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TEMPERATURE DEPENDENT FLEXURE OF THE BELGRADE VERTICAL CIRCLE

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SUMMARY: In this paper it is demonstrated that a significant fraction of the random error of a single declination determination (ε_δ), as well as of the seasonal error ($\Delta\delta_\alpha$) in declination, has its origin in the temperature dependent variability of the residual flexure of the Belgrade-Vertical-Circle. The temperature coefficient of flexure is equal to $0''.054/^\circ$C.

1. INTRODUCTION

As well known, with the majority of the classical vertical circles the absolute declinations have been obtained with significant systematic errors of the types $\Delta\delta_\alpha$ and $\Delta\delta_\delta$ (for more details e. g. Zverev, 1950). In this paper the main attention is paid to the disclosing of the basic cause of significant systematic deviations of the $\Delta\delta_\alpha$ type in the case of the Belgrade Vertical Circle (BVC). This time in order to obtain completely independent results we use only the observations performed with this instrument, in distinction from previous analysis (Bozhichkovich, 1991). Namely, then the author analysed the difference $\delta_{BVC} - \delta_{FK4}$ of preliminary mean declinations of 212 fundamental stars (to which 52 at lower culmination are added making a total of 264) obtained by his own observations and declinations taken from the FK4. This previous analysis indicated, among others, also a significant dependence of the BVC residual flexure on the temperature (coefficient is about $0''.04/^\circ$C). In addition, the reason for a new approach to this problem is contained in the fact that 157 series of horizontal-flexure- component determinations with the collimators performed during 113 nights, when the observations of stars were also carried out, yield relatively small values for the BVC flexure (mean value, $\Sigma b_c/157 = 0''.20 \pm 0''.34/\sqrt{157}$) showing practically no dependence on the temperature.

2. THE STATISTICS

In order to understand the necessity of this analysis, the reality of its results and the effect of the obtained results on the final declination values we present Table I, Table II, Table III and Table IV, which contain the statistics of the quantities used in the analysis.

The first column of Table I contains the scale, in arcsec, of the deviations ($\Delta\delta_{ij} = \delta_{ij} - \delta_j$) between the measured and mean declination of the same star j.

In the second column we present the number of observations ($N(\Delta\delta_{ij})$) deviating by $\Delta\delta_{ij}$ from the mean value. As may be seen, all the deviations analysed here (there are 2143 of them) are within the limits of $\pm 2''.1$, whereas 96% of them are within $\pm 1''.0$.

The third column ($N(\Delta\delta_{ij})'$) contains the number of the homonymous deviations, but now corrected by the results of the present analysis. A cursory look shows that the general concentration about zero is increased with respect to column 2. On the average practically all deviations, both large and small, are somewhat reduced.

Table I. Distribution of the deviations $\Delta\delta_{ij}$ obtained in the analysis, depending on their amount.

(1) $\Delta\delta_{ij}$	(2) $N(\Delta\delta_{ij})$	(3) $N(\Delta\delta_{ij})'$
$-2''.05$	1	0
−1.95	1	2
−1.85	1	1
−1.75	3	0
−1.65	5	4
−1.55	0	1
−1.45	0	0
−1.35	3	1
−1.25	5	5
−1.15	8	4
−1.05	15	5
−0.95	20	11
−0.85	21	15
−0.75	32	37
−0.65	49	51
−0.55	87	71
−0.45	102	110
−0.35	129	150
−0.25	179	170
−0.15	210	202
−0.05	220	245
+0.05	190	211
+0.15	201	215
+0.25	163	182
+0.35	150	135
+0.45	108	95
+0.55	62	74
+0.65	42	44
+0.75	34	38
+0.85	30	21
+0.95	33	14
+1.05	15	12
+1.15	4	6
+1.25	2	4
+1.35	8	3
+1.45	6	0
+1.55	2	3
+1.65	1	1
+1.75	0	0
+1.85	0	0
+1.95	1	0
+2.05	0	0

Table II. Statistical survey of quantities involving temperature.

