On the Alaskan Stream

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ABSTRACT

The Alaskan Stream boundary current south of the uniformly curving coastline formed by the Alaskan peninsula-Aleutian Island chain is examined analytically via steady, barotropic frictional theory. It is shown that, as a result of the changing zonal orientation of this boundary, there is an alteration in the characteristic vorticity balance in the current as it progresses westward from the Gulf of Alaska. Where the curving coastline becomes approximately zonal, this vorticity distribution is such that, unless the clockwise vorticity generated at the coast by the no-slip condition is balanced by a vorticity source external to the current, instabilities and separation of the Alaskan Stream from the coast will occur.

1. Introduction

In analytical and numerical studies of the flow in oceanic boundary layers, continental coastlines are usually modeled crudely by straight lines. This is acceptable in most cases since such coastlines do not change their orientation to parallels of latitude enough to affect the basic dynamics of the boundary layer flow. Thus, the relative importance of the various terms making up the balance of vorticity in a boundary current does not alter as the flow progresses along a given boundary.

Contrarily, the unique coastline formed by the Alaskan Peninsula-Aleutian-Komandorski Island chain can be excellently modeled by a boundary of uniform curvature whose changing zonal orientation gives rise to an alteration in the dynamics of the adjacent boundary current. The westward flowing Alaskan Stream, which forms the boundary current off this coast in the partially bounded subarctic region of the North Pacific Ocean, can then be shown to possess the interesting feature of a progressively altering balance in the various vorticity effects. This feature, resulting from the westward geographical variation of the coast from a western-like boundary through a zonal-like boundary to an eastern-like boundary, enables one to give a qualitative explanation for the observed separation of the Stream from the Aleutians.

Although much success has been achieved by inertial models in describing the detailed nature of the intense boundary currents such as the Gulf Stream (e.g., Robinson, 1963; Warren, 1963; Robinson and Niiler, 1967; Niiler and Robinson, 1967), the present work will be based on linear, frictional boundary layer theory. This is justifiable since the emphasis here is on the boundary current as a whole and not on the detailed dynamics within it. As the "no-slip" condition will be applied at the coast, the Alaskan Stream is bounded on either side by a line of zero velocity. Using Stewart's (1964) argument, this means that, integrated over the width of the current, there is no net vorticity or transport of vorticity to order δ/r (where δ is the width of the boundary layer and r is the mean radius of curvature of the boundary). Therefore, considered over the width of the boundary layer, the inertial terms play no part in the net vorticity balance. Similarly, the complicated relationship between the mean rate of strain and the Reynold's stress does not need to be known in detail since the basic role of the latter is to bring the tangential velocity component to zero at the boundary and not to explain the effect of friction within the current itself. [A recent numerical study by Blandford (1971) shows the importance of the viscous no-slip condition in producing realistic ocean circulation patterns even for highly nonlinear boundary layer models.] Furthermore, we will consider only the upper baroclinic layer of the ocean, whereby the effects of bottom topography can be omitted. This is particularly justified by the fact that the Aleutian-Komandorski Island chain is part of an island-arc, deep-sea trench system in which depths go to 5000–8000 m within 200 km of the coast.

In Section 2, curvilinear coordinates are introduced to replace the usual east and north Cartesian coordinates on the β plane. The time-averaged, vertical component of the vorticity equation is then presented for this coordinate system. In Section 3, the linearized version of this vorticity equation is integrated vertically from a depth where the time-averaged horizontal motions are assumed negligible to the ocean surface. This allows one to treat the upper baroclinic layer barotropically. An equation for the streamfunction,

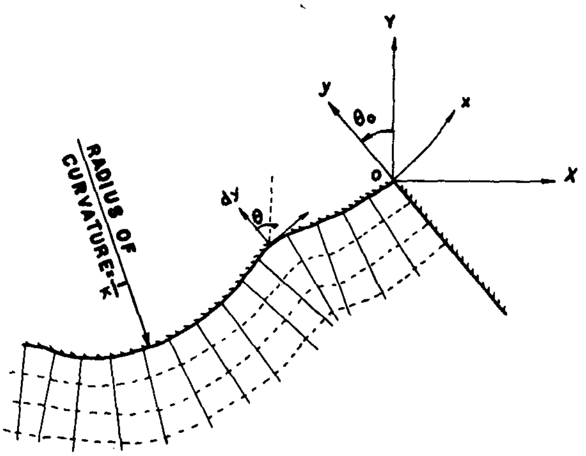

FIG. 1. Coordinate systems; X, Y, Cartesian coordinates; x, y, curvilinear coordinates.

based on the integrated mass transport, is then obtained for the interior and boundary layer regions of the subarctic Pacific. Characteristic scales, determined from observed data, are then introduced to derive the nondimensional form of the equations of which only the zeroth order flow is considered.

A distinct change in the equations characterizing the dynamics of the Alaskan Stream is found to occur as the flow moves westward. The ability of the Stream to remain attached to the coast in the zonal-like region is further found to depend upon its downstream spatial variation. Finally, Section 4 is devoted to a summary and physical interpretation of the findings of Section 3 and to a comparison of the theoretical results with observation.

2. Equations of motion

We consider a uniformly rotating ocean of variable depth and density, having solid land barriers to the east and west and a partial land barrier to the north. In Cartesian coordinates the east and north axes are X and Y, respectively, while z is the vertical coordinate, positive upward; the corresponding velocity components are U, V and w. The origin is taken at mean sea level at the intersection of the northern and eastern boundaries in the corner of the Gulf of Alaska (Fig. 1).

