TANK MODEL AND ITS APPLICATION TO PREDICTING GROUNDWATER TABLE IN SLOPE

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Abstract：Tank model is a helpful tool for rainfall-groundwater-runoff analysis since it can represent a nonlinear transport behavior and get solutions very quickly. It is known that the successful application of one conceptual model mostly depends on how well its parameters can be calibrated. Recently，in many literatures，it is indicated that by use of existing calibration methods，the calibration process with many parameters(such as multi-tank model proposed in this paper has parameters over 20) is typically difficult，sometimes even impossible to obtain unique optimal parameters. A new random optimization approach called dynamically dimensioned search(DDS) algorithm is introduced and improved for parameters calibration of tank model. DDS is designed for calibration problems with many parameters，requires no complicated algorithm parameter to be adjusted，and automatically scales the search to find good solutions within the maximum model evaluations. Tank model with 27 parameters is applied to the actual case；and DDS algorithm is adopted to find optimal solutions. The calculated runoff roughly agrees with the measured values. Finally a comparison between finite element method(FEM) and tank model is conducted，which shows that during rainfall infiltration，the multi-tank model has advantages over FEM in the simulation process of predicting groundwater table. It is clarified that the multi-connected tank model is useful in groundwater table prediction of the basin especially when the slope stability analysis is necessary there.

Key words：slope engineering；multi-tank model；dynamically dimensioned search；conceptual model；rainfall；parameter optimization method；groundwater table prediction

CLC number：P 642.22　　**Document code**：A　　**Article ID**：1000－6915(2008)12－2501－08

Tank 模型及其在边坡水位预测中的应用

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摘要：Tank 模型可以模拟非线性的降雨－地下水运移过程，并且能迅速得到解答。基于现有的单列 tank 模型，提出新的复合水箱模型。由于新模型参数超过 20 个，应用传统优化算法难以快速找到最优解，一种新的启发式自搜索算法(变维数搜索算法)被引入并改进后用于模型最优解的寻找。变维数搜索算法能够根据搜索进程的变化自动改

Received date：2008－08－07；**Revised date**：2008－09－15

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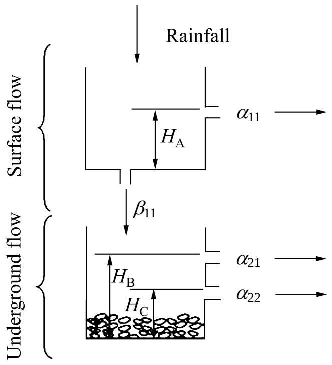
变搜索维数并且快速找到最优解。27 个参数的复合 tank 模型被应用于日本国道九号线的一个边坡，计算结果表明：变维数搜索算法能够在 10 min 左右找到合适的最优解；降雨过程复合 tank 模型计算的地下水位变化和观测值非常接近。最后通过和有限单元法计算结果的比较表明，有限单元法的计算结果受地质渗透特性的影响很大，而复合 tank 模型不存在这种问题。工程实例计算表明，该方法和监测结果比较一致，但其适应性更强，特别适用于没有进行足够地质结构探查的边坡。它能够快速反映降雨过程中地下水位的运移过程，可以推广使用。

关键词：边坡工程；复合 tank 模型；动态变维搜索；概念模型；降雨；参数优化方法；地下水位预测

1 INTRODUCTION

It is known that heavy rain may cause landslide，incurring significant damage to the local area. In Japan，it is reported that the number of slopes that are at high risk of downpour landslide is over 500 000[1，2]. So in order to prevent the slope from collapsing during rainfall process，it is necessary to understand the groundwater transport behaviors beforehand.

In most cases，these downpour landslides are caused not only by rainfall infiltration but also surface erosion. And landslides can be divided into two groups：slip failures，which are the result of a rise in groundwater level，and shallow landslides，which are the result of an increase of saturation degree. So in order to prevent such kinds of slopes from collapsing，it is important to understand the groundwater transport behaviors in the slope beforehand. Generally，the ground conditions are extremely unclear. In this case，if traditional numerical seepage analysis methods are adopted，the two- or three-dimensional geological models and also many stratum parameters should be provided，which are time-consuming work，and also sometimes are even impossible. M. Sugawara et al.[3，4] proposed a simple tank model. Since it can represent a nonlinear stream flow behavior and get solutions very quickly，it can be used for long-term runoff and water table analysis. Tank model is based on the water balance analysis，which is an accounting model that tracks flows of water into and out of the particular hydrologic system of interest. Its conceptual model is shown in Fig.1. It can reproduce actual water table fluctuations after identifying parameters，and the reproduced water table values can match the measured one well. But according to the circumstances，it can only give water table at one point or the average water table of the slope，which can not reflect the actual water table behavior of the whole slope，so it must be improved.



