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Self-enforcing contracts in agriculture

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Summary

This article examines a self-enforcing contract between a risk-neutral principal and a risk-averse agent who is able to hold up values *ex post*. It shows that risk aversion and variance can only partly explain the contract's incentive intensity. *Ex post* bargaining power and outside options will also determine contract choice. If the agent's *ex post* bargaining position is weak, the principal cannot commit to high-powered incentives, whereas if the agent's *ex post* bargaining position is good, the agent cannot commit to low-powered incentives. The model may thus explain some puzzles in agricultural contracts, in particular why risk-averse agents sometimes accept to be governed by high-powered incentive contracts that are quite similar to fixed rental contracts, and why risk-neutral agents are sometimes offered share contracts with lower-powered incentives.

Keywords: relational contracts, hold-up, moral hazard, risk aversion

JEL classification: D82, L14, Q19

1. Introduction

Agricultural contracts between landlord and tenant, or between farmer and integrator company, are often simple and informal, and in many cases not even written (see Allen and Lueck, 2003). The reason for this contract incompleteness is twofold. First, there may be variables that cannot be easily verified by the court in case of breach. For instance, the parties may contract on the quality of the crop, but even if quality is observable by the contracting parties, it may not be easy for an uninitiated court to assess whether the quality is equal to the one described in the contract. However, the performance of farmers can often be measured quite well. Thus, a second, and more natural explanation for the simplicity of agricultural contracts is that simplicity is efficient. It has been shown by several authors that even if parties are able to write complete contracts, it may be less costly to engage in simple (informal) contracting and to rely on market enforcement—or self-enforcement—instead of court protection.[1] A self-enforcing *relational contract* (also called an implicit

1 The point that it may be efficient for contracting parties to deliberately leave contracts incomplete is shown by Boot *et al*. (1993), Bernheim and Whinston (1998) and Kvaløy and Olsen (2005). Allen and Lueck (2003) discuss the existence of seemingly excessive contract simplicity in agriculture.

contract) must be designed such that the parties have economic incentives to honour it in all contingencies. In economic theory, a relational contract is modelled in a repeated game framework where the contracting parties write a contract on future transactions, and where the present value of honouring the contract versus the present value of reneging decides the contract's self-enforcing conditions.

An interesting feature in modern agriculture is the farmer's attempt to balance aspects of traditional employment and self-employment. On the one hand, the farmer runs his own firm; he owns physical assets, such as the land on which he works, and he is responsible for generating his own income. On the other hand, he is a risk-averse agent who seeks protection through contracting with larger integrator companies (or processors) that can provide him with critical assets such as management services, veterinarian services, feed and other inputs, as well as marketing and sales.

This balance is not captured in the traditional principal–agent literature. The agent is typically modelled as a 'non-owner' where values accrue directly to the principal in the process of production. Once the agent is modelled as an independent owner, he is by definition a firm and thus modelled as risk-neutral (because firms can 'share' risk). In this paper, it is assumed that the agent is both risk-averse and (potentially) an owner of critical assets. Hence, it studies the self-enforcing conditions of an incentive contract between a risk-neutral principal and a risk-averse agent who exerts unobservable effort and who is able to hold up values after production has taken place.

This hold-up opportunity affects the feasible intensity of incentives. In the classical analysis of agricultural incentive schemes (that is, the sharecropping analysis by Cheung (1969) and Stiglitz (1974)), the optimal share of output, or 'bonus-level',[2] is determined exclusively by the risk-insurance trade-off. If the principal is risk-neutral, and the agent is risk-averse, the principal should bear the risk of variance in output. But if the agent exerts unobservable effort, the principal must provide him with incentives to do so, and thus to some extent make pay conditional upon output. The optimal level of performance pay is then a negative function of the agent's risk aversion and the variance of output,[3] and a positive function of the agent's responsiveness to incentives.

When contracts are constrained by the requirement of being self-enforcing, however, risk sharing and variance can only partly explain contract choice. Bargaining power and *ex post* outside options are also important: in an incentive contract, there is a fixed transfer paid *ex ante*, and a contingent payment paid *ex post* that depend on observed output. *After* production, but *prior to* contingent payment, one of the parties can find it profitable to renegotiate

2 The terms 'bonus' and 'performance pay' are not common in agricultural contracts, but they are used here as they are commonly found in the incentive literature.

3 This relationship holds if the state of nature is revealed *after* effort has been expended. If the state of nature is revealed *after* the contract signing, but *before* effort is expended, optimal performance pay can be a positive function of variance (see Prendergast, 2002; Baker and Jorgensen, 2004).

this performance pay. Such renegotiations will, however, be regarded by the other party as a deviation from the relational contract, and therefore ruin future relational contracting. To implement a relational contract the parties must *ex ante* design the incentive scheme such that *ex post* renegotiations do not take place.[4] Because *ex post* bargaining power and outside options will determine the outcome of *ex post* renegotiations, they will also determine the *ex ante* design of the incentive scheme.

The model in this paper shows that if the agent's *ex post* bargaining position is weak, it may be impossible to implement high-powered incentives, because if the agent is entitled to a large share of output, this may lead the principal to renegotiate the terms of the contract *subsequent to* value realisations. In terms of agricultural contracts, this implies that if the farmer or tenant is unable to hold up the crop *ex post*, it may be impossible to implement fixed rental contracts, i.e. a contract that asks the tenant (farmer) to pay a fixed amount to the landlord (integrator) irrespective of the level of output, and keep the residual himself. In one-sided moral hazard, such a contract is optimal if the tenant (or farmer) is risk-neutral. The analysis shows that it may be impossible to implement fixed rental contracts if the landlord (or integrator) is able to expropriate the crop *ex post*, or have strong *ex post* bargaining power.

