Yield Criterial for Layered Rock Mass with Bending Effect

She Chengxue
(Wuhan University of Hydraulic and Electric Engineering, Wuhan 430072 China)

Abstract Two simplified yield functions are proposed for the layered rock mass with bending effect on the basis of two different hypotheses. By being compiled into the FEM program, they are used to evaluate the deformation of a cantilever. The results show the two functions present different properties: one has brittle property, the other ductile, and show that they can be combined or singly used to correctly model different kind of deformation of layered rock mass.

Key words yield function, bending effect; layered rock mass
Category O344.5

1 INTRODUCTION

In open pit mine or hydraulic engineering, the rock slope with rock layer dipping inside into slope is always met. This kind of slope will present flexural buckling deformation in natural state or after excavation. The interface slips and rock layer presents tensile bending yield. By introdution of the Cosserat type theory (She et al, 1994, 1996), a finite element fomulation has been established, the bending effect of rock layer in elastic-rcok layer in elastic state and slipping in interface have been considered in elastic-plastic constitutive relationship. However the tensile bending yield mechanism of the rock layer is also needed to be modelled in the constitutive relationship. The author of this paper proposed a criterion for this tensile bending yield mechanism (She et al, 1994). The criterion extends the calculation for layered rock mass with bending effect to elasto-plastic stage, but the criterion is complex, and it does not discriminate ductile and brittle failure pattern. To overcome these defects, two new simple criteria will be proposed based on new hypotheses. Because the element for rock layers has definite thickness, the yield criterion will discribe the yield state on the element face with this thickness. So the yield criterion is different from the conventional criterion in elasto-plastic theory. The conventional concept of yield is based on 'point yield', but the yield criterion for layers is based on 'face yield'. In this paper, two new criteria will be established for the tensile bending yield of rock layers on the basis of two kinds of 'point yield' concepts.

2 CRITERION BASED ON POINT PERFECT TENSILE YIELD HYPOTHESIS

Fig. 1 is the element of a rock layer. The dimension of the element in $x1$ direction is infinite small, this is to say $dx1$ is very small. On the face normal to $x2$, there are normal stress σ_{22} and shear stress σ_{21} but no couple stress exists. However in $x2$ direction the element has a thickness which is equal to the thickness of the rock layer d. On the face normal to the direction of $x1$, there are normal uniform stress σ_{11}, shear stress σ_{12} and couple stress,

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作者 佘成学 简介：男，35岁，博士，1986年毕业于武汉水利电力大学水电系施工专业，现任副教授，主要从事水工结构工程中的数值模拟方面的教学和研究工作。

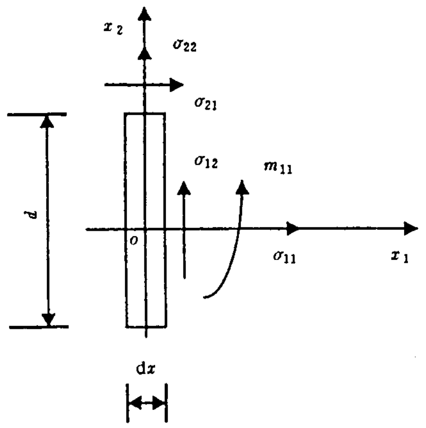


Fig. 1 Element of rock layer

m_{11}. Due to the united action of normal uniform stress σ_{11} and couple stress m_{11} on the face normal to the axis $x1$, there is a linear distribution of normal stress which is

$$\sigma = \sigma_{11} \pm 12m_{11}x_2/d^2 \tag{1}$$

We assume that the element does not yield in the direction of $x2$ and only tensile bending yield occurs in the direction of $x1$, and assume that the rock is a kind of perfect plastic material. The first assumption let us only consider the yield occurs on the face normal to $x1$ and the second assumption makes us think that the yield of the point on the face normal to $x1$ is perfect plastic yield. Fig. 2 shows the stress-strain relationship of the rock material. Based on these assumptions the initial yield function can be expressed as