β_{11}—Vertical seepage coefficient；H_A，H_B，H_C—Heights of lateral outlets；α_{11}，α_{21}，α_{22}—Discharge coefficients of each outlet

Fig.1 Sketch of tank model

In this paper，a new analytical model system called multi-tank model is proposed to simulate the rainfall infiltration process. With more tanks，more complex groundwater transport behaviors can be represented. And a new random optimization approach called dynamically dimensioned search algorithm is introduced and improved for parameters calibration of tank model. Finally the multi-tank model is practically verified by case studies.

2 DEFINITION OF MULTI-TANK MODEL

2.1 Multi-tank model for slopes

Conventionally，numerical saturated-unsaturated seepage analysis needs to deal with rainfall infiltration problem. However，even though such kind of numerical analysis has the advantage of being able to incorporate complex and inhomogeneous physical properties and geologic structure，it also has the disadvantage that the accuracy of the prediction is greatly affected by the given parameter values. However，sometimes it is difficult to investigate geologic structure completely and to identify enough good parameter values. If the designated parameter values are not so accurate，correspondingly，the analytical results are not so good either.

Considering the factors mentioned above，the rainfall response of groundwater table under the slope is focused on，and a new multi-tank model system is developed to simulate the water transportation of slopes. In the past research，simple tank model has often been applied to water runoff analysis[3]. On the other hand，the multi-tank model is the extension of tank model of one sery，which is based on the in-situ measurement data of water table or surface flow. It can be applied to the evaluation of groundwater table fluctuations for any specific slope. In other words，it is thought that using this method to simulate the transport process of rainfall in the slope makes it possible to identify slope behavior relatively easily[3]. And also it is believed that the development of such an analytical methodology can evolve into a new assessment tool of being incorporated into the slope stability analysis.

As shown in Fig.2，part of rainfall on the slope becomes surface flow，and at the same time the rest is turned into seepage flow. The reason that rainfall results in surface flow on the slope is generally categorized into three types，namely，generation of Horton′s flow，return flow and crusting. Horton′s flow is automatically generated when the rainfall intensity exceeds the infiltration capacity of the slope. Return flow is generated when the unconfined groundwater effuses to the slope surface again，which is caused by the rise of groundwater table due to continuous rainfall. Crusting is the result of rapid deterioration of infiltration

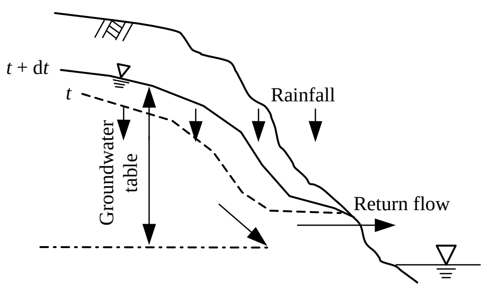


Fig.2　Hydraulics of slope groundwater

capacity because of raindrop′s impact on the Earth′s surface. Thus it is important to consider the interaction among all kinds of water balance factors including all components(rainfall，evapotranspiration and flow discharge，etc.).

The behavior of groundwater under the slope is greatly affected by rainwater infiltration，regardless of slope size in a broad sense. And also rain water infiltration process is influenced by many other factors，such as the slope gradient，soil components of slope，geological structures，properties and moisture state. So it is necessary to develop a simple and effective way to simulate this process.

As illustrated in Fig.3，multi-tank model proposed here is a two-dimensional triplet tank model，which consists of three two-dimensional two-tired tanks to simulate rainfall and runoff responses. One of these inter-connected tanks is set at the highest position to serve as the base point，which is followed by another one at the medium position and the last one at the lowest position. These tanks represent the vertical flow in the upper part，lateral flow in the middle part and return flow in the lower part respectively. In this way，they can be designed to simulate the evaluation of major groundwater behavior of the slope.