If the agent's *ex post* bargaining position is good, however, it may be impossible to implement contracts with low-powered incentives, as the agent has an incentive to renege on the contract and plea for a renegotiation. Hence, if the agent is risk-averse and prefers something like a fixed-wage contract, a good *ex post* bargaining position creates a lower bound on the 'bonus' level that lies above the desirable level. This result casts some light on the shift from tenancy to independent farming. When the farmer actually owns the land on which he works, this automatically increases the farmer's incentives to exert effort, as he (to some extent) is residual claimant of the crop. But, as stated by Holmström (1999), 'market incentives do not protect at all against risk'. Hence, if the farmer is risk-averse, he faces a dilemma: as asset-owner he is exposed to the incentives of the market. But as a risk-averse agent he would prefer a secure employment relationship with a fixed salary and a low degree of performance pay.

Even though bargaining power and *ex post* outside options matter, comparative statics show that the optimal bonus level of the relational contract is still a negative function of risk aversion and variance, and a positive function of incentive responsiveness. Hence, the repeated game approach is robust to the standard results from static incentive contracts. However, in contrast to static contracts, the optimal bonus level of the relational contract is affected by the value of future surplus. Second-order effects show that the optimal

4 In this paper, renegotiation always means renegotiating the terms of the contract *after* production (that is, when output is known, but *before* the transfer of the performance pay). Renegotiations *post* bonus payment (that is, in the beginning of a new period) never occur in stationary contract equilibria that are considered here.

bonus level's sensitivity to risk aversion and incentive responsiveness decreases with a lower discount factor. This, together with the general problem of implementing optimal performance pay, may explain why the empirical support for risk aversion and variance, as an explanation for agricultural contract choice, is not as strong as expected (see Allen and Lueck (1995, 1999, 2003) on the critique of the risk-sharing approach).

As a 'special case' of the general model, this paper considers the self-enforcing conditions of a simple linear contract. In agriculture simple linear incentive contracts can sometimes be observed. The landlord–tenant relationship that we still observe in rural economies is typically governed by linear piece-rate contracts, or so-called share contracts, that trade off the landlord's need to provide incentives with the tenant's need for insurance. In production contracts for broilers, turkeys, hogs and pigs as well as some fruits and vegetables, one may observe simple linear incentive schemes based on either piece-rates or cardinal tournaments (Levy and Vukina, 2002; Dubois and Vukina, 2004).[5] And linear crop-share contracts have at least until recently still been quite common in modern farming (see Allen and Leuck, 1993).

The literature on relational contracts has not examined the self-enforcing conditions of the linear contract because this contract is in general not optimal. Holmström and Milgrom (1987) showed that linear contracts *can* be optimal, but under some strict assumptions, including normally distributed noise. However, with normally distributed noise, there is no self-enforcing contract equilibrium, as the best and worse contingencies are infinite. So, to study the self-enforcing condition of the linear contract, one has to make the reasonable assumption that the noise term has bounded support.

It should be added that the results in this paper do not depend on linearity. The basic insight—that bargaining power and outside options constrain the feasible set of incentive pay when contracts are incomplete—applies to a variety of individual contracts (as indicated by Proposition 1), as well as tournaments and team settings.[6] However, in analysing models with risk aversion, we prefer the linear contract approach because of its simplicity and unambiguous properties in standard verifiable settings.

1.1. Related literature

Influential models of self-enforcing relational contracts include Klein and Leffler (1981), Shapiro and Stiglitz (1984), Bull (1987), Thomas and Worrall (1988), Kreps (1990) and Baker *et al.* (1994). MacLeod and Malcomson (1989) generalise the case of symmetric information, whereas Levin (2003)

5 Although linearity is still found, a variety of contractual arrangements have now developed in integrator–farmer relationships, from two-part piece-rate tournaments quite common in broiler production, to fixed performance standards, prevalent in swine production (see Tsoulouhas and Vukina, 1999).

6 The self-enforcing conditions of tournaments and team incentives are analysed in Kvaløy and Olsen (2006) in a model with risk-neutral agents and no agent hold-up.

makes a general treatment of relational contracts with asymmetric information, allowing for incentive problems due to moral hazard and hidden information. Levin shows how the self-enforcement constraint limits the feasible intensity of incentives. But Levin considers only risk-neutral parties, and because he uses assumptions where the Mirlees' (1974) step contract is optimal, he does not analyse linear contracts.

To our knowledge, the present paper is the first to study relational contracts in a context that simultaneously includes moral hazard, risk aversion and agent hold-up. Most relational contracting models assume that values accrue directly to the principal in the process of production, so that the agent cannot hold up values *ex post*. Notable exceptions are Halonen (2001) and Baker *et al*. (2002), who integrated relational contracting with the hold-up literature, but with risk-neutral agents. In the model presented here, the risk-averse agent is able to hold up values *ex post* (although I also allow for no hold-up). This is a key feature of the model, as it makes outside options and bargaining power essential in determining the feasible intensity of incentives.

In most relational contract models, risk neutrality is assumed.[7] A complicating problem with risk aversion is that it does not ensure the optimality of stationary contracts. In a stationary contract, the principal promises the same *contingent* (expected) compensation in each period. Levin (2003) showed that given risk neutrality, if an optimal contract exists, there are stationary contracts that are optimal (theorem 2).[8] The present paper considers equilibria in stationary contracts where one of the players (the agent) is allowed to be risk-averse. We can, however, restrict our attention to stationary contracts by saying that a stationary contract can achieve any surplus between $\bar{s}$ and the optimal surplus (see Levin, 2003, footnote 10). In sum, by allowing for risk aversion, and restricting our attention to stationary contracts, we may lose some superior equilibria. On the other hand, we explore some interesting aspects of the relationship between risk aversion, incentives, and the possibility to hold up values *ex post*.

