Some Consideration on Elastic Pile Foundation Analysis by Poulos

Cheng Yungming

(*Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hong Kong, China*)

Abstract The elastic pile foundation analysis by Poulos is well developed and has been adopted by some countries in their design guideline. In the vertical load analysis, the author has discovered that the approach by Poulos is sensitive to the number of segments used for analysis and can lead to some unreasonable results for pile in finite depth of soil. In the lateral load analysis, the group interaction factor which is claimed to be a linear relation of the departure angle β by Poulos is found to be a linear relation of $\sin^2\beta$ by the author.

Key words pile, elastic analysis, vertical and horizontal load

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1 INTRODUCTION

Pile foundation is an extremely important element which carry vertical load, horizontal load as well as moment from super-structure. While some empirical formulae have been used to estimate the movement of single pile foundation and pile group, the use of analytical and computational techniques have become more popular in recent year. Elastic analysis based on finite difference, finite element and boundary integral methods have been used and many computer programs based on these methods are available in the commercial market. In Hong Kong, the programs by Poulos[1] and Randolph[2] are two of the commonly used computer programs. The results obtained from the two programs have been shown to be small by Randolph[2]. The elastic method by Poulos is well famous and has been adopted in the design guidelines in some countries[3]. It is also a better documented method which is known by many local engineers, hence it is chosen to study by the author. The author has discovered that there are many printing errors in the famous text 'Pile Foundation Analysis and Design'. The errors include many various printing errors in equations and some description of the equations and matrices for the sections of vertical and horizontal loads on pile.

The elastic method of pile foundation analysis is based on the Mindlin's equation. Some of the important assumptions which are used by Poulos are:

(1) The vertical or horizontal stress is constant within each segment of pile.

(2) A planar surface is used in the integration of Mindlin equation under horizontal load.

(3) For finite layer of soil, the displacement-influence factor can be obtained by the Steinbrenner approximation.

(4) For a pile group analysis subjected to a vertical or horizontal load, the stresses on each pile are assumed to be equal so that a group interaction factor can be obtained.

For the assumptions as used by Poulos, the first one appears to be reasonable as long as the length of each segment is small enough. For the second assumptions, the author views that a planar surface is not reasonable because many piles are circular in shape. Although a

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Cheng Yungming: Male, PhD, Associate Professor, Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hong Kong, China.

circular surface integral of the Mindlin equation is more complicated, the author has adopted the Gaussian numerical integration scheme using 5 to 10 integration points (the actual number depends on the choice of the user). The Gaussian integration scheme is also used by the author in vertical and lateral load analysis which can reduce the time of computation by several times as compared with the integration scheme by Poulos. For the other two assumptions by Poulos, the author will demonstrate in the next section that they may not be suitable in some cases. During the programming development, the author has found that the use of single precision computation is not adequate as the rounding-off error from the solution of equations can be very great. For pile group analysis, a negative vertical load may even come out from the analysis if single precision is used in the analysis. Furthermore, the computer time required for the analysis of horizontal load on pile group is much greater than that required for vertical load analysis. For a large pile group subjected to horizontal load, the method by Poulos needs tremendous computation time and appears to be not a practical tool to the engineers. The author is working on a combination of finite element and finite difference scheme which requires less matrix inversion to reduce the time of computation.

2 LIMITATIONS OF ELASTIC METHOD BY POULOS (VERTICAL LOAD)

The program developed by the author has been validated carefully with the design charts by Poulos and is found to be in close agreement with the results by Poulos. While Poulos's method is suitable for most practical cases, there are some cases where the input parameters can lead to unacceptable results which may not be commonly noticed. To illustrate the limitations of the Poulos's method under such cases, several cases will be examined. Case 1 is a floating pile in uniform soil where the rock stratum can be taken as infinity. The basic information for analysis are given below:

$E_p = 0.3\times10^8$ kPa $E_s = 0.3\times10^5$ kPa $\mu = 0.5$ $l = 25$ m

$R_A = 1.0$ $d_b = d = 0.5$ $P = 5\,000$ kN

These symbols are used in the text by Poulos and are self-explained. The variation of the pile top settlement and the loading at pile base for various number of segments is given in table 1.