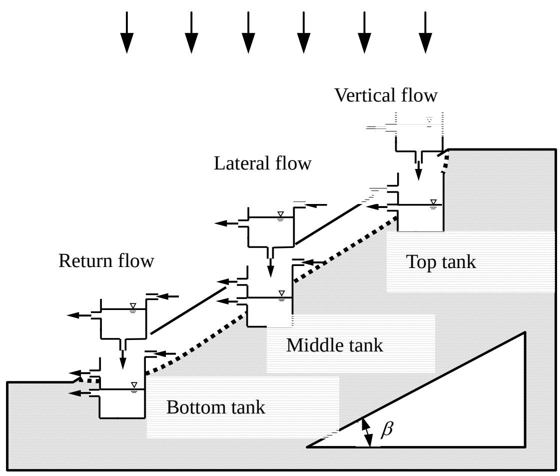


Fig.3　Configuration of multi-tank model

2.2　Flow patterns in tank model

Fig.4 shows assumed flow patterns in tank model. Here，P is rainfall intensity and E means evaporation on the same day.

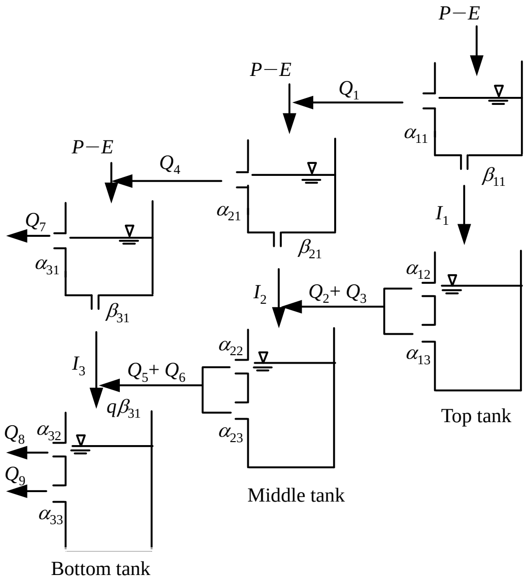


Fig.4　Assumed flow patterns in tank model

Then the lateral flow discharge $(Q_i(t)$, i = 1 – 9) and the vertical seepage volume $(I_i(t)$, i = 1 – 5) at one specific time can be evaluated by the following equations:

$$Q_i(t)=\begin{cases}\alpha(i)(WL_i(t)-H_i) & (WL_i \geqslant H_i)\\ 0 & (WL_i < H_i)\end{cases} \tag{1}$$

$$I_i(t)=\beta(i)WL_i(t) \tag{2}$$

$$\left.\begin{aligned}\mathrm{d}h_1/\mathrm{d}t &= R(t)-Q_1(t)-I_1(t)\\ \mathrm{d}h_2/\mathrm{d}t &= I_1(t)-Q_2(t)-Q_3(t)\\ \mathrm{d}h_3/\mathrm{d}t &= R(t)+Q_1(t)-Q_4(t)-I_3(t)\\ \mathrm{d}h_4/\mathrm{d}t &= I_2(t)+Q_2(t)+Q_3(t)-Q_5(t)-Q_6(t)\\ \mathrm{d}h_5/\mathrm{d}t &= R(t)+Q_4(t)-Q_7(t)-I_3(t)\\ \mathrm{d}h_6/\mathrm{d}t &= I_3(t)+Q_5(t)+Q_6(t)-Q_8(t)-Q_9(t)\end{aligned}\right\} \tag{3}$$

where $WL_i(t)$ is the water table of corresponding tank, H_i is height of the lateral outlet, $R(t)$ is rain intensity, and $\alpha(i)$ and $\beta(i)$ are discharge coefficients. Then the water levels of the three lower tanks(h_2 , h_4 and h_6) are related to groundwater table. $GWL_i(t)$ at a specific time t can be calculated if reference water level(RWL) is added:

$$GWL_i(t)=GWL(0)+h_i^{\mathrm{bot}}(t)/v \quad (i = 1，2，3) \tag{4}$$

where $GWL(0)$ is the reference groundwater level, v is effective porosity of soil, and $h_i^{\mathrm{bot}}(t)$ is calculated water levels of three lower tanks.

2.3　Definition of optimization function

In order to identify the appropriate parameters, the following optimization equation is adopted:

$$J_{\mathrm{XS}}=\frac{1}{M}\sum_{i=1}^{M}\frac{[Q_{\mathrm{c}}(i)-Q_{\mathrm{o}}(i)]^2}{Q_{\mathrm{c}}(i)} \tag{5}$$

where $Q_{\mathrm{o}}(i)$ is the measured value, $Q_{\mathrm{c}}(i)$ represents calculated result, and M is number of measurements.