In order to model the curved northern coastline of the subarctic Pacific region, it is convenient to transform to curvilinear coordinates (see e.g., Goldstein, 1965, p. 119). In these coordinates, the local curvature κ depends only upon the distance x along the boundary formed by the Alaska peninsula and the Aleutian-Komandorski island chain (henceforth referred to as the Northern Boundary) and not upon the distance y perpendicular to the boundary, nor z (Fig. 1). Increments in x and y are given by $dx[1-\kappa(x)y]$ and dy, respectively, so that lines of constant y are parallel curves of curvature κ, while lines of constant x are straight lines. Convergence of the y-coordinate lines will not be considered important, thereby confining our model to small curvatures. The east coast, north of about 47N, will be approximated by a straight line rotated by an angle θ_0 to constant X. Since this study will concentrate on the flow structure east of the Kuroshio and Oyashio current systems, there will be no attempt to model the Asian coastline.

The inverse transformations, from x, y to X, Y are given by

$$\left.\begin{aligned} X&=\int_0^x [1-\kappa(\omega)y]\cos\theta(\omega)d\omega - y\sin\theta_0 \\ Y&=\int_0^x [1-\kappa(\omega)y]\sin\theta(\omega)d\omega + y\cos\theta_0 \end{aligned}\right\},$$

in which $\theta(x)$ measures counterclockwise the angle between lines of constant X and x, $\theta(0)=\theta_0$, and where

$$\kappa(x)=\frac{d\theta(x)}{dx}.$$

If we then time-average (denoted by an overbar) the general vector form of the conservation of momentum equation and take the curl of the result, we obtain

$$\bar{\mathbf{u}}\cdot\nabla(f+\bar{\zeta})-(f+\bar{\zeta})\frac{\partial\bar{w}}{\partial z}+\nabla\bar{w}\times\frac{\partial\bar{\mathbf{u}}}{\partial z}\cdot\mathbf{k}=A_j\nabla\cdot\nabla\bar{\zeta} \tag{2.1}$$

for the vertical component of the vorticity equation, where $\mathbf{u}=(u,v,w)$ is the velocity in curvilinear coordinates, $f(x,y)=f_0+\beta_0Y(x,y)$ is the Coriolis parameter (f_0 and β_0 are constants), ζ the relative vorticity, $\mathbf{k}$ a unit vector in the positive z direction, and A_j a concise notation for the eddy viscosity coefficient. In deriving (2.1), the kinematic viscosity has been neglected compared to the eddy viscosity, and the conservation of mass equation,

$$\nabla\cdot\mathbf{u}=0 \tag{2.2}$$

has been used. As usual (e.g., Fofonoff, 1962) it will be assumed that $A_j=A_h$ for horizontal components and $A_j=A_v$ for vertical components, with $A_v\ll A_h$. The vertical term of the right side of (2.1) can be related to a horizontal shearing stress, $\boldsymbol{\tau}=(\tau^x,\tau^y,0)$, by

$$\rho A_v\frac{\partial\bar{\zeta}}{\partial z}=\nabla\times\boldsymbol{\tau}\cdot\mathbf{k}.$$

Representation of the turbulent processes in the ocean by a constant eddy viscosity coefficient is, to say the least, very crude. In fact, eddies can feed momentum laterally into boundary currents (e.g., Starr and Gaut, 1970), thereby giving rise to a negative viscosity. Nevertheless, as the effect is local and requires an external source of energy which is not available in the present steady-state treatment, use of a constant positive eddy viscosity serves as a useful approximation to the net effect of turbulence.

3. The linearized streamfunction equation

If one does a nondimensional analysis of the terms in (2.1) based on mean flow scales, it can be shown that the effect of both nonlinearities and horizontal friction will be small over most of the subarctic Pacific region being considered. However, neglect of the above terms, which are of O(Ro) and O(E) (where Ro and E are the Rossby number and Ekman number), only gives a first-order velocity equation though there are two horizontal boundaries to be considered. This loss of a boundary condition requires the existence of a boundary layer along at least one of the coasts. In this paper, the required higher order terms will be obtained by retaining the horizontal turbulent friction rather than the nonlinearities. The choice is based on the arguments given in the Introduction and on the requirement that there be no slip along the boundary. A more detailed analysis of the structure of the Alaskan Stream could be achieved by assuming that the current consists of an inner frictionally controlled boundary layer (see, e.g., Carrier and Robinson, 1962). Indeed, the ability of the boundary current to hold together after leaving the coast is a nonlinear effect. As previously mentioned, though, the fact that the current is bounded by lines of zero velocity means that one need not consider the detailed structure in order to understand the breakaway feature associated with the flow.

Since we are interested in the horizontal distribution of flow only, (2.1) can now be integrated vertically from the depth $z=-H(x,y)$ where vertical motions are assumed negligible to the mean ocean surface. Using the mass transport,

$$\mathbf{M}=(M^x, M^y, 0)=\int_{-H}^{0}\rho(\bar{u},\bar{v},0)dz, \tag{3.1}$$

and defining a mass transport streamfunction ψ by

$$\left.\begin{aligned} M^x &= -\partial\psi/\partial y \\ (1-\kappa y)M^y &= \partial\psi/\partial x \end{aligned}\right\}, \tag{3.2}$$

the integrated continuity equation, *viz.*,

$$\frac{\partial M^x}{\partial x}+\frac{\partial}{\partial y}[(1-\kappa y)M^y]=0$$

is satisfied identically, while the integrated version of (2.1) becomes

$$J(\psi,f)=A_h\nabla_h\cdot\nabla_h\psi+G,$$

where J is the Jacobian with independent coordinates x, y, G is minus the curl of the wind stress $(-\nabla\times\boldsymbol{\tau}\cdot\mathbf{k})$, and ∇_h is the horizontal gradient operator. The nondimensional form of the above equation based on scales for the interior mean flow is then (Thomson, 1971)

$$J(\tilde{\psi},\tilde{f})=\mathrm{E}_*\tilde{\nabla}_h\cdot\tilde{\nabla}_h\tilde{\psi}+\tilde{G}, \tag{3.3}$$

where the tildes signify nondimensional variables and

$$\mathrm{E}_*=\frac{A_h}{\beta_0 L^3}\ll 1 \tag{3.4}$$

represents a modified Ekman number. The horizontal length scale L is defined through the assumption that

