega}{H}\frac{dH}{dr}, \tag{32}$$

where ω is the angular velocity of the model and H the

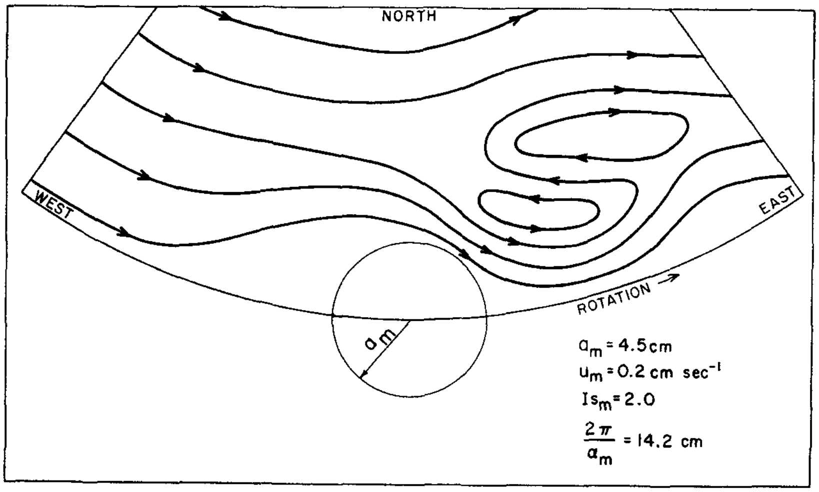


FIG. 5. Stationary flow pattern in the laboratory model, where $\mathrm{Is}_m = 2.0$.

depth of fluid at radial distance r from the axis of rotation. In order to obtain a model Island number of 2.0 with $u_m = 0.2$ cm sec^{-1} and $a_m = 4.5$ cm, we require that β_m be about 0.039 cm^{-1} sec^{-1}. Thus, for a rotation rate of 10 rpm ($\omega = 1.05$ rad sec^{-1}), we require a fractional rate of change of H of

$$\frac{1}{H}\frac{dH}{dr} = 0.0185. \qquad (33)$$

In the actual model the bottom was taken in the form of a conical surface with constant slope such that (33) is satisfied in the vicinity of the model island at which $H \approx 23$ cm so that $dH/dr = 0.43$.

The theoretical wavelength of the waves downstream from the cylinder in the model is given by

$$\lambda = 2\pi\left(\frac{u_m}{\beta_m}\right)^{\frac{1}{2}} = 14.2 \text{ cm}. \qquad (34)$$

b. Results of the experiment

The results of this experiment are given in Fig. 5. It illustrates the steady-state flow pattern that is constructed from a time series of photographs showing the path of dye patches from their introduction upstream of the model island to their extension past the island. Due to the close proximity of the model island to the outer wall of the annulus, only the circulation pattern poleward of the island is investigated.

Fig. 5 shows a pattern of flow quite similar to that of Fig. 3b or 3c. Near the island boundary, the speed is reduced on the upstream side and intensified on the downstream side. Associated with this downstream intensification is the formation of an eddy pair. This is similar to that observed in Fig. 3c, except that the eddies are more zonally elongated in the laboratory model. Unlike the prototype picture, no eddies are formed farther away from the island. This may be due to the presence of the inner wall of the annulus. Downstream from the eddy pair, the flow meanders rather gently with an approximate wavelength (not shown in the figure) of 12 cm, somewhat less than the predicted Rossby wavelength (14.2 cm).

4. Summary

We have restricted our analysis to a steady barotropic flow regime around a vertical circular cylinder in a system where f varies linearly with y. Moreover, friction has been neglected and the possibility of boundary layer separation was not allowed for in this work. A solution for the case of uniform eastward flow upstream from the island is obtained. Because of the inherent indeterminacy of the problem, it cannot be claimed that the present solution is physically unique. However, comparison of the theory with laboratory experiments shows qualitative agreement.

An important nondimensional parameter in the theory is the Island number defined by (30). For sufficiently large values of Is the disturbance downstream from the island shows vigorous meandering with closed eddies adjacent to the island, a feature which seems to be confirmed by the limited laboratory experiments.

Although not investigated here, it is plausible that the influence of turbulent transfer of momentum would limit the latitudinal and downstream extent of the disturbance; without frictional attenuation the disturbance decays only as $\rho^{-\frac{1}{2}}$ on the downstream side.

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