2.4　Parameters identification

The tank model proposed by M. Sugawara et al. is structurally simple and useful, but it also has many parameters, so it is very important to determine them correctly. In previous literatures[5－7], there are some methods to find solutions for one series of four-tank model(16 parameters). T. Yasunaga et al.[8，9] tries sequential estimation using Kalman filter; H. Tanakamaru et al.[10，11] and others use genetic algorithm(GA) as an efficient search procedure. In this paper, multi-tank model is even more complicated than theirs, three series of tanks are introduced, and 27 parameters(sometimes even more) generally need to be estimated from measurements by use of optimization functions. For three series of tank distribution, using GA is very time-consuming and also the solutions are not good. So in order to facilitate calibration process, the development of a multipoint random optimization approach called dynamically dimensional search(DDS)[12] is studied.

Basically, during the studies of estimating parameters, the calibration function of tank model is replaced with a nonlinear optimization function, for example, minimizing the errors between calculations and measurements is the most popular. In this paper, some groundwater table measurements are used as criterion function. If GA is used to find the estimation of these parameters, because of too many parameters, it has a significant computational burden. This is because that too many dimensions will lead to distribution order increase with geometric series. In fact, once a limited number of model evaluations are considered, the idea of achieving global optimality becomes unreasonable in most of automatic calibration process. So here a new DDS as one of such good algorithms that are focused on identifying good calibration results in relatively short time is proposed.

The DDS algorithm is a novel and simple stochastic single-solution method，and it is based on heuristic global search. The algorithm searches globally firstly，and becomes more and more local as the number of iterations gradually approaches the maximum allowable number of function evaluations. The adjustment from global to local search is achieved by dynamically and randomly reducing the number of searching dimensions in the neighborhood.

The only parameter to be set in DDS algorithm is the scalar neighborhood size perturbation parameter r that defines the random perturbation size as a fraction of the decision variable range. An initial value of the parameter r is set as 0.2，and with the calculation process going on r will reduce step by step，the minimal value of r is 0.05，which is different from the one obtained by B. A. Tolson and C. A. Shemaker[12]. This initial sampling region size is designed to allow the algorithm to escape regions around poor local minima. In the final stage，because the current solution is close to final results，in order to avoid big perturbation，the value of r must decrease. And also in order to accelerate the rate of convergence，its update algorithm is also improved compared with that of B. A. Tolson and C. A. Shemaker[12]. The complete calculation process of the improved DDS algorithm is provided as the follows：

(1) Define initial neighborhood perturbation size parameter r (0.2 is default) and the maximum evaluation step.

(2) Set counter from 1 to D(number of parameters)，and give initial solution x^0.

(3) In each counter，randomly select J of the decision variables D for inclusion in neighborhood $\{N\}$.

(4) For $j=1, 2, \cdots, J$，decision variables x_j^{best} in $\{N\}$ perturb using a standard normal random variable $N(0, 1)$，reflecting at decision variable bounds if necessary，and get new solution x^{new}.

(5) Evaluate $F(x^{\text{new}})$ and update the current best solution if necessary：

① If $F(x^{\text{new}}) \leqslant F^{\text{best}}$，update new best solution：

$$F^{\text{best}} = F(x^{\text{new}}), \quad x^{\text{best}} = x^{\text{new}} \tag{6}$$

② If $F(x^{\text{new}}) > F^{\text{best}}$ and $\exp[-(F^{\text{new}} - F^{\text{best}})/f(j)] >$ random(P_{n})(P_{n} is the selected probability of solution x^{new})，update new best solution according to Eq.(6).

(6) Update iteration counter，$i = i+1$，and check stopping criterion：

① If i reaches the maximum iteration counter，stop calculation and print output(e.g. F^{best} and x^{best}).

② Else go to step (2).

3　CASE STUDY OF AN ACTUAL SLOPE

The multi-tank model is applied to the slope along Japanese national road No.9 to simulate the fluctuations of groundwater table induced by rainfall.