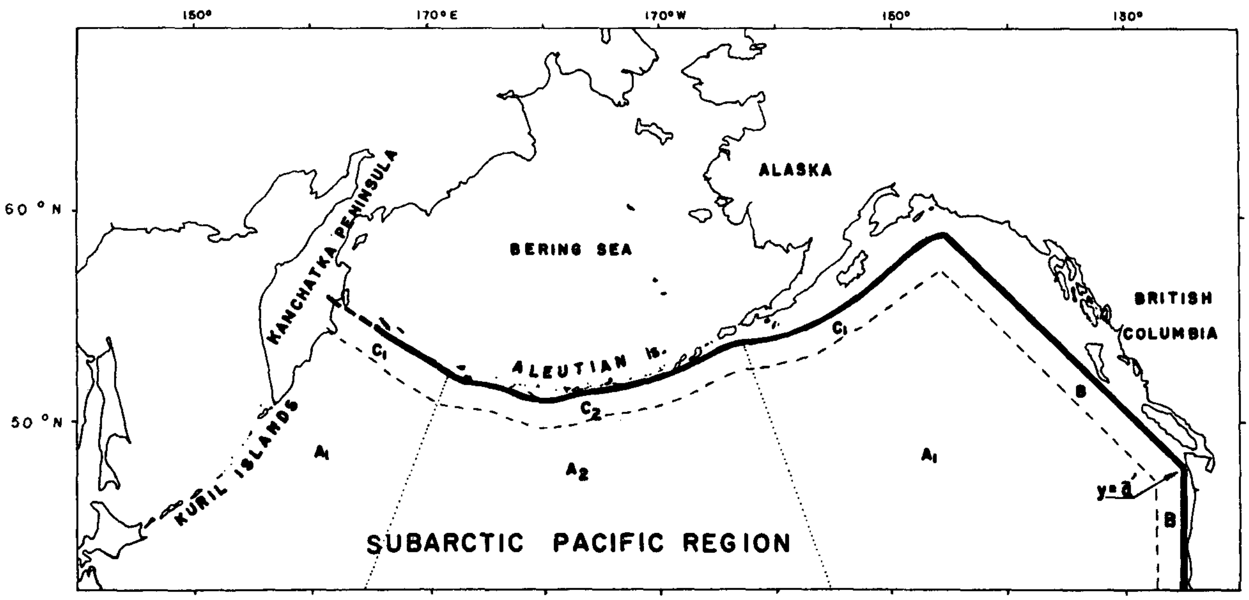

FIG. 2. Schematic diagram of the subregions used in the mathematical model: land barrier (heavy solid line), approximate position of the subregion boundaries (dotted lines), approximate limit of the boundary layer regions (dashed line).

the wind-stress curl scale G_0 satisfies

$$G_0 \sim O\left(\frac{\tau_0}{\rho_0 D U_0 L \beta_0}\right) \sim O(1), \tag{3.5}$$

where τ_0, ρ_0, D and U_0 are scales in the oceanic interior for the surface wind stress, the density, the depth over which the baroclinic motions exist, and the mean horizontal velocities, respectively.

Solutions to (3.3) can be obtained using the method of a power series expansion in the dependent variables in each of three basic regions. As shown in Fig. 2 these regions are: (A) the interior region, away from the boundaries at $x=0$ and $y=0$; (B) the eastern boundary region, near $x=0$ north of point a' and near $LX=a' \sin\theta_0$, south of point a'; and (C) the northern boundary region, near $y=0$. No attempt to model the subarctic Pacific west of 165E is made. Furthermore, if there is detachment of the Stream (or part of the Stream) from the coast, inertial effects will allow the "jet" to penetrate into the interior region thereby destroying the basic divisions assumed here. Such a jet will also divide the northern part of the oceanic region into two (or more) independent regions which may then be treated individually. As this paper is essentially concerned with the break-away feature of the Alaskan Stream, and not the details of the subarctic Pacific as a whole, no analytical discussion will be presented for the current once it leaves the coast.

a. The interior region (A)

In the oceanic interior, away from the influence of the boundaries, both the horizontal viscous terms and the inertial terms will be negligible. The steady flow will, therefore, be determined by the climatological distribution of the surface forcing. As subsequent analysis will show, the requirement that the frictional northern boundary layer flow match to the interior flow, which here is the eastward drifting subarctic current, makes it convenient at this point to divide the latter into two subregions, labeled A_1 and A_2 in Fig. 2. This division of the interior is purely for convenience and serves only to make presentation more coherent; by its very definition, the interior region cannot be directly dependent upon the form taken by the boundary flow. Furthermore, the wind-stress curl determines the zeroth-order streamlines in the interior explicitly, so that $\tilde{G}$ will be considered to be a zeroth-order function only.