In the literature on agricultural incentive contracts, self-enforcing contracts are not yet analysed. A reason for this (except for methodological concerns) may be that agricultural contracts often are short-term, and self-enforcing contracts are based on long-term agreements. However, agricultural contracts are increasingly becoming long-term (hog contracts, for instance, typically have long durations of up to 10 years). More importantly, the repetition of short-term contracts often develops into what the contracting parties interpret as long-term informal agreements.[9] Annual agreements are most often

7 Exceptions are Thomas and Worall (1988), Pearce and Stacchetti (1998) and MacLeod (2003), but none of them allow for agent hold-up, as I do.

8 Stationarity implies that the parties should respond to variable outcomes by adjusting wage, not the underlying incentives. Stationarity does not imply, however, that the underlying incentive structure is unaffected by technological shifts. An optimal contract is stationary for a given technology.

9 Landlord–tenant relationships often develop into long-term informal contracts where eviction threats are used as an incentive device (Banerjee and Ghatak, 2004).

automatically renewed unless one party makes an early commitment not to renew (Allen and Lueck, 2003). Renegotiations over the standard short-term agreements are then potentially viewed as opportunistic deviations from the informal relational contract.

The remainder of the paper is organised as follows: Section 2 first presents the general model, and linearity is introduced in Section 2.1. Analysis is performed in Section 3, and Section 4 offers some concluding remarks.

2. The model

Consider a relationship between a risk-neutral principal and a risk-averse agent. The agent makes an unobservable choice of effort e, which stochastically determines the agent's output. A random variable x with $E(x) = 0$ and variance V represents noise between the level of effort e and the observed output $Y(e, x) = e + x$. It is assumed that x has bounded support $x \in (x_L, x_H)$.

The principal offers a wage contract $w(\alpha, \varpi)$ where α is a fixed transfer to be made prior to production and ϖ is a payment that is contingent on performance, and thus paid *ex post*. If $\alpha > 0$ it can be interpreted as a fixed wage component, whereas if $\alpha < 0$ it can be interpreted as a fixed rent component.

Assume that the agent's utility from his wage is given by $u(w)$, where u is three times continuously differentiable, and $\bar{w} = E[w]$. Assume that the agent's total utility U is separable in income and effort, and is thus given by $U = u(\alpha, \varpi, r, V) - C(e)$ where $r = r(\bar{w}) = -u''(\bar{w})/u'(\bar{w})$ is the agent's coefficient of absolute risk aversion, and $C(e)$ is the personal cost of making effort, where $C'(e) > 0$ and $C''(e) > 0$. The principal is risk-neutral and his expected payment can thus be written $\pi = e - w(\alpha, \varpi)$.

Assume now that the agent's output is not verifiable, and thus not enforceable by a court of law. The parties then have to agree on a self-enforcing relational contract.[10] The relational contract is self-enforcing if the present value of honouring it is greater than the present value of reneging on it. Now, consider a *stationary* relational contract where the following game proceeds in *each* period. First, the principal offers a wage scheme $w(\alpha, \varpi)$. Second, the agent makes a choice of effort, e. Third, the principal and the agent observe Y. They now decide whether they still want to accept the contingent payment ϖ or to renegotiate a spot price, S. If the principal is able simply to take the good without paying, then $S = 0$. But if the agent is able to hold up values *ex post*, then S is determined by bargaining power and outside options. The agent can then choose to take the good and sell it in the alternative market to a price $\theta Y(e, x)$, or he can bargain with the principal. Here we assume that the alternative value is lower than the principal's valuation, that is $\theta \in (0, 1)$, so the agent will choose to bargain with the principal.

10 Note that even if we now enter into the study of repeated relationships, the moral hazard problem cannot be solved as in Radner (1981), Rogerson (1985) and Fudenberg *et al.* (1990), because, in contrast to these models, the parties cannot write verifiable contracts.

We assume Nash bargaining: The agent receives the alternative value θY plus a share, η, from the surplus from trade, $Y - \theta Y$, i.e. $S = \theta Y + \eta(Y - \theta Y) = \gamma Y$, where $\gamma = \theta + \eta - \eta\theta$. Hence γ is the agent's share of output if the price is renegotiated *ex post*. For example, if 50:50 Nash bargaining decides the renegotiated (spot) price,[11] then $\gamma = (1 + \theta)/2$.

In a single-period relationship, the agent will choose to renegotiate if $\varpi < S$, and the principal will choose to renegotiate if $\varpi > S$, so the players will *ex ante* agree to compensation $w(\alpha, \varpi) = S$. In other words, a relational contract where $w(\alpha, \varpi) \neq S$ is not enforceable. To be able to implement a relational incentive contract with $w(\alpha, \varpi) \neq S$ the players must have an infinite horizon, or uncertainty with respect to when the relationship ends. To formalise this, we consider an *infinitely* repeated relationship between the agent and the principal, where they both play trigger strategies. The principal begins by offering a wage scheme $w(\alpha, \varpi)$. The principal will continue to do so unless the agent or the principal chooses to renegotiate *ex post*, in which case they refuse to agree on anything other than the spot price, S, hereafter called a spot contract.[12] The agent's utility from a spot contract can then be written $U^S = S - C(e^S)$ where e^S maximises U^S.

The agent will now honour the contract if

$$\varpi + \frac{\delta}{1-\delta} U^R(\alpha, \varpi, r, V, e^R) \geq S + \frac{\delta}{1-\delta} U^S(\gamma, r, V, e^S)$$

where δ is the discount factor and e^R maximises U^R (superscript R denotes relational contracting). The principal will honour the contract if

$$-\varpi + \frac{\delta}{1-\delta} \pi^R(e^R, \alpha, \varpi) \geq -S + \frac{\delta}{1-\delta} \pi^S(e^S, \gamma).$$

If $\omega \neq S$ is desirable, it follows that bargaining positions and outside options, expressed in γ, determine the feasible level of performance-contingent payment. Formally we can state

Proposition 1. *The feasible level of contingent payment* $\varpi \in (\varpi_H, \omega_L)$ *is a function of* r, V, γ.