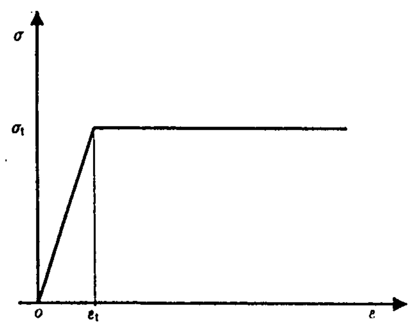


Fig. 2 Stress strain relationship of rock point

$$F = \sigma_{11} \pm 6m_{11}/d - \sigma_t = 0 \tag{2}$$

This function means that under the united action of σ_{11} and m_{11}, the rock point at the edge of the face normal to $x1$ is in tensile yield state. The function can be intuitively shown by the initial yield lines in Fig. 3.

After the face enter initial tensile bending

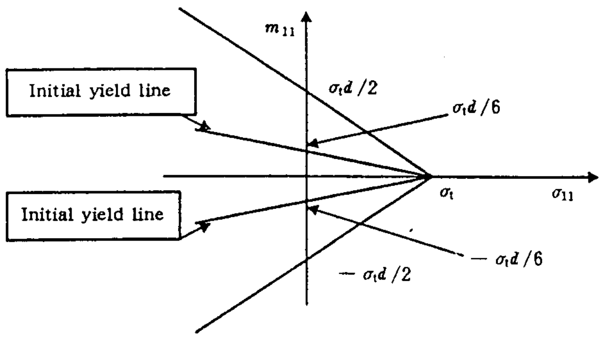


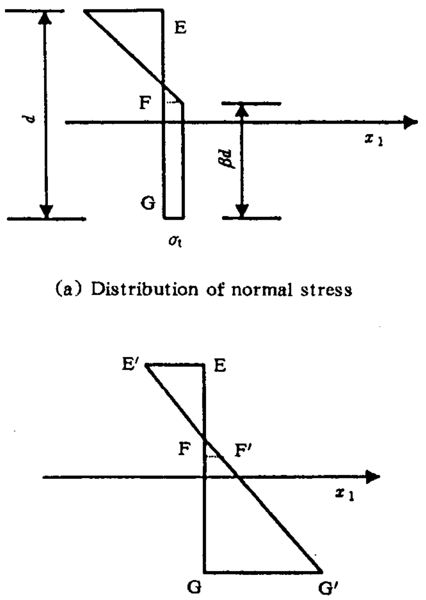
Fig. 3 Schematic plot of the yield function

yield, if σ_{11} and m_{11} increase subsequently, it will yield further. Because the other part of the face hasn't yielded, the further tensile bending yield will develop until all the face is totally yielded. So we should also give the yield function after the face initially yields.

Fig. 4 gives the schematic tensile bending yield situation of the face after initial yield occurs. β is the proportion of the face in yield state, $\beta \leqslant 1$. By using the equilibrium condition, we can deduce the yield function after it is initially tensile yielded, which can be written as

$$F = \sigma_{11} \pm 6m_{11}/[d(1 + 2\beta)] - \sigma_t = 0 \tag{3}$$

This is a linear function for variables σ_{11} and m_{11}. Fig. 4 shows schematically the change of the function with β. From the plot it can be seen that after the face initially yields, with the development of tensile bending yield , the yield zone on the face



(a) Distribution of normal stress

(b) Distribution of normal strain

Fig. 4 The schematic yield state of the surface become bigger and bigger and the value of β increases, correspondingly the yield lines in the plot extend outwards. This process shows a strengthening phenomenon.

When $\beta = 0$, the expression (3) is equal to (2), this means that the expression (3) includes expression (2).