The resulting balance of vorticity for the interior region (Sverdrup, 1947) is then, to $O(E_*)$,

$$J(\tilde{\psi}_i, \tilde{f}) = \tilde{G}_i,$$

with solution

$$\tilde{\psi}_i(\tilde{X},\tilde{Y}) = \int_{g(\tilde{Y})}^{\tilde{X}(\tilde{x},\tilde{y})} \tilde{G}_i(q,\tilde{Y}) dq, \tag{3.6}$$

where $i=1, 2$ refer to subregions A_1 and A_2, respectively. The function $g(\tilde{Y})$ is determined only after consideration of the eastern boundary region. If a total (or partial) separation of the Alaskan Stream does occur somewhere along the northern boundary, then an equation of the form (3.6) will still apply to those regions of the ocean in which nonlinear and frictional terms are small. The functions $g(\tilde{Y})$ for each region would then be determined by considering the appropriate eastern type boundary of that particular region.

b. The eastern boundary region (B)

Formally, one could perform a boundary layer expansion near the eastern coast by introducing a stretched variable for the coordinate direction perpendicular to the boundary. However, from Munk (1950) we find that the one term that makes the eastern boundary layer solution distinct from the interior solution goes as

$$\exp[-E|s-s_0|],$$

where $E=(\beta_0/A_h)^{\frac{1}{3}} \ll 1\ \text{km}^{-1}$, s is the coordinate distance perpendicular to the boundary, and s_0 the value at the coast (south of point a', $s_0=a' \sin\theta_0$). This term is very small and as with most authors we choose to neglect it [see Carrier and Robinson (1962) for a good discussion of this point]. The function $g(\tilde{Y})$ in (3.6) is then determined by requiring that the east coast be the zero streamline.

Thus, in effect, we have neglected the eastern boundary layer so that the interior solution now satisfies the eastern boundary condition of zero normal transport.

The southward flowing California current, south of point a' in Fig. 2, was determined by Munk (1950) to be of the eastern boundary type. Such currents depend upon large, local onshore or offshore transport for their existence.

c. Boundary layer region (C)

In this region, higher order terms are important. Observationally, this is supported by the existence of a concentrated westward flow along the northern boundary.

If the passes connecting the North Pacific Ocean to the Bering Sea are neglected, the northern boundary [approximately represented by the 1000 m depth contour (see Fig. 3)] can be closely modeled by a curve of constant radius. In this case, the form taken by (3.3) in the boundary layer will change as $\theta(x)$ varies from θ_0 through zero to negative values, necessitating the two subregions outlined in Fig. 2. Because the radius of curvature of the Northern Boundary is so much larger than the characteristic width of the boundary layer, the curvature terms will not appear in the zeroth order equations.

As cross-stream (y) variations in the flow are expected to be much larger than the downstream (x) variations, the ordered form of (3.3) appropriate to the northern boundary layer (circumflexes) is obtained by transforming to the inner variable $\hat{y}$,

$$\hat{y}=\frac{\tilde{y}}{\phi(\mathrm{E}_*)}, \tag{3.7}$$

where, in particular, we assume

$$\phi(\mathrm{E}_*)=\mathrm{E}_*^{\mu_i},$$

$i=1,2$. The exponents μ_1 and μ_2 are determined by requiring that the largest frictional term be the same order as the largest Coriolis term along the coast. Expanding ψ in a power series in $\mathrm{E}_*^{\mu_i}$ ($\hat{G}$ is not ordered) and substituting this together with (3.7) into (3.3), the zeroth-order equations for the streamfunction in the two subregions C_1 and C_2 are obtained.

d. Subregion (C_1)

The subregion for $i=1$ is that in which $\cos\theta$ is at most of the same order as $\sin\theta$ in the boundary layer. With the assumption that the largest frictional and Coriolis terms balance in this layer, it is easily shown that $\mu_1=\frac{1}{3}$ and that

$$\frac{\partial^4\hat{\psi}_1}{\partial\hat{y}^4}+\sin\theta(\tilde{x})\frac{\partial\hat{\psi}_1}{\partial\hat{y}}=\hat{G}_1, \tag{3.8}$$

where $\theta(x)=(1+\tilde{x})\theta_0$, $\hat{G}\sim O(\mathrm{E}_*^0)$ if there is an $O(\mathrm{E}_*^{-\mu_1})$ increase in the wind-stress curl in the coastal region and of $O(\mathrm{E}_*^{\mu_1})$ if there is not; and for which the appropriate boundary conditions are:

the coast is the zero streamline, $\hat{\psi}(\tilde{x},0)=0$
there is no slip along the boundary, $\partial\hat{\psi}/\partial\hat{y}(\tilde{x},0)=0$
the solution is bounded for large negative $\hat{y}$
the solution matches to the interior solution

(3.9a–d)

Thus, if the local wind curl is comparable to that over the interior region, it can be shown that the necessary condition for the existence of a frictional boundary current is that (3.8) has at least two homogeneous solutions which decay rapidly with distance away from the boundary. The above condition is fulfilled for the case of a western type boundary where $\sin\theta>0$ (i.e., $\theta>0$) and for which the complementary solutions decay as

$$\exp[(\sin\theta)^{\frac{1}{3}}\hat{y}]=\exp[(\mathrm{E}_*^{-1}\sin\theta)^{\frac{1}{3}}\tilde{y}].$$

In the region where $\sin\theta<0$ (i.e., $\theta<0$), the case of an eastern type boundary, only one decaying type solution exists so that a boundary current can develop only if the local wind curl is much greater than that over the interior.