2.1. Linear contract

To deduce more specific results, we will now restrict attention to linear contracts. Holmström and Milgrom (1987) showed that CARA (constant absolute risk-aversion) preferences, and normally distributed noise terms are sufficient for linear incentive contracts to be optimal. Here, the noise term $x \in (x_L, x_H)$ does not fulfil the normal distribution requirement. But even so, the choice of linear contracts can still be justified on both theoretical and empirical grounds.

11 The 50:50 Nash bargaining solution is quite common in the literature on incomplete contracts (see, e.g. Grossman and Hart, 1986; Baker *et al.*, 2002).

12 This trigger strategy is common in the relational contract literature.

First, non-linear incentive contracts have the disadvantage of being susceptible to gaming. As Gibbons (2005) argues, the main contribution of the Holmström–Milgrom model is not that it justifies linear contracts, but rather that it implicitly demonstrates the gaming-problem of non-linear contracts. For example, Mirrlees' (1974) famous step contract, where the agent earns w_H if $Y \geq \bar{Y}$, but L if $Y < \bar{Y}$, would induce no effort once the agent's aggregate output passes $\bar{Y}$. Linear incentive contracts have the advantage of preventing this kind of dynamic moral hazard problem within a period. A growing body of evidence is consistent with the prediction that non-linear contracts create history-dependent incentives (see e.g. Healy (1985) on bonus plans with ceilings and floors, and Asch (1990) and Oyer (1998) on bonuses tied to quotas).

Moreover, the simplicity and observed occurrence of linear contracts makes it reasonable to assume that costs associated with the implementation of such contracts may be lower than the costs associated with more complex non-linear contracts. As noted by Stiglitz (1991), complicated incentive formulae are rarely observed; and when piece-rates are used, linear piece-rate systems are prevalent. Hence, for the rest of this paper we assume that excessive costs associated with the implementation of non-linear incentive contracts exceed the possible benefits. This assumption is particularly reasonable in risk-averse environments such as are considered in this paper. Because risk aversion and variance increase the complexity of non-linear contracts and also make the gaming problem more severe, the costs associated with implementing non-linear incentive contracts are probably a positive function of these variables.

Assume $\varpi = \beta Y$ so that $w = \alpha + \beta Y$.

The agent's certainty equivalent is then (from Taylor approximation, see the Appendix):[13]

$$CE_w = \alpha + \beta e - C(e) - \frac{1}{2} r \beta^2 V$$

and the principal's certainty equivalent can be written

$$\pi = CE_e = e - (\alpha + \beta e).$$

Total certainty equivalent (TCE) is then $CE_w + CE_e$, that is,

$$TCE = e - C(e) - \frac{1}{2} r \beta^2 V.$$

If the parties could write a verifiable contract on output level and the ownership of the output, they could easily implement the optimal division of incentives and insurance. The agent maximises his certainty equivalent.

13 This expression is exact only if $u(.)$ is CARA. Otherwise it is a second-order approximation.

The first-order condition yields the following incentive constraint:

$$\beta = C'(e). \tag{1}$$

The principal now maximises the total certainty equivalent by choosing e, subject to the incentive constraint. That is, he solves the problem $\max_e\,(e - C(e) - \frac{1}{2}r\beta^2 V)$, subject to (1).

Solving this for β yields

$$\hat{\beta} = \frac{1}{1 + rV\,C''}. \tag{2}$$

We know that $1/C''$ can be interpreted as the agent's responsiveness to incentives ($\mathrm{d}e/\mathrm{d}\beta = 1/C''(e)$). From (2) we obtain the classic result that the optimal level of performance pay is a negative function of risk aversion and variance, and a positive function of incentive responsiveness. The principal must set the fixed wage component, α, sufficiently high so that the individual rationality constraint (IR) holds, i.e. α is such that the agent will choose to work for the principal. Of course, profits are maximised when IR binds, that is $\alpha + \hat{\beta}e - C(e) - \frac{1}{2}r\hat{\beta}^2 V = w_a$, where w_a is alternative expected wage.

Let us then return to the relational contract. The agent honours the contract if

$$\beta(e^R + x) + \frac{\delta}{1-\delta}CE_w^R \geq \gamma(e^R + x) + \frac{\delta}{1-\delta}CE_w^S, \quad \forall x \tag{3}$$

where e^R maximises the certainty equivalent such that $CE_w^R = \max_e(\alpha + \beta e - C(e) - \frac{1}{2}r\beta^2 V)$, and e^S maximises the agent's surplus from spot transactions, such that $CE_w^S = \max_e(\gamma e - C(e) - \frac{1}{2}r\gamma^2 V)$.

The principal honours the contract if

$$(1-\beta)(e^R + x) + \frac{\delta}{1-\delta}CE_e^R \geq (1-\gamma)(e^R + x) + \frac{\delta}{1-\delta}CE_e^S, \quad \forall x \tag{4}$$

where $CE_e^R = e^R - (\alpha + \beta e^R)$ and $CE_e^S = e^S - \gamma e^S$.

Combining (3) and (4) yields a necessary condition for the relational contract to be self-enforcing:

$$|\gamma - \beta|\Delta x \leq \frac{\delta}{1-\delta}\left(e^R - C(e^R) - \frac{1}{2}r\beta^2 V\right) - \left(e^S - C(e^S) - \frac{1}{2}r\gamma^2 V\right)$$

where $\Delta x = x_H - x_L$.

