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网格基任向偏导实现及层析应用*

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摘　要:将光线偏折方程中的任向偏导转化为数值差分形式,并应用于层析线性运算. 网格化待测场,将微分待测场的每一正方形网格及相应折射率近似为曲面底的正圆锥体,圆锥体顶端的折射率值对应该网格的折射率,在底面的投影对应网格的中心. 假设紧邻三网格中心间的折射率分布共平面,在一个网格宽度内将偏导转化成数值差分形式. 结果发现:基于上述近似和假设,可以将任意探测光线相关的偏导转化为数值差分形式,将非线性偏导方程转化为线性差分方程,建立层析方程. 于是,偏折角可以作为投影直接重建.

关键词:层析;折射率;投影;偏导;差分

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0　引言

利用光学计算机层析技术[1-2]将流场可视化[3-4]是一种重要的流场可视化方案,人们已做了大量研究[5-6],开发了干涉层析技术可以成功可视化低温流场[7-8],但随温度升高,干涉条纹判读越来越困难. 高温流场层析,需要探索新的投影方法. 偏折投影对温度变化不是很敏感,但探测光线的偏折角作为层析的重建数据,建立的是待测场折射率的一阶偏导数非线性方程组[9],不能直接用层析的一般解法直接解偏折问题[10].

本文通过对网格的曲底正圆锥近似;紧邻三网格中心间区域的折射率分布共面假设,将偏导转化成数值差分运算,为应用层析的迭代算法[11]解偏折问题克服了偏导算符"非线性"障碍.

1　偏导方程

光线穿过非均匀折射率场时,受场的扰动,传播方向发生偏折. 光线离开折射率场后测试段出口处的偏折角为[9]

$$\alpha=\int\frac{1}{n}\frac{\partial n}{\partial y}\mathrm{d}z \tag{1}$$

式中,α 是偏折角,n 是折射率.

将待测场网格化,并建立层析坐标系如图 1.

设待建场按图 1 划分为 H 行、W 列,共 $N=H\times W$ 个足够小的相等网格,设每一个小网格内的折射率是均匀的,则对于某一射线ray,按照式(1)

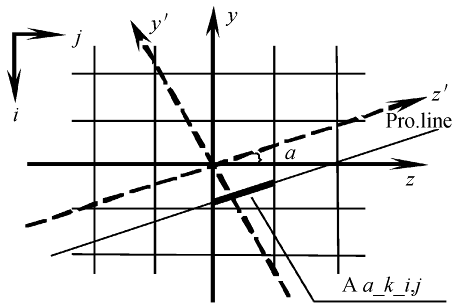

图 1　层析坐标系

Fig. 1　Reference frames for tomography

可得[12]

$$\sum_{i=0}^{H-1}\sum_{j=0}^{W-1}A_{\mathrm{ray}\_i,j}\frac{1}{n_{i,j}}\left(\frac{\partial n}{\partial y'}\right)\Big|_{i,j}=\varphi_{\mathrm{ray}} \tag{2}$$

式中 $A_{\mathrm{ray}\_i,j}$ 为射线 ray 在第$[i,j]$个网格内的截距,i,j 分别是当前网格的纵、横坐标.

2　偏导差分计算

如果将层析的迭代解法应用于式(2),则需要将式(2)中的偏导数$(\partial n/\partial y')|_{i,j}$转化为数值差分形式. 图 2 给出了一种转化方法. 将网格$[i,j]$近似为

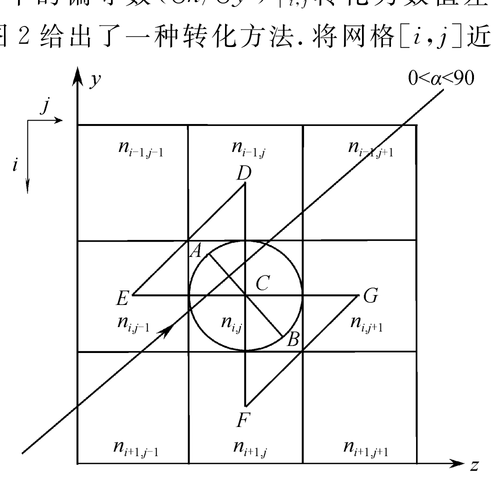

图 2　计算折射率 n 关于 y' 偏导数的方法

Fig. 2　The schematic diagram showing a method to calculate the partial derivative of refractive index n versus y'

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一个底面半径为 r，底面为曲面的圆锥体，圆锥的顶点在网格的中心，值为 $n_{i,j}$，此时通过中心 C 对经过该网格的射线做垂线交底面圆于 A，B 两点，则 $(\partial n/\partial y')|_{i,j}$ 可以表示为 $(\partial n/\partial y')|_{i,j} = (n_A - n_B)/2r$，考虑网格[$i$,j]周围的 8 个相邻网格，图 2 示例在投影方向 $0<\alpha<90°$ 的情况下，用 C、D、E 三点构成的平面计算 A 在该平面上的值，用 C、F、G 构成的平面计算 B 点的值.