(1) °C	(2) $N(t_{ij})$	(3) $N(t_{ij}-t_j)$	(4) $N(t_j)$
−17.5	0	1	0
−16.5	0	1	0
−15.5	0	1	0
−14.5	0	3	0
−13.5	0	13	0
−12.5	5	19	0
−11.5	23	20	0
−10.5	0	19	0
−9.5	0	15	0
−8.5	0	28	0
−7.5	0	55	0
−6.5	0	39	0
−5.5	3	64	0
−4.5	14	84	2
−3.5	26	133	10
−2.5	42	131	2
−1.5	31	183	1
−0.5	73	175	13
+0.5	84	242	3
+1.5	62	186	6
+2.5	16	204	10
+3.5	18	148	6
+4.5	10	125	2
+5.5	18	95	3
+6.5	23	55	19
+7.5	12	39	17
+8.5	41	26	4
+9.5	69	14	2
+10.5	85	2	6
+11.5	81	11	33
+12.5	87	5	5
+13.5	145	0	20
+14.5	191	4	20
+15.5	141	1	11
+16.5	167	1	15
+17.5	170	1	23
+18.5	134	0	25
+19.5	137	0	6
+20.5	108	0	0
+21.5	61	0	0
+22.5	33	0	0
+23.5	19	0	0
+24.5	9	0	0
+25.5	4	0	0
+26.5	1	0	0

The first column of Table II contains the scale, in °C, of temperature values involving t_{ij} and t_j and temperature difference $(t_{ij}-t_j)$, t_{ij} being the measured and t_j mean temperature.

In the second column ($N(t_{ij})$) we present the number of observations at given temperatures t_{ij}. As may be seen, throughout these observations the temperature ran within a very wide range of nearly 40°C. During the execution of this small programme it was intended to achieve this range to be as large as possible for this research to be successful, since on these latitudes it is almost impossible to have a range narrower than 30°C, so that the knowledge of temperature dependent characteristics of the instrument, perforce necessary for any serious work, may be assured. Besides, an increased number of observations at relatively high temperatures is noticeable. Namely, in 1983 and 1984 a relatively uniform distribution of the observations over the seasons is characteristic and this would have yielded their relatively uniform distribution also of the temperatures if the programme, first conceived as a biannual one, because of missing of several winter observations had not been prolonged also into 1985. Due to the observing of the outer planets in that year the number of observations was significantly increased in the summer-autumn period.

The third column contains the number of observations with the difference between the measured t_{ij} and the mean temperatures t_j ($N(t_{ij} - t_j)$), related to the same star. As seen, the differences are within a very wide range (±18°C) but with a relatively symmetric concentration about zero. This will provide, as will be seen later on, high certainty in the temperature-coefficient determination for the flexure of BVC.

In the fourth column is presented the number of stars ($N(t_j)$) at whose transits mean temperature measured inside the pavilion is t_j ($\Sigma t_j/264 = +10.61$°C). As evident the t_j are within the range of 24°C whereby, as in column 2, the additional observations from 1985 violated the relatively simmetric temperature distribution.

Table III. Distribution of alidad axis inclination ($N(I_{ij})$) depending on its amount (I_{ij})

(1) I_{ij}	(2) $N(I_{ij})$
$-6''.5$	0
-5.5	3
-4.5	32
-3.5	93
-2.5	183
-1.5	279
-0.5	412
$+0.5$	529
$+1.5$	381
$+2.5$	147
$+3.5$	61
$+4.5$	19
$+5.5$	4
$+6.5$	0

The first column of Table III contains the scale, in arcsec, of values of half the inclination difference of the alidade axis in the two (E, W) positions of BVC necessary in the zenith-distance determination of particular star (hereafter inclination only) according to the readings of the two BVC levels.

The second column contains the number of observation $N(I_{ij})$ according to inclination I_{ij}. It can be seen that all the inclinations are within the limits of $\pm 6''$ with a relatively symmetric concentration with respect to zero and it seems that they are sufficient to detect any inaccuracy in the applied division values of the two levels of BVC.