That is (see Appendix),

$$|\gamma - \beta|\Delta x \leq \frac{\delta}{1-\delta}(TCE^R - TCE^S) \tag{5}$$

The parties can choose the fixed transfer, α, to make the condition sufficient (see Baker *et al.* (2002) for a similar argument). Note that if Δx goes to infinity, then (5) never holds, i.e. the relational contract is never self-enforcing. That is why we need bounded support to achieve equilibrium in self-enforcing linear contracts.

3. Analysis

From (5) we observe that there are upper and lower bounds on the feasible level of performance pay. Define $\beta^R \in (\beta_L, \beta_H)$ as the feasible levels of performance pay in a linear relational incentive contract.

Proposition 2. *The feasible levels of performance pay $\beta^R \in (\beta_L, \beta_H)$ in a linear relational incentive contract are given by* (5).

Proposition 2 is a special case of Proposition 1 and clarifies the limits of relational contracting. In a verifiable contract, any level of $\beta \in (0, 1)$ is feasible, and the optimal choice is independent of bargaining positions, outside options and discount factors. In a relational contract relying on self-enforceability, however, *ex post* outside options and the value from future trade, constrain the feasible β. The proposition is similar in spirit to Levin (2003). He showed that if the agent is risk-neutral, the optimal relational incentive contract is non-linear, where a bonus is paid if output exceeds a critical level. As a result of risk neutrality, the strongest possible incentives are desirable, but self-enforcement imposes a lower and an upper bound on the critical output level. We show that if the agent is risk-averse, and the parties stick to linear contracts, the feasible levels of performance pay have a lower and an upper bound $\beta \in (\beta_L, \beta_H)$. From the concavity of TCE, we have

Lemma. *The optimal β of a linear relational incentive contract is given by $\hat{\beta}$ iff $\hat{\beta} \in (\beta_L, \beta_H)$ β_L iff $\hat{\beta} < \beta_L \leq \gamma$ and β_H iff $\hat{\beta} > \beta_H \geq \gamma$, where $\hat{\beta}$ is given by* (2); *$\beta_L | \beta_L \geq \hat{\beta}$ is given by*

$$v(\gamma - \beta_L) = TCE^R - TCE^S, \quad v = \Delta x \frac{\delta}{1-\delta}, \tag{6}$$

and $\beta_H | \beta_H \leq \hat{\beta}$ is given by

$$v(\beta_H - \gamma) = TCE^R - TCE^S. \tag{7}$$

Hence,

Corollary. *There exist levels of γ, Δx, δ, r, V and C'' where the optimal level of performance pay, β, in a verifiable linear incentive contract cannot be implemented in a relational linear incentive contract, that is $\hat{\beta} \notin (\beta_L, \beta_H)$.*

It is most interesting to study the properties of the relational contract when $\hat{\beta} \notin (\beta_L, \beta_H)$, that is when the optimal β cannot be implemented in the relational contract. When $\beta > \gamma$, the principal has short-term gains from contract deviation; and if $\hat{\beta} > \beta_H > \gamma$, the principal cannot provide the agent with sufficient high-powered incentives[14] because the principal cannot commit to the contract if $\beta > \beta_H$.

However, the problem can also be that the parties cannot implement the optimal β when this is low. When $\beta < \gamma$, it is the agent who has the short-term gains from contract deviation. If $\hat{\beta} < \beta_L < \gamma$, the principal must offer $\beta = \beta_L$. To earn a profit he then has to reduce the fixed transfer α. Hence, if the agent is risk-averse, and $\hat{\beta} < \beta_L$, then the optimal balance between incentives and insurance cannot be implemented and the good *ex post* bargaining position becomes a 'burden' for the agent: Even though the agent prefers a wage contract with a higher fixed salary, the *ex post* realisation of value added automatically creates a lower bound on the bonus level, which again reduces the feasible fixed transfer that the principal can afford to pay.

Hence, we see that the agent's expected share of output γ after deviations will determine whether the incentives are too high-powered or too low-powered. It is natural to assume that γ is lower for tenants than for independent growers. Recall that $\gamma = \theta + \eta(1 - \theta)$, where η is (Nash) bargaining power and θ decides alternative value. As opposed to independent growers, tenants do not own the land on which they work, hence they cannot sell to alternative buyers, i.e. $\theta = 0$, and $\gamma = \eta$. Hence, for a given η, independent growers have a higher γ if $\theta > 0$. A low γ may explain why we see share contracts with low β between landlords and tenants, even if high-powered incentives are desirable: the landlord cannot *commit* to high-powered incentives.[15]

In modern farming, the reverse may be the case. If *ex post* outside options are high, independent growers (or farmers) cannot commit to honour relational contracts with low-powered incentives. In fact, even when the farmers' *bargaining power is* low due, for example, to integrator oligopsony or monopsony power, they have (to some extent) residual control rights and can therefore hold up values *ex post*. Now, the empirical evidence on integrator market power is scarce. For instance, Inoue and Vukina (2006) cannot reject the null hypothesis of no integrator market power in the swine industry, while some monopsony power is evident in the broiler industry (Lewin-Solomons, 2000; Vukina and Leegomonchai, 2006). In the hog industry, it is in fact a sellers' market, as integrators compete for available growers.