3.1　Outline of the slope

From the borehole survey results[13]，it is revealed that in the slope，the thickness of the weathered colluvia changes from 3 to 5 m，and its saturated permeability coefficient is 1.7×10^{-4} m/s. The other saturated permeability coefficients and tanks distribution are listed in Fig.5. With the history of collapses，it is urgent to determine the groundwater level fluctuations and to evaluate the stability of slope. Top tank is assumed on the top hill，middle tank is at the center，and bottom tank lies on the lowest part of the slope. There are two observation systems for this cut-slope at two positions，one is at the center and the other is on the upper position.

3.2　Application of multi-tank model to an actual slope

The representative slope′s cross-section is illustrated in Fig.5. As mentioned above，in order to evaluate tank parameters，historical data of rainfall and groundwater table are required. According to the field investigation，rainfall intensity and groundwater level have been recorded. In this case，two boreholes on the slope，namely，boreholes No.1 and No.2，are drilled to monitor the groundwater level at the two locations；their depths are $L = 20$ and 29 m respectively. The period of 7 days including two rainstorms is selected

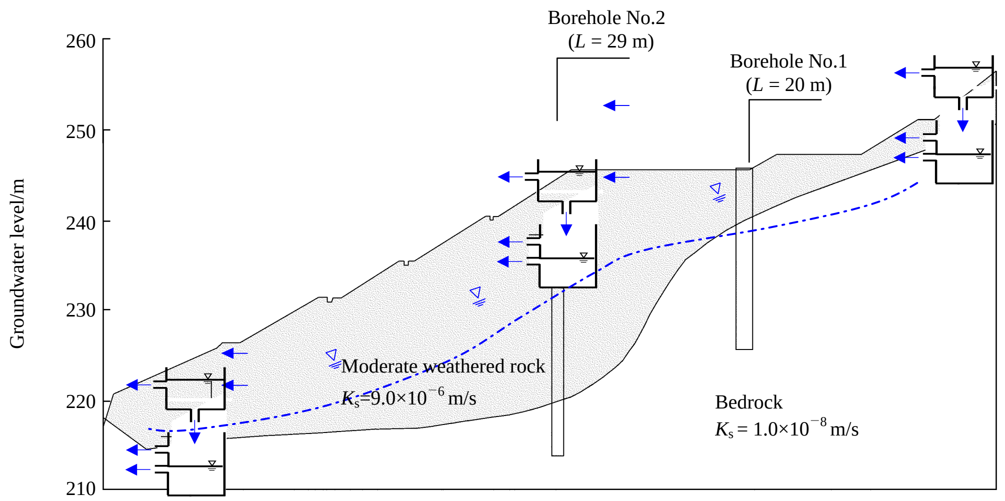


Fig.5　Configuration of multi-tank model in the slope

to demonstrate the calculation results of multi-tank model. The parameters in multi-tank model are calibrated by minimizing the difference between the observed values of groundwater table obtained from borehole No.2 and the calculated values at the middle tank. The results of borehole No.1 are just used to check rationality of results. The calibration parameters yields the identified parameters as listed in Table 1. Such parameters then can be used to estimate the groundwater table at other slopes in this study area.

Table 1　Summaries of tank model's parameters

Tank	RWL /m	Surface flow				Underground flow				
		H_{01} /mm	α_{11}	H_A /mm	β_{11}	H_{02} /mm	α_{11}	α_{12}	H_B /mm	H_C /mm
Top tank	247.0	0.000	0.100	25.00	0.100	0.000	0.160	0.010	110.0	15.00
Middle tank	232.0	0.000	0.100	20.00	0.350	0.000	0.050	0.025	200.0	10.00
Bottom tank	221.0	0.000	0.400	15.00	0.450	0.000	0.080	0.050	60.00	10.00

Note：H_{01} and H_{02} are initial water levels in each tank.

Simulation results of multi-tank model are shown in Fig.6，and the histogram on the bottom represents rainfall intensity in 2001. According to results，good agreement between the observed values(at borehole No.2) and the calculation results(in the middle tank) is obtained with $J_{XS}=0.012$；compared with peak value of rainfall，the lagged effect of groundwater level peak is also reproduced in each tank. From the results shown in Fig.6，it can be concluded that during the rainfall，