具体计算为

2.1 确定 A、B 点的坐标

根据图 2，按照内切圆近似，确定 A，B 点的位置坐标分别为

$$\begin{cases} A(z_0-\dfrac{rk}{\sqrt{1+k^2}},y_0+\dfrac{r}{\sqrt{1+k^2}}) \\ B(z_0+\dfrac{rk}{\sqrt{1+k^2}},y_0-\dfrac{r}{\sqrt{1+k^2}}) \end{cases} \quad (k>0) \tag{3}$$

或

$$\begin{cases} A(z_0+\dfrac{rk}{\sqrt{1+k^2}},y_0-\dfrac{r}{\sqrt{1+k^2}}) \\ B(z_0-\dfrac{rk}{\sqrt{1+k^2}},y_0+\dfrac{r}{\sqrt{1+k^2}}) \end{cases} \quad (k<0) \tag{4}$$

式中，(z_0, y_0) 是当前网格中心的坐标，k 是探测光线的斜率.

2.2 确定 A、B 点的折射率

当 $k>0$ 时，A 点折射率. 参考式(3)，A、C、D、E 点空间坐标分别为：$A(z_0 - rk/\sqrt{1+k^2}, y_0 + r/\sqrt{1+k^2}, n_A)$、$C(z_0, y_0, n_{i,j})$、$D(z_0, y_0+D_y, n_{i-1,j})$、$E(z_0-D_z, y_0, n_{i,j-1})$，其中，$D_z$，$D_y$ 分别是当前网格 z 轴、y 轴方向边长，通常取等值. 假设 A 点的折射率与周围紧邻的 8 个点的折射率相关，并且这种关系是简单的共平面关系，即 A 点的折射率 n_A 在 C，D，E 三点的折射率 n_C，n_D，n_E 决定的平面上，则可由 n_C，n_D，n_E 决定的平面求得 n_A. 参考图 2 计算得

$$n_A = n_{i,j} + \frac{rk}{\sqrt{1+k^2}}\,\frac{n_{i,j-1}-n_{i,j}}{D_z} + \frac{r}{\sqrt{1+k^2}}\cdot \frac{n_{i,j}-n_{i-1,j}}{D_y} \tag{5}$$

同样的方法可得 B 点的折射率 n_B 为

$$n_B = n_{i,j} - \frac{rk}{\sqrt{1+k^2}}\,\frac{n_{i,j}-n_{i,j+1}}{D_z} - \frac{r}{\sqrt{1+k^2}}\cdot \frac{n_{i,j}-n_{i+1,j}}{D_y} \tag{6}$$

由式(5)和(6)得

$$\left.\frac{\partial n}{\partial y'}\right|_{i,j} = \frac{n_A-n_B}{2r} = \frac{k}{\sqrt{1+k^2}}\,\frac{n_{i,j-1}-n_{i,j+1}}{2D_z} + \frac{1}{\sqrt{1+k^2}}\,\frac{n_{i-1,j}-n_{i+1,j}}{2D_y} \quad (k>0) \tag{7}$$

同样的方法可得 $k<0$ 时的偏导数为

$$\left.\frac{\partial n}{\partial y'}\right|_{i,j} = \frac{n_A-n_B}{2r} = \frac{k}{\sqrt{1+k^2}}\,\frac{n_{i,j+1}-n_{i,j-1}}{2D_z} + \frac{1}{\sqrt{1+k^2}}\,\frac{n_{i+1,j}-n_{i-1,j}}{2D_y} \quad (k<0) \tag{8}$$