For $\theta>0$, therefore, it is possible to have a zeroth-order boundary layer flow in the absence of local wind forcing, while for $\theta<0$ such a boundary layer flow is possible only if there is a large, local wind forcing. As Stewart (1964) pointed out, western-type boundary currents are realizable since the vorticity gained from the coast by the intense flow can be balanced by a corresponding gain in planetary vorticity in the absence of wind-generated vorticity. For an eastern-type boundary current to exist, however, wind-generated vorticity is needed since the planetary vorticity and coastal-generated vorticity gains can no longer balance one another. It should also be clear that a western-type boundary current would not be possible if $\sin\theta_0\ll\cos\theta_0$ (where θ_0 is the rotation angle at $x=0$), that is, if there is no significant rotation of the x, y axes relative to the X, Y axes. Since we are modeling the Alaska peninsula and the coast of British Columbia it can be seen from Fig. 2 that $30°<\theta_0<45°$ and a western boundary current can exist.

e. Subregion (C_2)

As the current of subregion C_1 progresses westward, it begins to move along a nearly zonal boundary. In this second distinct subregion we find

$$(\mathrm{E}_*)^{-\mu_2}\sin\theta\sim O(\cos\theta)\sim O(1).$$

A balance of horizontal friction and Coriolis terms in (3.3) now yields $\mu_2=\frac{1}{4}$, for which the zeroth order equation is

$$\frac{\partial^4\hat{\psi}_2}{\partial\hat{y}^4}+(\mathrm{E}_*)^{-\frac{1}{4}}\sin\theta(\tilde{x})\frac{\partial\hat{\psi}_2}{\partial\hat{y}}-\cos\theta(\tilde{x})\frac{\partial\hat{\psi}_2}{\partial\tilde{x}}=\hat{G}_2, \tag{3.10}$$

with boundary conditions (3.9a–d). In contrast to subregion 1, the local wind-stress curl need only be of the same order of magnitude as the wind-stress curl in the oceanic interior in order to play an important role in the overall vorticity balance. Also, the longshore character of the Stream [which in (3.10) is related to the offshore transport of planetary vorticity] now influences its offshore character.

Strictly speaking, $\sin\theta$ must be expanded in terms of $(\mathrm{E}_*)^{-\frac{1}{4}}$ in (3.10) in order that $\hat{\psi}_2$ be dependent upon $\tilde{x}$ and $\hat{y}$ alone, i.e.,

$$\sin\theta(\tilde{x})=(\mathrm{E}_*)^{-\frac{1}{4}}[\gamma_0(\tilde{x})+\gamma_1(\tilde{x})(\mathrm{E}_*)^{\frac{1}{4}}+\cdots],$$

where the $\gamma_j(\tilde{x})$ are of $O(1)$. We can then write (3.10) as

$$\frac{\partial^4\hat{\psi}_2}{\partial\hat{y}^4}+\gamma_0(\tilde{x})\frac{\partial\hat{\psi}_2}{\partial\hat{y}}-\frac{\partial\hat{\psi}_2}{\partial\tilde{x}}=\hat{G}_2 \tag{3.11}$$

to zeroth order. The angles for which (3.11) is valid are less than 10–15°, or so, depending upon the longshore distance over which such an equation can be used to describe the flow. Obviously, there is no specific longshore position at which the character of the boundary layer changes from one type to another. In fact, the western-type boundary subregion and the zonal-type boundary subregion can be expected to be separated by a kind of transition zone in which neither type of approximation is actually valid.

If we now consider $\gamma_0(\tilde{x})$ to be a slowly varying function, whereby we ignore the small changes to the vorticity balance brought about by the longshore advection of planetary vorticity, then the method of separation of variables may be used to obtain homogeneous solutions to (3.11). These will have the form

$$\hat{\psi}(\tilde{x},\hat{y};\lambda,\alpha) = \text{Real}\{\exp[(\lambda+i\alpha)\tilde{x}]\sum_{n=1}^{4} D_n \exp(d_n\hat{y})+C\}, \quad (3.12)$$

where λ, α are real, $0<\alpha$, $|\lambda|<O(1)$, and C, D_n, d_n are constant (there is no loss of generality to the results in taking $\alpha>0$). As for subregion 1 the necessary condition for a boundary layer is that at least two of the roots (d_n) give rise to solutions which decay rapidly with distance away from the boundary. Substituting (3.12) into (3.11) yields the characteristic equation

$$d_n^4+\langle\gamma_0\rangle d_n-(\lambda+i\alpha)=0 \quad (3.13)$$

for the four roots d_n, in which $\langle\gamma_0\rangle$ represents a local spatial average of $\gamma_0(\tilde{x})$.

As the Stream enters subregion 2, the term involving γ_0 will affect the vorticity balance in such a way as to represent a stabilizing influence since it has the sign required to balance, at least partially, the coastal generated vorticity. Contrarily, this term will represent a destabilizing factor on the western side of the zonal region. In order to see how the term involving the along-shore derivative affects the boundary current, however, it is necessary to take $\gamma_0\approx 0$ since the four roots of (3.13) are much too complicated to facilitate any simple discussion. With this approximation, the roots are given trigonometrically by

$$d_n=(\lambda^2+\alpha^2)^{1/8}\cos\left[\frac{\phi+2(n-1)\pi}{4}\right] + i\sin\left[\frac{\phi+2(n-1)\pi}{4}\right], \quad (3.14)$$

where

$$\phi=\begin{cases}\arctan\left(\dfrac{\alpha}{\lambda}\right), & \text{for } \lambda>0\\[2ex] \pi-\arctan\left(\dfrac{-\alpha}{\lambda}\right), & \text{for } \lambda<0\end{cases}$$

and $0\leqslant\arctan(|\alpha/\lambda|)\leqslant\pi/2$. The real part of (3.14) determines the exponential decay of the solution in the offshore direction.