Table 1 Influence of number of segments for floating pile for $\mu = 0.5$

No. of segments	Pile top movement/mm	Pile base load/kN
10	20.06	104.0
20	19.74	5.7
30	20.07	−223.5

It appears that the number of segment used in the analysis has a great influence on the pile load at base but a small effect on the pile top movement. However, if we change the Poisson ratio to 0.3, the results will be different(table 2).

Table 2 Influence of number of segments for floating pile for $\mu = 0.3$

No. of segments	Pile top movement/mm	Pile base load/kN
10	20.13	122.00
20	19.28	44.04
30	19.29	−3.36

When the Poisson ratio is 0.3, the effect of number of segment on the pile base load is much smaller. Actually when μ is near or equal to 0.5 or the soil is compressible, the elastic method by Poulos may not be reliable. Even when finite element is used, this problem will probably still be present. Actually, it is well known that a Poisson ratio of 0.5 or incompressible material will usually cause many computation problem. The calculated pile base load by elastic method is only a small portion of the total load (around 2%~3% in the above examples). The number of segments used in the finite difference analysis and the rounding-off error has an total influence of several percent, hence the estimation of pile base load from Figs. 5. 11 to 5. 13 of Poulos's text can be questionable. On the other hand, the design figures used for the estimation of the pile top movement can be used with confidence.

Case 2 is the effect of finite soil layer on the analysis. The soil layer is assumed to have a depth of 30, 40 and 50 m. The results are shown in tables 3 to 6.

Table 3　Influence of the finite soil layer (30 m depth, $\mu = 0.5$)

No. of segments	Pile top movement/mm	Pile base load/kN
10	8.00	−48.62
20	8.07	−198.92
30	8.12	−854.93

Table 4　Influence of the finite soil layer (40 m depth, $\mu = 0.5$)

No. of segments	Pile top movement/mm	Pile base load/kN
10	8.21	−12.91
20	88.31	−126.32
30	8.48	−529.70

Table 5　Influence of the finite soil layer (50 m depth, $\mu = 0.5$)

No. of segments	Pile top movement/mm	Pile base load/kN
10	8.31	4.14
20	8.42	−94.84
30	8.60	−420.78

Table 6　Influence of the finite soil layer (40 m depth, $\mu = 0.3$)

No. of segments	Pile top movement/mm	Pile base load/kN
10	8.15	−14.51
20	8.04	−98.42
30	8.12	−188.69

From this example, it is clear that the elastic method by Poulos can lead to negative base load which is clearly impossible. This error possibly come from the use of Steinbrenner approximation. Furthermore, it is also clear that the effects of Poisson ratio and number of segments are even more important than the effects of finite layer depth. For pile resting on a firm layer, the author has found that the variation of the base load with the number of segments is relatively small and Poulos's approach is acceptable. From the study above, it is clear that the number of segments used in the analysis and the Poisson ratio has great effects on the calculation of the pile base load. The pile top movement is however basically independent of these two factors.

3　LIMITATIONS OF THE ELASTIC METHOD BY POULOS (HORIZONTAL LOAD)

Poulos has adopted a rectangular surface for the numerical integration of the Mindlin equation. Since circular pile is more common than other shape, the author has adopted a circular surface in the integration and the forces associated with each narrow strip has to be resolved to the component parallel to the horizontal load in the integration. After such modification, the author has extended the program developed to the analysis of pile group. Based on the assumption that the horizontal stresses on each pile are the same and the pile top horizontal movement is also equal and constrained by the pile cap, some design charts for free head and fixed head conditions have been developed by the author. Fig. 1 is some typical results which are slightly different from that by Poulos. The most critical problem in these design figures is that a negative value of horizontal pile load can develop at the middle of the pile group which is definitely impossible. Furthermore, after the determination of individual horizontal pile load, the horizontal stresses on each pile calculated from the pile movement are actually different for each pile. These discrepancies are the limitations of the method by Poulos. Similar problems actually also exist for the case of vertical load analysis. Furthermore, Poulos has claimed that the interaction factor is a linear function of the departure angle β. The author has however found that the interaction factor is actually a linear function of the $\sin^2\beta$ which is shown in Fig.2. From the results obtained, it is clear that the relation of the interaction factor with departure angle has to be used with caution.