Hence, when we observe agricultural incentive contracts that are higher-powered than the typical *wage contract* in many other industries, a natural explanation is that farmers own more critical assets and have a larger hold-up power, than an average (blue-collar) employee. That is, even if the independent farmers strive to achieve a low-risk employment relationship through

14 A similar point is made in Baker *et al.* (2002), but with binary output and a risk-neutral agent.

15 An alternative explanation for low-powered incentives in sharecropping is provided by Dubois (2002), who shows how weak incentives may be used to mitigate land over-use.

contracting with integrator companies, they cannot easily commit to contracts with low-powered incentives, as their hold-up opportunities yield them *ex post* bargaining power to renegotiate a better bonus after good performance.[16]

Let us now return to the formal analysis. We see from (2) the effect of risk aversion, variance and incentive responsiveness on the optimal β when output is verifiable. Obviously, this also applies when the contract is not court-enforceable, but $\hat{\beta} \in (\beta_L, \beta_H)$. It can also be shown that when $\hat{\beta} \notin (\beta_L, \beta_H)$, these basic relationships still apply. Let k be a parameter in the cost function, and a proxy for incentive responsiveness in the following sense: for $e(\beta, k)$ given by $\beta = (\partial C/\partial e)(e, k)$, we have $\partial^2 e/\partial k \partial \beta > 0$. That is, the incentive responsiveness $\partial e/\partial \beta$ increases with increasing k (see Appendix for more details). We obtain

Proposition 3. *The optimal β of a relational linear incentive contract is a negative function of risk aversion and variance, and a positive function of incentive responsiveness. That is $\partial\hat{\beta}/\partial r = \partial\hat{\beta}/\partial V < 0$, $\partial\hat{\beta}/\partial k > 0$ and $\partial\beta_i/\partial r = \partial\beta_i/\partial V < 0$, $\partial\beta_i/\partial k > 0$, $i = H, L$.*

Proof. See Appendix.

This is not a surprising result, but it demonstrates that the standard effects of risk aversion, variance and incentive responsiveness still apply when contracts are not court-enforceable. In a way, it also demonstrates the robustness of the infinite repeated game approach. However, even if the 'sign of the effects' is the same, the optimal bonus level's sensitivity to parameter changes is not the same. In relational contracts, as opposed to verifiable contracts, the optimal bonus sensitivity to changes in risk aversion, variance and incentive responsiveness is affected by the discount factor. On low discount factors, the range of feasible bonus levels is smaller ($\beta_H - \beta_L$ is smaller). This implies that the optimal bonus level is less sensitive to parameter changes when $\hat{\beta} \notin (\beta_L, \beta_H)$:

Proposition 4. *When $\hat{\beta} \notin (\beta_L, \beta_H)$, the lower the discount factor δ, the weaker is the effect of risk aversion, variance and incentive responsiveness on the optimal bonus level of the relational contract. That is $|\partial^2\beta_i/\partial r \partial\delta| = |\partial^2\beta_i/\partial V \partial\delta| > 0$ and $\partial\beta_i/\partial k \partial\delta > 0$, $i = H, L$.*

Proof. See Appendix.

4. Concluding remarks

This article applies the risk-neutral principal/risk-averse agent approach to agricultural contracting, but opens the possibility that contracts are incomplete and that the agent owns critical assets. A general proposition is formulated whereby outside options and bargaining positions will affect incentive intensity as long as desirable incentives deviates from spot incentives.

16 It should be noted that hold-up and renegotiations will not occur in a self-enforcing contract equilibrium (because then it is not a self-enforcing contract). But this does not mean that hold-up does not affect equilibrium payoffs; in fact, the parties will design their contract just to avoid hold-up.

Contract linearity is then introduced in order to deduce more explicit results and to make fruitful comparisons with the well-known properties of the linear contract in the verifiable setting. However, our general results do not depend on the linearity assumption.

Like Baker *et al*. (2002), who combined the relational contract literature with the property rights approach (Grossman and Hart, 1986; Hart and Moore, 1990), we assume that the agent is potentially able to hold up values after production has occurred. A condition for agent hold-up to be possible is that the principal cannot appropriate output as soon as it is realised. If the principal owns all assets involved in the agent's production, it is natural to assume that such asset-ownership conveys ownership of the output produced (as in Baker *et al*., 2002), and that the principal can thus take the output as soon as it is produced. So for agent hold-up to be possible, there must be assets involved that are owned by the agents. Technically, by allowing for $\gamma > 0$, the agent is given some *ex post* bargaining power, which can be interpreted as either farmers owning critical assets, or tenants possessing indispensable human capital. Hence, the model in this paper is suitable for analysing modern farming as well as traditional landlord–tenant relationships (allowing also for $\gamma = 0$).

Some authors, most notably Allen and Lueck (1995, 1999, 2003), question the risk-sharing approach, as they cannot find the expected relationship between variance and level of sharing in their empirical studies. But this does not imply that the assumption of risk-averse agents is false.[17] In fact, the model in this paper shows that the risk-neutral principal/risk-averse agent assumption does not necessarily imply the same incentive–insurance balance that the classical model predicts, as self-enforcement constraints limit the feasible provision of incentives. Thus, the model in this paper provides another reason why the empirical support for risk sharing as an explanation for agricultural contract choice is not as strong as expected.

That said, risk aversion is not a necessary condition for the analysis to be interesting. One result is that if the agent has a weak *ex post* bargaining position, the principal cannot commit to high-powered incentives, even if the agent is risk-neutral. This result may explain why share contracts in developing countries often are less high-powered than incentive-based considerations would suggest.[18] But equally interesting is the model's prediction when the

17 Several studies show that farmers are risk-averse. A recent study by Kumbhakar and Tveterås (2003) found a substantial degree of absolute and relative risk aversion among salmon farmers. Barrett *et al*. (2004) noted that the Arrow–Pratt coefficient of relative risk aversion is typically in the 1.0–5.0 range for farmers. Dubois and Vukina (2004) found that hog-growers are heterogeneous in terms of risk aversion, and also that risk aversion affects the principal's contract choice. In addition, Allen and Leuck (1999) has been criticised for not controlling for self-selection, i.e. that agents choose contracts that fit with their risk tolerance (see Ackerberg and Botticini, 2002).