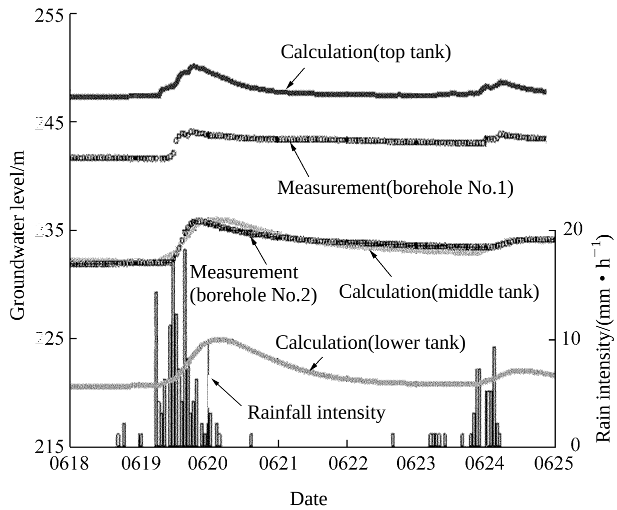


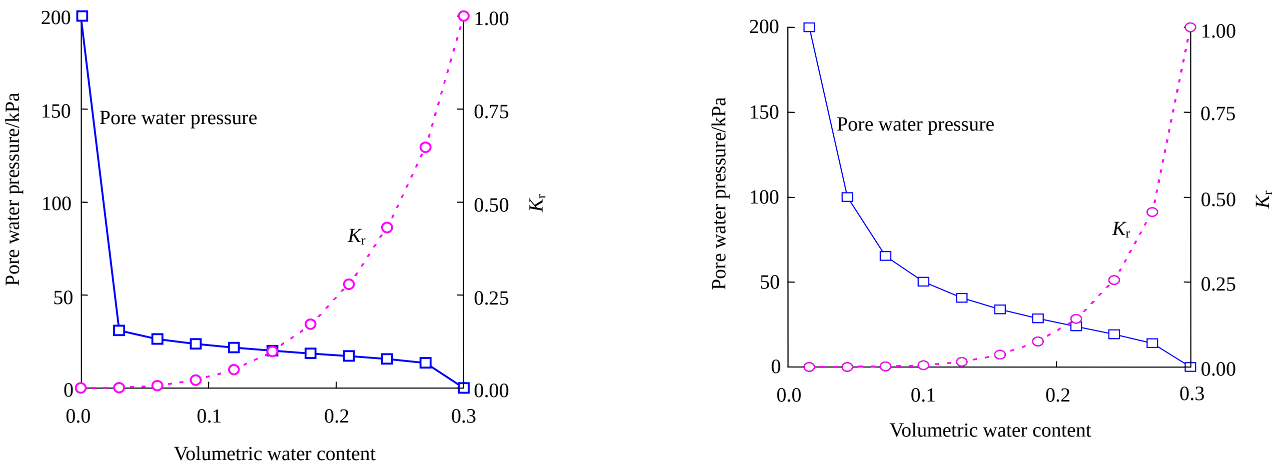
Fig.6　Comparison between measurements and calculations (in 2001)

multi-tank model with three continuous tanks can effectively simulate the transport behavior of groundwater table in the slope.

3.3　Comparison with numerical analysis

It is well known that the capacity of soil to conduct water can be viewed in terms of hydraulic conductivity(or the coefficient of permeability). So basically，for numerical analysis methods，such as finite element method or finite difference method，it is very important to consider the unsaturated characteristic of surface layer. In this context，the hydraulic conductivity is dependent on the water content. Since the water content is a function of capillary pressure and the hydraulic conductivity is a function of water content，it follows that hydraulic conductivity is also a function of capillary pressure. In order to show how the unsaturated characteristic influences the infiltration process，Fig.7 presents two types of curves to show two types of typical relationships between relative hydraulic conductivity K_r and pore water pressure (type A is assumed).

The saturated conductivities of different strata are shown in Fig.5；the relative permeability K_r of two cases are listed in Fig.7；and its saturated water content is 0.3. The results of comparison are shown in Fig.8，and the histogram on the bottom records the rainfall intensity from June 19 to July 22 in 2001. From Fig.8 it is obvious that，compared with the tank model，numerical analysis is not so good. Especially for type A，its biggest error is about 4－5 m；for type B，it also has bigger errors. That is because for finite element method，analytical results are greatly dependent on the unsaturated characteristic of layers. So if the sufficient field investigation is not done，FEM will not give enough good predictions，but it is well known that，because of budget or other reasons，in many cases it is very difficult to do many field investigations. On the other hand，results of tank model have relatively good correlation with the real field data，which shows that during rainfall infiltration，the multi-tank model has some advantages over finite element analysis in the simulation process of predicting groundwater table.