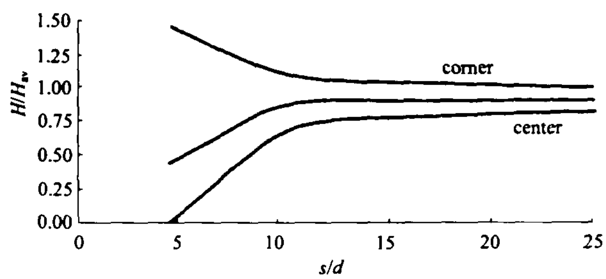

Fig.1　Horizontal loads distribution in free head floating pile group (3×3)

4　FINAL CONCLUSION AND DISCUSSION

Since the author has found that the truncation error from Gaussian elimination for this problem is very high,

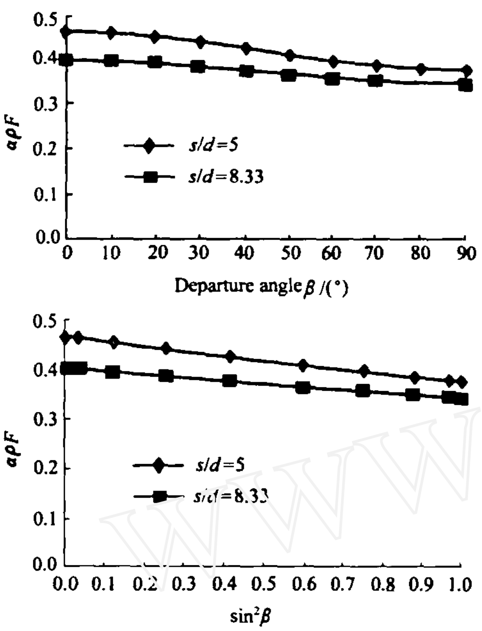

Fig. 2 Relation of interaction factor with departure angle

the author has adopted both double precision calculation as well as full-pivoting scheme in order to reduce the truncation error to a minimum. The strange behaviour as mentioned above is hence not related to the truncation error but is a fundamental feature of Poulos method. Poulos's approach requires double precision to achieve the required accuracy or else the results of analysis may have great error.

From the above study, it is clear that the use of elastic method suffer from the limitation that the results (in particular the pile base load) are sensitive to the number of segments, Poisson ratio and finite depth of soil used for analysis. Actually the author notices that there is a large fluctuation of shear stress at the base of the pile and negative skin friction is actually obtained in some special cases. This is probably due to the generation of tensile strain from pile tip movement. Under the finite difference formulation, it is very difficult to eliminate such problem and finite element may be a better solution in this respect. Since the pile settlement is largely controlled by the top portion of the pile unless the pile is very short, the pile settlement is hence not sensitive to the number of segments used for analysis and can be used with confidence in design. In general, the user can use either the design charts by Poulos or computer programs for design and the accuracy is good enough for any number of segments used. However, the user should not expect the design charts or the program to give an accurate prediction of the pile base load.

The assumption of equal stress distribution between piles in a pile group for vertical as well as horizontal loads analysis is approximate only and inconsistent results are significant if the pile spacing is close. The design charts as developed by Poulos have to be used in caution and good prediction should not be expected for piles spaced close together.

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《岩石力学与工程学报》1999 年的即年指标及其在全国科技期刊中的排名

一种期刊某年的即年指标是指当年发表在该刊上的论文被同年其他刊物（称为统计用期刊或来源期刊，也包括该刊在内）所引用的次数与该刊当年发表的论文总数之比。此值越高，表明该刊论文的被引用速度越快，因而是评价刊物学术质量和学术水平的一个重要指标。因为那些让人先睹为快的刊物，显然也在一定程度上反映了刊物的质量和水平。

2001 年 1 月在中国科技信息研究所出版的《中国科技期刊引证报告》(扩刊版)中，公布了约占全国自然科学学术类和技术类 90%的 2 804 种科技期刊 1999 年的即年指标，《岩石力学与工程学报》的这一指标值为 0.256，位居当年 2 840 种刊物的第 13 名和 12 种“力学”类专业期刊的首位。

(范文田供稿)