When $\sigma_{11} = \sigma_t$, there is a corner point. At this point the normal stress distribution on the face is uniform and the couple stress is zero. The yield function can be written as

$$\sigma_{11} = \sigma_t \quad (\sigma_{11} \geqslant \sigma_t,\ m_{11} = 0) \qquad (4)$$

The parameter β in expression (3) needs to be deduced further. Fig. 4 (b) shows the deformation of the points on the face corresponding to Fig. 4 (a), we assume that after part or total of the face yields, the face remains in a plane, this means that before the face yields E, F and G are in the same plane, EFG is a straight line. After the face yields, E, F and G move to E', F' and G' respectively. $E'F'G'$ are also a straight line. From Fig. 4(a), it can be seen that EF part of the face are in elastic state. By using the Hoek law in elastic theory, equilibrium condition and geometric relationship between point E and point F, we can derive the expression of β which can be written as

$$\beta = 1 - \sqrt{2(\varepsilon_t - \varepsilon_{11})/[k_s + |k^p|)d]} \qquad (5)$$

where: ε_t—the elastic tensile normal strain limit of rock material.

ε_{11}—the total elastic normal strain, $\varepsilon_{11} < \varepsilon_t$.

$|k^p|$—the absolute value of the total plastic curvature.

$$k_s = 2\sigma_t(1 - \mu^2)/(Ed) \qquad (6)$$

where: E—elastic modulus.

μ——poisson's ratio.

$\varepsilon_{11} = \varepsilon_t$ means the face is totally in tensile yield state, $\beta = 1$. $\varepsilon_{11} \neq \varepsilon_t$, $\beta < 1$.

3 CRITERION BASED ON POINT BRITTLE TENSILE BREAKAGE YIELD HYPOTHESIS

In this section, we keep the first assumption stated above, modify the second assumption and assume that the rock is a kind of brittle tensile breakage material. This means that the point on the face normal to $x1$ will occur tensile breakage when the normal stress σ is bigger than the tensile strength σ_t of the rock material. The σ-ε relationship of the rock material is shown in Fig. 5.

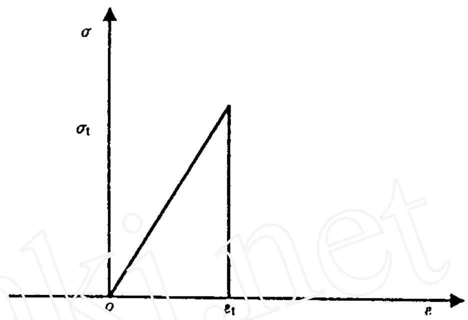


Fig. 5 σ-ε relation of rock point

Fig. 6 gives the schematic plot of normal stress distribution on the face normal to $x1$. The βd part of the face is broken by tensile stress so the stress on this part falls to zero.

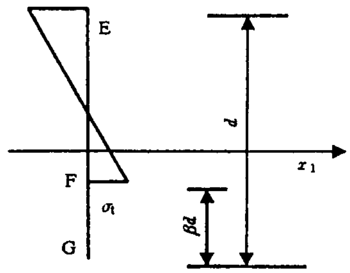


Fig. 6 Distribution of normal stress after yield

σ_{11} and m_{11} are the normal uniform stress and couple stress on the whole face respectively. After FG part of the face in Fig. 6 is broken, the real normal stress and couple stress are

$$\begin{cases} m'_{11} = m_{11}/(1 - \beta)^2 \\ \sigma'_{11} = \sigma_{11}/(1 - \beta)^2 \end{cases} \qquad (7)$$

Substitute σ'_{11} and m'_{11} in (7) into σ_{11} and m_{11} in (2), and substitute $d/(1 - \beta)$ into d in (2), we can get

$$F = \sigma_{11} \pm 6m_{11}/d - \sigma_t(1 - \beta) = 0 \qquad (8)$$

If $\beta = 0$, equation (8) is equivalent to (2), this means the face is in initial yield state. So, equation (8) includes (2). If $\beta = 1$, the face is totally broken. Fig. 7 schematically illuminates the yield function. From the plot it can be seen that the yield function is also a set of linear lines and after initial yield, the yield lines shrink inwards showing a weakening tendency. When $\beta = 1$, the face is totally broken, at this time, only $\sigma_{11} < 0$ can the face sustain bending deformation.