18 There are also other explanations of why sharecropping may emerge in risk-neutral environments. Eswaran and Kotwal (1985) showed that with two-sided moral hazard (i.e. both the agent and the principal take important non-observable decisions that affect output) sharecropping naturally emerges as a way to maximise total incentives. Basu (1992), Sengupta (1997) and Ghatak and Pandey (2000) all showed that if the agent has limited liability and moral hazard in risk-taking, the principal must limit incentives such that the agent does not take too much risk. Sharecropping may thus emerge even with one-sided moral hazard and risk neutrality.

agent is risk-averse and has high *ex post* bargaining power. Even if he prefers a fixed wage contract, he cannot commit to low-powered incentive contracts, because he is tempted to renege on the relational contract and plead for renegotiation. For a given level of risk aversion, one would thus expect to see a positive relationship between the agent's *ex post* bargaining power, and the contract's incentive-intensity. Assuming that a land-owning farmer who contracts with an integrator has higher *ex post* bargaining power than a tenant who works for a landowner, the model predicts higher-power incentives in modern integrator contracts than in traditional crop share contracts.

More generally, the self-enforcing contract approach may explain why integrator companies do not to a larger extent offer individualised contracts or discriminate between growers by strategically allocating inputs of different quality. When the parties contract repeatedly, the integrator will learn the relative ability of the growers, yet they seldom individualise contracts (see, e.g. Levy and Vukina, 2002), and only to a small extent discriminate on input allocations (see Leegomonchai and Vukina, 2005). This can be explained by transaction costs (e.g. costs of screening and contract matching), but a second and more general explanation, underscored by Leegomonchai and Vukina, is reputation effects, i.e. the costs in loss of future goodwill. This view is supported by the repeated game approach to agricultural contracting, presented here.

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Appendix

1. Deducing (5)

Because x is continuous, (3) and (4) include an infinite number of restrictions. By using bounded support on x, we can find the binding constraints, analysing (3) and (4) for extreme realisations of x.

When $\beta \leq \gamma$, (3) is weakest for $x = x_H$ and (4) is weakest for $x = x_L$. The binding constraints are thus

$$\beta(e^R + x_H) + \frac{\delta}{1-\delta} CE_w^R \geq \gamma(e^R + x_H) + \frac{\delta}{1-\delta} CE_w^S \tag{A1}$$

$$(1-\beta)(e^R + x_L) + \frac{\delta}{1-\delta} CE_e^R \geq (1-\gamma)(e^R + x_L) + \frac{\delta}{1-\delta} CE_e^S. \tag{A2}$$

A necessary condition for the relational contract to hold is that the sum of (A1) and (A2) holds. This yields

$$(\gamma - \beta)(x_H - x_L) \leq \frac{\delta}{1-\delta}(TCE^R - TCE^S). \tag{A3}$$

When $\beta \geq \gamma$, (3) is weakest for $x = x_L$ and (4) is weakest for $x = x_H$. The binding constraints are thus

$$\beta(e^R + x_L) + \frac{\delta}{1-\delta} CE_w^R \geq \gamma(e^R + x_L) + \frac{\delta}{1-\delta} CE_w^S \tag{A4}$$

$$(1-\beta)(e^R + x_H) + \frac{\delta}{1-\delta} CE_e^R \geq (1-\gamma)(e^R + x_H) + \frac{\delta}{1-\delta} CE_e^S \tag{A5}$$

and the sum of (A4) and (A5) yields

$$(\beta - \gamma)(x_H - x_L) \leq \frac{\delta}{1-\delta}(TCE^R - TCE^S). \tag{A6}$$

As (A3) is relevant for $\beta \leq \gamma$ and (A6) is relevant for $\beta \geq \gamma$, we can write these two restrictions in one expression using absolutes:

$$|\gamma - \beta|\, \Delta x \leq \frac{\delta}{1-\delta}(TCE^R - TCE^S) \tag{5}$$

where $\Delta x = x_H - x_L$. As noted, the parties can choose the fixed salary, α, to make the condition sufficient.

2. The measure of incentive responsiveness

For $e(\beta, k)$ given by $\beta = \partial C/\partial e(e, k)$, we have $\partial^2 e/\partial k \partial \beta > 0$. That is, the incentive responsiveness $\partial e/\partial \beta$ increases with increasing k. This holds if the cost function satisfies $\partial^2 C/\partial e \cdot \partial k \partial^3 C/\partial e^3 - \partial^2 C/\partial e^2 \cdot \partial^3 C/\partial e^2 \partial k > 0$. (For example, the condition holds for a cost function of the form $C(e, k) = A(k)e^n$, $n \geq 2$, where $A'(k) < 0$.) With this condition, the gain from a marginal increase in β increases with the level of incentive responsiveness. That is $\partial TCE/\partial k\partial\beta = (1-\beta)\partial^2 e/\partial k\partial\beta > 0$ for $\beta < 1$.

3. Proof of propositions 3 and 4

When $\beta \in (\beta_L, \beta_H)$, the optimal β is given by (2), showing that the optimal level of performance pay is a negative function of risk aversion and variance and a positive function of incentive responsiveness. From the Lemma, we have that β_L is optimal iff $\hat{\beta} < \beta_L \leq \gamma$ where β_L is given by

$$v(\gamma - \beta_L) = TCE^R - TCE^S, \quad v = \Delta x \frac{1-\delta}{\delta}. \tag{6}$$

When $\hat{\beta} < \beta_L < \gamma$, we must have (for simplicity I exclude functional arguments):

$$-\left.\frac{\partial TCE^R}{\partial \beta}\right|_{\beta=\beta_L} < v. \tag{A7}$$

Differentiating (6) with respect to r yields

$$\left(-v - \frac{\partial TCE^R}{\partial \beta}\right)\frac{\partial \beta_L}{\partial r} = \frac{\partial TCE^R}{\partial r} - \frac{\partial TCE^S}{\partial r}. \tag{A8}$$

From (A7), the bracket on the left-hand side is negative, and the difference on the right-hand side is positive, as $\beta_L < \gamma$ and $\partial^2 TCE/\partial r\partial\beta < 0$. This yields $\partial\beta_L/\partial r < 0$, which also implies $\partial\beta_L/\partial V < 0$.