For $\lambda<0$, it can easily be seen that the criterion for the existence of a boundary layer is satisfied for all magnitudes of α and λ, except perhaps for the limit $\alpha/|\lambda|\rightarrow\infty$. On the contrary, for $\lambda>0$ two rapidly decaying solutions only become possible for large α/λ. In the latter solution, the general solution (3.12) contains frictional terms which do not decrease rapidly in amplitude away from the boundary. As this means that horizontal friction would be important in the interior, contrary to assumption and to scaling results, such a situation suggests a breakdown of the frictional boundary layer with associated separation from the coast. Although the second term in (3.13) has not been included explicitly in the above discussion, it would be expected to decrease the separation tendency of the Stream when $\lambda>0$ in the more eastern part of the zonal region and to increase the separation tendency when $\lambda<0$ in the western part.

As the above discussion establishes the criterion for which λ gives boundary layer solutions, the next step is to determine what such λ imply about the downstream character of the current. To do this, however, it is necessary to first consider subregion 1. The reason for this is that the flow is observed to move westward along the northern boundary. The wind-induced northward transport M^Y is greater than zero over the interior and boundary regions of subregion 1 since the mean wind curl is positive (counterclockwise) there. Thus, in curvilinear coordinates, M^y is greater than zero also. Then, since $\hat{M}^y=\partial\hat{\psi}_1/\partial\tilde{x}$, we have $\partial\hat{\psi}_1/\partial\tilde{x}>0$. But, $\hat{\psi}_1=0$ at the coast $(\tilde{x}=0)$ by boundary condition (3.9a), whereby $\hat{\psi}_1<0$ in subregion 1. Following the current westward we see that there are two possibilities: First, if there is a change of sign in $\hat{\psi}_1$ in the negative x direction, it must go to zero, which implies separation of the frictional boundary layer from the coast; there would be no continuous boundary layer flow. Second, if no sign change occurs in $\hat{\psi}_1$, then $\hat{\psi}_2<0$ as the flow enters subregion 2. The latter condition corresponds to a continuous boundary layer from subregion 1 to subregion 2. Using (3.12) it can be seen, then, that the case $\lambda<0$ corresponds to an increasing downstream transport and an accelerating current while the case $\lambda>0$ corresponds to a decreasing downstream transport and a decelerating current. Furthermore, condition (3.9d) requires the parameters α and λ to be found by matching to the interior region at the outer edge of the boundary current. But since the interior form of the streamfunction is determined by the wind-stress curl, these parameters must be directly related to the form taken by the time-averaged winds over the region. The case of increasing downstream transport $(\lambda<0)$ corresponds to a positive wind-stress curl driving water into the boundary layer while a decreasing downstream transport $(\lambda>0)$ corresponds to a negative wind-stress curl driving water away from the boundary region. In the former situation, the Alaskan Stream would remain attached to the coast until at least the western part of

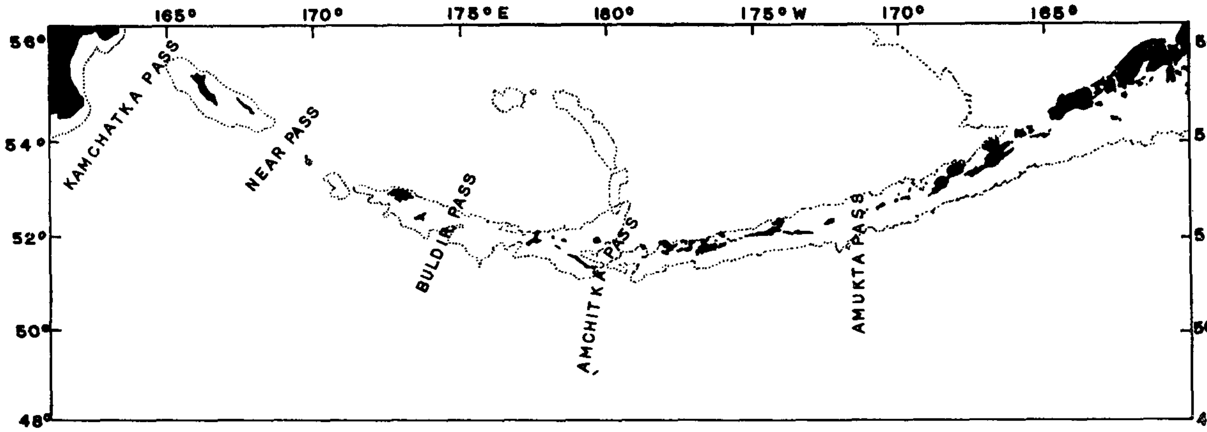

FIG. 3. Passes and 1000 m depth countour (dotted lines) in the vicinity of the Aleutian Islands.

subregion 1 where it could remain as a boundary current only if there was a large increase in the northward transport of water by the wind-stress curl over the adjacent interior region.

4. Discussion

In summary, then, for the general range $-\infty<|\alpha|/\lambda<0$ (accelerating flow), the boundary layer transport entering subregion 2 from the direction of the Gulf of Alaska can continue westward along the Aleutians into the western part of the subregion where $\theta(x)$ becomes negative. Provided that the transport of water into the Alaskan Stream in this subregion is sustained by the wind-stress curl, the current will continue as a boundary layer for a distance of approximately 300–500 km [$=\kappa^{-1}$ (actual)$\cdot$sin 10–15°] west of where the coast reaches its maximum southern extent near 180° longitude. For angles $\theta(x)$ less than $-15°$ or so, however, continued existence of a frictional boundary layer requires an order-of-magnitude increase in the local mean winds above those of the interior oceanic region, a requirement that the mean winds do not appear to fulfill [in reality, the coast is interrupted by large passes in this region and can no longer be considered continuous (see Fig. 3)]. For the general range $0<|\alpha|/\lambda<\infty$ (decelerating flow), a fully stable frictional boundary layer within the zonal region would not be possible. Theoretically, part of the concentrated boundary current in subregion 2, upon encountering the above condition, should breakup and separate from the boundary. This branch could take with it, perhaps, the excess vorticity that was initiating the instability within the Stream and leave behind a still attached portion of the current if there was a regained balance of vorticity. It is expected that inertial effects would tend to hold any detached part of the Stream together a short distance downstream of any such separation region.