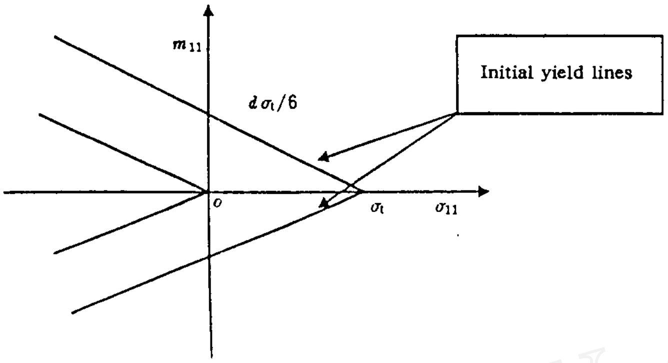


Fig. 7 Schematic plot of yield function (8)

Just as the yield function (3), there are corners at $\sigma_{11} = \sigma_t(1-\beta)$ ($m_{11} = 0$). At the corners the yield function can be written as

$$\sigma_{11} = 0 \quad (\sigma_{11} \geqslant \sigma_t(1-\beta),\ m_{11} = 0) \qquad (9)$$

The parameter β is deduced by the same method as (5), it can be written as

$$\beta = 1 - 2(\varepsilon_t - \varepsilon_{11})/[(k_s + |k^p|)d] \qquad (10)$$

where ε_{11}, k_s and $|k^p|$ have the same meaning as stated above. ε_t is the elastic tensile normal strain limit of rock material. if $\varepsilon_{11} = \varepsilon_t$, the face is totally in tensile breakage state, $\beta = 1$. if $\varepsilon_{11} \leqslant \varepsilon_t$, then $\beta < 1$.

4 SIMPLE EXAMPLES

The yield criteria expressed by equation (3), (4) and (8), (9) are compiled into elasto-plastic FEM program based on the Cosserat theory. By using the program to calculate some simple tensile bending problems, we can get some knowledge of the criteria.

Fig. 8 shows a layered rock beam. The length is 4 m, and height 0.4 m. On top of the beam, there is an evenly distributed load. M1 and M2 are two kinds of rock material with the same height. The thickness of every rock layer is 0.02 m. Every two rock layers draw a layer of FEM mesh. Totally ten layers of FEM mesh are drawn.

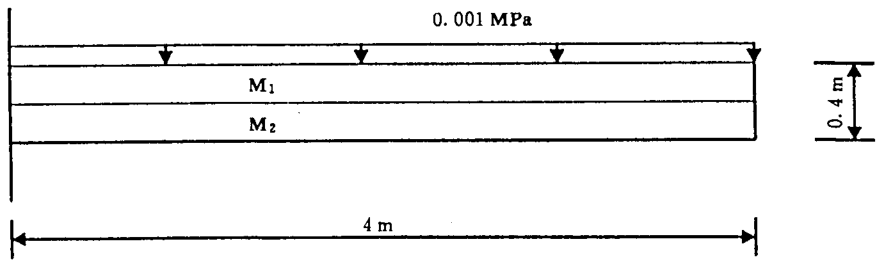


Fig. 8 Plot of a cantilever

Firstly, it is assumed that M1 is the same as M2. Elastic modulus $E = 0.1$ GPa, poisson's ratio is 0.3, $\sigma_t = 0.30$ MPa, $\varepsilon_t = 0.003$, the parameters on layer face are normal stiffness $k_n = 10$ MPa, shear stiffness $k_s = 4$ MPa, $c = 0.001$ MPa, $\varphi = 25°$.

By using the yield criteria expressed by (3), (4) to model the yield of rock layers, the calculation is convergent. The yield zones are on the left end of the beam. The top layer is in tensile yield state, $\beta \approx 1$. From top to bottom, β become smaller to zero. Correspondingly, the rock layers are in tensile bending yield, tensile bending and compressed bending state. The deformation is ductile.