Differentiating (6) with respect to k yields

$$\left(-v - \frac{\partial TCE^R}{\partial \beta}\right)\frac{\partial \beta_L}{\partial k} = \frac{\partial TCE^R}{\partial k} - \frac{\partial TCE^S}{\partial k}. \tag{A9}$$

From (A7), the bracket on the left-hand side is negative, and the difference on the right-hand side is also negative as $\beta_L < \gamma$ and $\partial^2 TCE/\partial k\partial\beta > 0$. This yields $\partial\beta_L/\partial k > 0$.

From the Lemma, we have that β_H is optimal iff $\hat{\beta} > \beta_H \geq \gamma$, where β_H is given by

$$v(\beta_H - \gamma) = TCE^R - TCE^S. \tag{7}$$

When $\hat{\beta} > \beta_H \geq \gamma$, we must have

$$\left.\frac{\partial TCE^R}{\partial \beta}\right|_{\beta=\beta_H} < v. \tag{A10}$$

Differentiating (7) with respect to r yields

$$\left(v - \frac{\partial TCE^R}{\partial \beta}\right)\frac{\partial \beta_H}{\partial r} = \frac{\partial TCE^R}{\partial r} - \frac{\partial TCE^S}{\partial r}. \qquad \text{(A11)}$$

From (A10), the bracket on the left hand side is positive, and the difference on the right hand side is negative as $\beta_H > \gamma$ and $\partial^2 TCE/\partial r \partial \beta < 0$. This yields $\partial \beta_H/\partial r < 0$, which also implies $\partial \beta_H/\partial V < 0$.

Differentiating (7) with respect to k yields

$$\left(v - \frac{\partial TCE^R}{\partial \beta}\right)\frac{\partial \beta_H}{\partial k} = \frac{\partial TCE^R}{\partial k} - \frac{\partial TCE^S}{\partial k}. \qquad \text{(A12)}$$

From (A10), the bracket on the left-hand side is positive, and the difference on the right-hand side is also positive, as $\beta_H > \gamma$ and $\partial^2 TCE/\partial k \partial \beta > 0$. This yields $\partial \beta_H/\partial k > 0$.

Proposition 4 can be verified by differentiating (A8), (A9), (A11) and (A12) with respect to v, noting that $\partial v/\partial \delta < 0$.

4. Deducing the worker's certainty equivalent $CE_w = \alpha + \beta e - C(e) - \frac{1}{2} r \beta^2 V$

The worker's utility from his wage is given by $u(w)$, where u is three times differentiable, and the expected wage is equal to its mean, that is $\bar{w} = E[w]$. Let us first leave out personal cost $C(e)$. The certainty equivalent is then approximately $\bar{w} - \frac{1}{2} r(\bar{w})\mathrm{Var}(w) = \hat{w}$, where $r(\bar{w}) = -u''(\bar{w})/u'(\bar{w})$.

Derivation (from Milgrom and Roberts, 1992):
According to Taylor's theorem, for any z we have

$$u(z) = u(\bar{w}) + (z - \bar{w})u'(\bar{w}) + \frac{1}{2}(z - \bar{w})^2 u''(\bar{w}) + R(z)$$

where $R(z) = u'''(\hat{z})(z - \bar{w})^3/6$ for some $\hat{z} \in [\bar{w}, z]$. This last term is assumed to be small and thus negligible. Hence, we can write approximately

$$u(z) \approx u(\bar{w}) + (z - \bar{w})u'(\bar{w}) + \frac{1}{2}(z - \bar{w})^2 u''(\bar{w}).$$

Substituting w for z and computing the expectation, we find, approximately

$$E[u(w)] \approx u(\bar{w}) + E[w - \bar{w}]u'(\bar{w}) + \frac{1}{2}E[(w - \bar{w})^2]u''(\bar{w}).$$

As $E[w-\bar{w}]=E[w]-\bar{w}=\bar{w}-\bar{w}=0$, we can write

$$E[u(w)] \approx u(\bar{w}) + \frac{1}{2}E[(w-\bar{w})^2]u''(\bar{w}). \tag{A13}$$

The certainty equivalent $\hat{w}$ is expected to be close to $\bar{w}$, so its utility is approximated differently, also using Taylor's theorem,

$$u(\hat{w}) = u(\bar{w}) + (\hat{w}-\bar{w})u'(\bar{w}) + \bar{R}(\hat{w}) \tag{A14}$$

where $\bar{R}(\hat{w}) = \frac{1}{2}u''(\hat{z})(\hat{w}-\bar{w})^2$ for some $z \in [\bar{w}, \hat{w}]$. If we apply the approximation only when $\hat{w}-\bar{w}$ is small, the remainder term is again negligible. As $\hat{w}$ is a certainty equivalent, we have $u(\hat{w}) = E[u(w)]$. So combining (A13) and (A14) yields

$$(\hat{w}-\bar{w})u'(\bar{w}) \approx \frac{1}{2}E[(w-\bar{w})^2]u''(\bar{w}).$$

This can be expressed in the form

$$\hat{w}-\bar{w} \approx \frac{1}{2}[u''(\bar{w})/u'(\bar{w})]E[(w-\bar{w})^2] = -\frac{1}{2}r(\bar{w})\mathrm{Var}(w)$$

which establishes $\hat{w} = \bar{w} - \frac{1}{2}r(\bar{w})\mathrm{Var}(w)$.

Subtracting personal cost, and inserting $\bar{w} = E[w] = \alpha + \beta e$, we obtain the worker's certainty equivalent.

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