With the results presented here and the vorticity arguments used by Stewart in describing oceanic circulation, we can now give a physical description of the subarctic circulation east of the Kuroshio current system.

The wind-stress curl north of about 45N adds net positive vorticity to the oceanic interior. The steady, Sverdrup-type flow that results from this accumulated wind-stress curl effect consists of a net northward transport to latitudes of more positive planetary vorticity. That part which concentrates into a western boundary current (3.8) in the vicinity of the eastern corner of the northern boundary is known as the Alaskan Stream. Since the boundary current is bounded by a line of zero velocity, it cannot contain or transport any relative vorticity. Therefore, considered over the width of the current, relative vorticity is not a determining factor in the overall vorticity balance. Also, as a western boundary current, the Alaskan Stream can exist in the absence of a local wind-stress curl. Therefore, the gain of more negative vorticity (planetary) by the current as it moves southward is balanced by the diffusion of negative vorticity from the coast. As the flow proceeds westward along the ever northward turning boundary, however, its gain of negative planetary vorticity decreases. This results in the formation of the zonal boundary current section of the Alaskan Stream. Eventually, the planetary vorticity variation can no longer balance the diffusion of negative vorticity from the boundary. If this accumulating vorticity is not balanced by other terms in the appropriate vorticity equation (3.11), the frictional boundary layer (or branches of it) would tend to separate from the coast. As the analysis of Section 3 shows, the required balance can be obtained by a transport of water into the boundary layer, or if one prefers, by a positive wind-stress curl over the boundary layer. Therefore, if the wind-stress curl over the boundary layer in this region goes from the positive values it has to the east, to negative values downstream, separation of the flow can occur. From the large-scale wind-stress curl data calculated by the author (Thomson, 1971), it is found that such changes in the sign of the wind-stress curl can occur in this region due to a buildup of high pressure on the Alaska Peninsula. Thus, a breakdown in the frictional boundary layer flow as a result of the change in sign of the wind-curl is possible and boundary layer separation of at least part of the flow can result. No exact prediction can be made, however, since the steady, frictional solutions breakdown in that case. If the wind-stress curl remains positive over subregion 2, a vorticity balance in the frictional layer in this subregion is possible. The concentrated coastal current can then exist until the western part of subregion 1 where the planetary vorticity effect is no longer small. Further existence of the frictional boundary layer into this region requires an order-of-magnitude increase in the local wind-stress curl over that of the interior.

The importance of nonlinearities to the frictional model being considered here can be deduced from a recent study by Blandford (1971). In that paper, application of the viscous no-slip condition to a numerically modeled inertial boundary layer produced large, time-varying eddies in the corner region between the western and northern boundaries. The attempt of the western boundary current to continue along the northern boundary was disrupted by these eddies which were highly effective in decreasing the relative vorticity associated with the flow and in diffusing it into the oceanic interior. A similar result could be expected to occur in the case of the Alaskan Stream. Upon the Stream's separation from the northern boundary, highly nonlinear eddies would result in a southward diffusion of the current's relative vorticity into the interior region.

As the Alaskan Stream is just beginning to be studied by oceanographers, there is still little historic data on the structure and variation of this current with time. Nevertheless, some observational studies do exist and their findings can be compared to the theoretical results obtained here. For example, we may compare the mean transport ($\sim 10\times 10^{12}$ gm sec^{-1}) and the mean surface speed (~ 50 cm sec^{-1}) of the Alaskan Stream, as obtained from geopotential topography (Ingraham and Favorite, 1968), with the zeroth-order theoretical values. Using the definitions (3.1), (3.2) and Eq. (3.5), the transport scale ψ_0 takes the form

$$\psi_0 \approx \rho_0 U_0 L D \approx \tau_0 \beta_0^{-1}.$$

With $\beta_0 \approx 1.3\times 10^{-13}$ cm^{-1} sec^{-1} at 55N we obtain

$$\psi_0 \approx 10\times 10^{12} \text{ gm sec}^{-1}$$

for reasonable mean wind stresses of only 1.3 dyn cm^{-2}. To obtain the surface speed C_0 within the boundary layer we assume an exponential decrease of the horizontal speed with depth with an e-folding scale z_0 of between 200 and 400 m. This, together with a boundary layer width of $b \sim O(100$ km) as obtained from the previous reference, yields for

$$C_0 \approx \psi_0 (\rho_0 z_0 b)^{-1}$$

the result

$$25 < C_0 < 50 \text{ cm sec}^{-1},$$

which again agrees with the geostrophically calculated values.

Fig. 4, which is based on the temperature distribution on the sigma-t (σ_t) surface 26.90 (below surface heating) in the summer of 1959 (Favorite, 1967), roughly indicates the western and southern extent of the Alaskan Stream within the zonal region. One branch is seen to be separating from the boundary while the rest appears to be flowing northward and then through Near Pass at about 170E.