(a) Type A　　　　(b) Type B

Fig.7　Unsaturated characteristic of surface layer

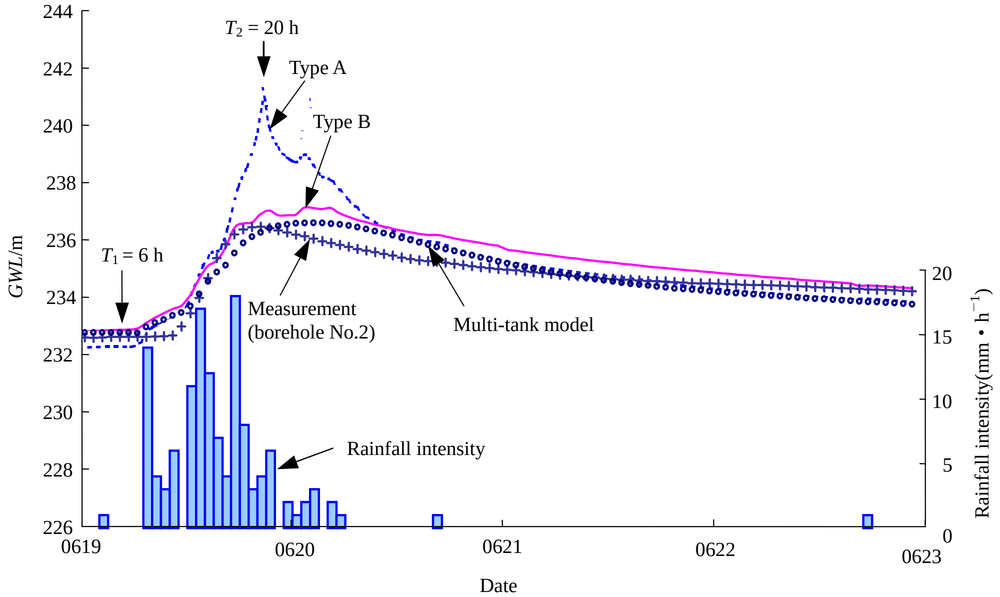


Fig.8　Results of FEM compared with multi-tank model(in 2001)

Consequently，from the comparison of the two analytical methods，the importance of the rainfall infiltration condition and the unsaturated characteristic curve to the groundwater numerical analysis(FEM or FDM) is reconfirmed. At the same time，with multi-tank model proposed by this research，there is no need of the complicate field investigations，and it has the advantage of reproducing the rainfall response(the groundwater level fluctuations) in a relatively easy way. Moreover，multi-tank model has the attributes of simulating nonlinear infiltration process in a comparative easy way；and with appropriate number of tanks，the new method can quickly obtain prediction results with the same accuracy with the traditional numerical methods，even if there are enough field investigations to determine permeability coefficients for FEM.

4 CONCLUSIONS

The paper focuses on the behavior of rainfall infiltration process and aims at developing a simple and quick analytical tool to evaluate groundwater table. The insights gained through this study are summarized as the follows：

(1) According to water balance，based on the storage characteristic of tank model，a multi-tank model that can reproduce the water movement behavior of rainfall infiltration process is developed.

(2) A new stochastic single-solution method called dynamically dimensional search is adopted and improved to identify the optimal solutions. Compared with genetic algorithm，the new method can find relatively good solutions in a shorter time.

(3) Multi-tank model is applied to an actual slope. Its consistency with field data is confirmed；and its practicability is proven. Compared with traditional numerical analysis，such as FEM or FDM，multi-tank model is not dependent on unsaturated characteristic and other geologic structures；and it can produce better results in shorter time on the basis of measurements.

(4) Although multi-tank model has the advantage of predicting water level or surface flow fluctuations，it is useful in groundwater table prediction of the slope，especially when stability analysis of slope is necessary there. But multi-tank model has a disadvantage，i.e. during the rainfall process it can not give the details of saturation degree evolution of the unsaturated zone. The authors are planning to work on the development of unsaturated tank model and even stability assessment methodology for the shallow landslide caused by rainfall. Combined with a new accurate rain gauge，the methodology will be evolved further into an assessment system for correctly predicting the hazardous amount of rainfall that may lead to slope failure.

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