3 偏折层析表达式

将式(7)、(8)分别代入式(2)，并整理得

$$\sum_{i=0}^{I-1}\sum_{j=0}^{J-1}\frac{k}{|k|}\Big[\frac{k}{D_z}(A_{a\_r\_i,j+1}-A_{a\_r\_i,j-1}) + \frac{1}{D_y}(A_{a\_r\_i+1,j}-A_{a\_r\_i-1,j})\Big]n_{i,j} + n_0\frac{k}{|k|}\cdot \Big[\frac{k}{D_z}(\sum_{\substack{i=0\\ j=0,\mathrm{left}}}^{I-1}A_{a\_r\_i,0} - \sum_{\substack{i=0\\ j=J-1,\mathrm{right}}}^{I-1}A_{a\_r\_i,J-1}) + \frac{1}{D_y}(\sum_{\substack{j=0\\ i=0,\mathrm{top}}}^{J-1}A_{a\_r\_0,j} - \sum_{\substack{j=0\\ i=I-1,\mathrm{bottom}}}^{J-1}A_{a\_r\_I-1,j})\Big] = \frac{n_0\varphi_{a\_r}}{\xi}(k>0 \text{ or } k<0) \tag{9}$$

式中 $\xi=1/2\sqrt{1+k^2}$. 式(9)是 $k>0$ 或 $k<0$ 时第 r 条探测光线的偏折投影方程. 同样的方法可得 $k=0$ 或 $k=+\infty$ 时的投影方程

$$\sum_{i=0}^{I-1}\sum_{j=0}^{J-1}\frac{A_{0\_r\_i+1,j}-A_{0\_r\_i-1,j}}{D_y}n_{i,j} + \frac{n_0}{D_y}(\sum_{\substack{j=0\\ i=0,\mathrm{top}}}^{J-1}A_{0\_r\_0,j} - \sum_{\substack{j=0\\ i=I-1,\mathrm{bottom}}}^{J-1}A_{0\_r\_I-1,j}) = 2n_0\varphi_{0\_r} \quad (k=0) \tag{10}$$

$$\sum_{i=0}^{I-1}\sum_{j=0}^{J-1}\frac{A_{90\_r\_i,j+1}-A_{90\_r\_i,j-1}}{D_z}n_{i,j} + \frac{n_0}{D_z}(\sum_{\substack{i=0\\ j=0,\mathrm{left}}}^{I-1}A_{90\_r\_i,0} - \sum_{\substack{i=0\\ j=J-1,\mathrm{right}}}^{I-1}A_{90\_r\_i,J-1}) = 2n_0\varphi_{90\_r} \quad (k=+\infty) \tag{11}$$

如果取 l 个投影方向，每个方向上 k 条采样射线，则共有射线 $l\times k$ 条，记为 $M=l\times k$. 则共有 M 个这样的偏折投影方程，这些方程作为一个方程组构成了偏折层析方程组.

4 结论

通过将正方形网格视为底面半径为 r，底面是曲面的圆锥体，圆锥的顶点在网格的中心，值为该网格折射率的近似；以及上下、左右紧邻的三网格中心间折射率分布共面假设，可将折射角投影的偏导数转化为数值差分形式. 从而可以解决偏折层析的非线性问题，可以直接使用偏折角作为投影建立层析方程.

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A New Technique to Calculate Partial Derivative in Random Direction on Micro-grid Frame Being Applied to Tomography

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Abstract: The random-direction partial derivatives in refraction equation were transformed into numerical differences in reference frames for tomography, and the nonlinear refraction equations were transformed into linear reconstructive equations. The diagnosed field was divided into tiny foursquare grids. Each grid and its relative refractive index were approximated to a correct cone with an irregular bottom. The peak of the cone corresponded to the refractive index of the grid, and its geometrical projection to the bottom located the grid center. With the approximation, the space partial increment calculation was much simplified at any grid, in any direction, and to any detecting ray. It was assumed that the refractive index distribution should be coplanar in the zone between three grid centers of the three close-adjacent grids. Here, the zone was a space triangle deriving from connecting the close-adjacent grid centers with straight lines. With the assumption, the refractive index partial increment could be calculated with a numerical difference function of close-adjacent grid refractive indexes. With the approximation and assumption, the partial derivative was transformed into numerical difference in a grid width. And based on the approximation and assumption, partial derivative related to any detecting ray could be transformed into numerical difference. Nonlinear refraction equations could be transformed into linear reconstructive equations. The deflection angles can be applied to reconstruction directly as projections.

Key words: Tomography; Refractive index; Projections; Partial derivative; Difference

SONG Yi-zhong was born in 1971. He received the Ph. D. degree in nonlinear optics from Shandong University in 2003. He has been a postdoctor at Nanjing University of Aeronautics and Astronautics since 2006. He is majoring in optical engineering and his research interests focus on optical computerized tomography, optical information processing and numerical image processing.