Finally, Favorite (1969) has used the accumulated data on surface salinity distributions in an attempt to

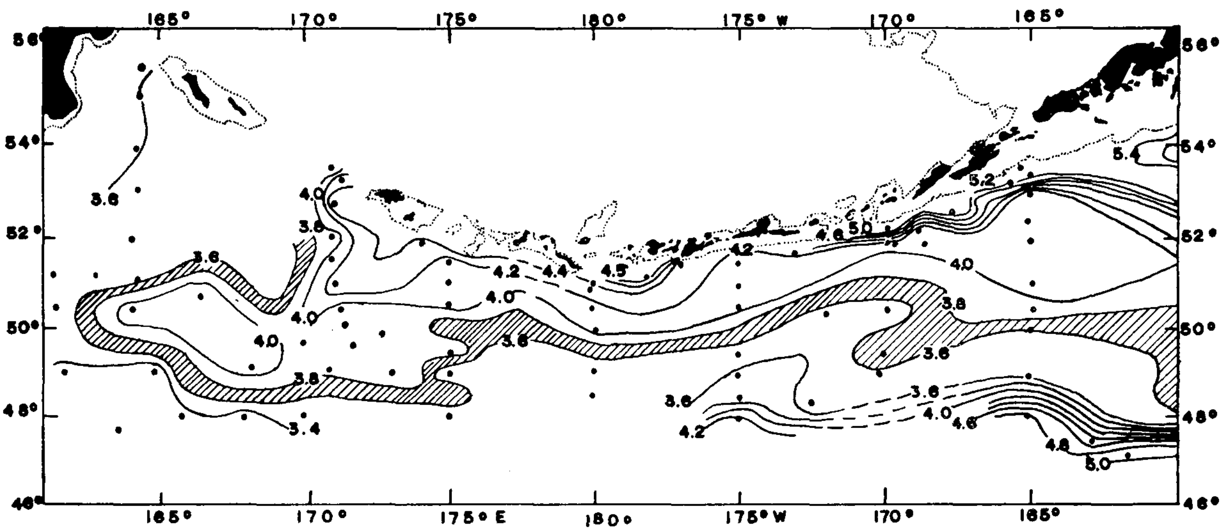

FIG. 4. Temperature (°C) distribution on the sigma-t surface 26.90, summer 1959. Shaded region shows approximate extent of the Alaskan Stream. The 300 m depth contour (dotted lines) is also shown. (From Favorite, 1967.)

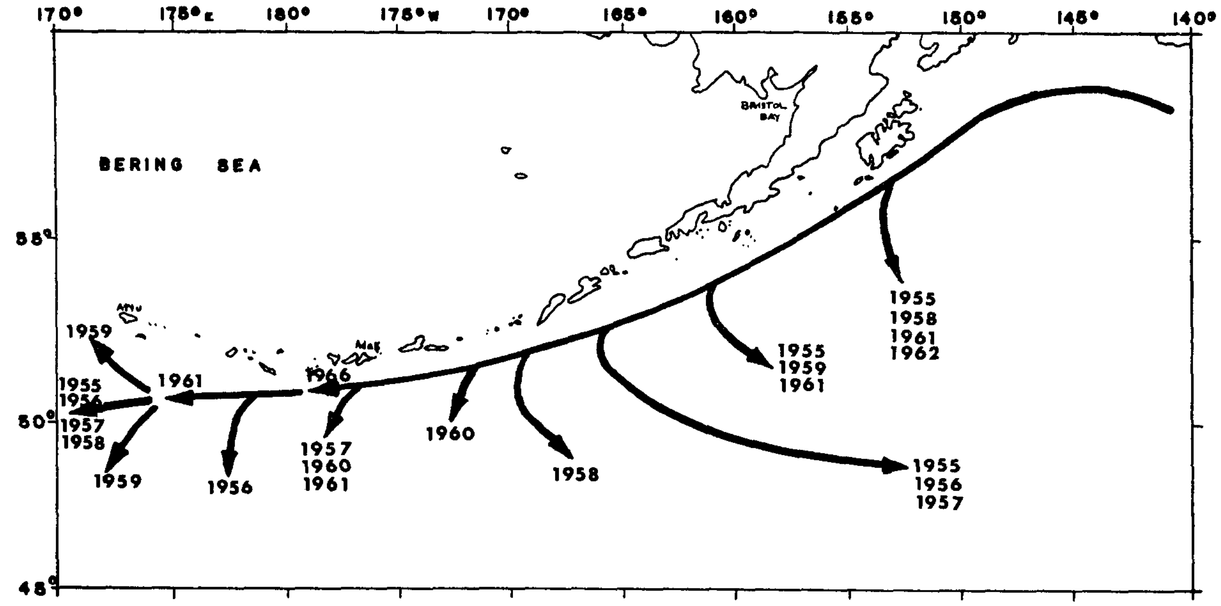

Fig. 5. Schematic diagram of the variability in location at which southern branches diverge from the main flow, as indicated by surface salinity. (From Favorite, 1969.)

determine where branches of the Alaskan Stream separate from the coast. Basically, he has assumed that because the water associated with the southern side of the Stream has a significantly lower salinity than that of the adjacent interior region, any detachment of the boundary current from the coast will be detectable by its dilution of the interior region waters. As seen in Fig. 5, there is a large amount of variability in the location at which these southern branches diverge from the main flow. Except, perhaps, for the branches near 155W which appear to have occurred within the western boundary region, the data in Fig. 5 are consistent with the results presented in which separated flows occur west of the transition of the boundary from a western to a zonal type. Those branches near 155W could possibly be the result of abnormal wind conditions. Also, it is possible that the branching has in fact occurred within the transition region and is therefore consistent with the analysis presented (as previously stated it is not possible to determine exactly where the western type boundary ends and the eastern one begins).

It is further noted that for the observations available, only in 1959 did the boundary current move westward toward the western section of subregion 2.

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