Secondly, if the values of the parameters are kept unchanged and the criteria expressed by (8), (9) are used to model the yield of the rock layers, the calculation is divergent. The failure begins from top. After the failure of the top layer, the load transferred to lower layers. This load transfer causes the increment of load on the lower layers. Then new failure happens. This process proceeds until all the rock layers are broken. The failure is brittle tensile bending breakage.

At last, between M1 and M2 set a layer of joint elements, the criteria expressed by (8), (9) are used to model rock material M1 and the criteria expressed by (3), (4) are used to model rock material M2. Keep the other parameters unchanged, gradually increase σ_t, ε_t of material M2 to 0.6 MPa, 0.006 respectively, the calculation is convergent. Though the top rock layer of M2 is totally broken, the lower layers of material M1 are in tensile bending breakage yield state, $\beta < 1$. This means the rock layers of material M1 are not totally broken. The reason is that with the development of deformation, part of the load sustained by material M1 is transferred to material M2. This causes some part of the layers of M2 enter tensile bending yield state. The yield presents composite pattern not only with brittle tensile bending breakage but also with ductile tensile bending yield.

5 CONCLUSION

In this paper, from two kinds of point yield assumption, the point perfect tensile yield and point brittle tensile breakage assumption, two types of tensile bending yield functions for layered rock mass are deduced. If the rock layer has good ductility, it can be modelled well with the first yield function. Conversely, if the rock layer is brittle material, the second yield function will be suitable. These two yield criteria can also be combined to model complicated yield pattern of complex rock mass. Because the yield criteria are simple in pattern, they can be easily extended to 3D problems.

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《岩石力学与工程学报》1999 年发行分布统计图

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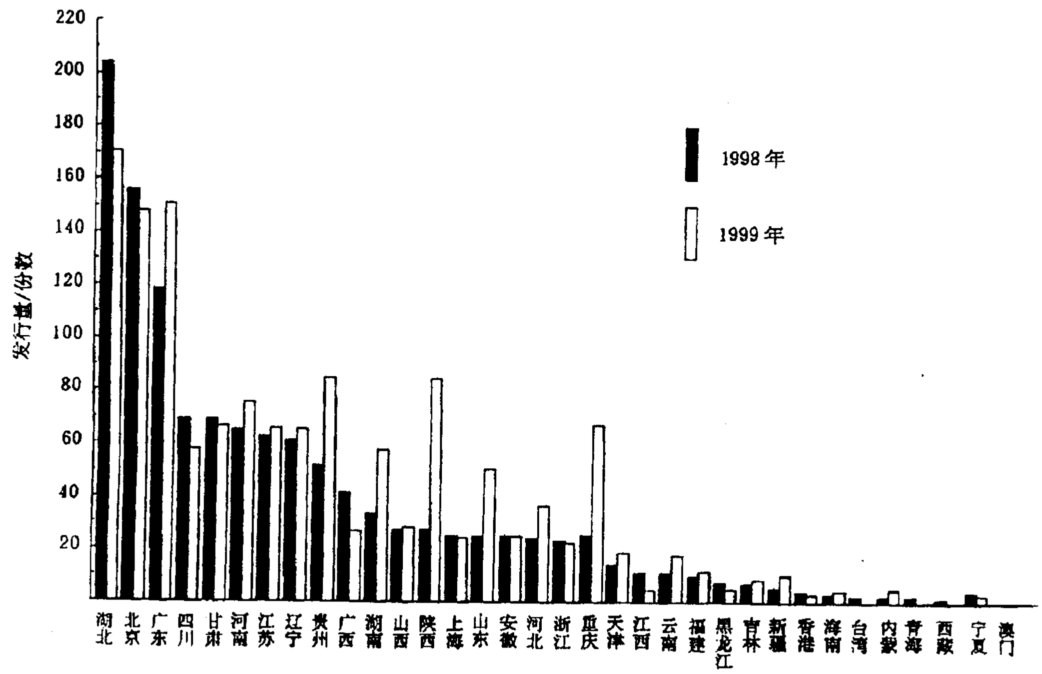

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