The first column of Table IV contains the scale, in arcsec, of values of the refraction corrections ($\Delta\rho_{ij}$), if the transition from the outer temperature to the one inside the pavilion is desired.

In the second column is presented the number of observations ($N_\delta(\Delta\rho_{ij})$) in which the correction to be added to the observed declinations (δ_{ij}) for the refraction calculated according to the inner temperature, has a value $\Delta\rho_{ij}$. As may be seen for a vast majority of observations this correction is within the limits of $\pm 0''.5$ and for more than half of them it is less than $\pm 0''.1$. One should bear in mind that its amount depends on both the difference between the temperatures inside and outside the pavilion and approximately on the tan of the zenith-distance ($Z_{ij} = -75°$ up to $+75°$).

In order to make the things more clear in the third column is given the number of observations ($N_Z(\Delta\rho_{ij})$) with the values of the corrections $\Delta\rho_{ij}$, but this time added to the measured zenith distance, now always positive in the calculation of this correction ($Z_{ij} = 0°$ up to 75°). As seen, this time too the concentration with respect to zero is present, but the positive values prevail, indicating that for the majority of observations the inner temperature was lower than the outer one. The reason for this is, as mentioned above, the prominent increase of the observations in the period summer-autumn 1985, these having been largely performed in the beginning of the evenings (when the outer temperature exceeds the inner one) because of observing the outer planets, unlike the winter-spring period when, on the average, we have a converse situation. About twenty observations with prominent $\Delta\rho_{ij}$ correspond to stars with high zenith distances, most often observed before the others during the same evening, sometimes early in the twilight, in a few cases even before the sunset when high temperature differences of 2 to 3°C (inside and outside the pavilion) are possible.

Yet, as may be seen from columns 2 and 3 of Table IV, above all due to the inadequate construction of the BVC pavilion, there is a systematic difference of temperatures inside ($\Sigma t_{ij}/2143 = +12.51$°C) and outside ($\Sigma t_{ij}/2143 = +12.78$°C) the pavilion. These differences lead to two conspicuously different declination systems following from the same observations. In spite of the existing asymmetry in the seasonal distribution of the observations we try in this analysis to solve this problem and the dilemma – which temperature yields the best representation of the field in and around the BVC in the refraction calculation.

Table IV. Distribution of the differences of the refraction $\Delta\rho_{ij} = (\rho(t_{in}) - \rho(t_{ex}))_{ij}$ used in the analysis, depending on their amount.

(1) $\Delta\varrho_{ij}$	(2) $N_\delta(\Delta\varrho_{ij})$	(3) $N_Z(\Delta\varrho_{ij})$
$-2''.05$	0	0
−1.95	0	0
−1.85	1	0
−1.75	0	0
−1.65	0	0
−1.55	1	0
−1.45	0	0
−1.35	0	0
−1.25	0	0
−1.15	3	0
−1.05	0	0
−0.95	7	0
−0.85	4	1
−0.75	4	0
−0.65	7	2
−0.55	19	2
−0.45	24	5
−0.35	66	19
−0.25	99	50
−0.15	207	116
−0.05	761	568
+0.05	525	718
+0.15	227	318
+0.25	82	131
+0.35	56	103
+0.45	16	35
+0.55	16	33
+0.65	4	9
+0.75	6	10
+0.85	3	6
+0.95	2	9
+1.05	0	0
+1.15	1	4
+1.25	1	1
+1.35	0	0
+1.45	0	0
+1.55	0	1
+1.65	0	0
+1.75	0	0
+1.85	0	1
+1.95	1	1
+2.05	0	0

3. THE APPLIED PROCEDURE

It has already been mentioned several times that in the present analysis an attempt will be made to determine the temperature coefficient of the flexure, a possible inaccuracy in the division values of the two BVC levels, as well as the temperature with which the refraction should be calculated. Due in the first place to the fact that all observations were performed within a relatively short interval, of two years only, the effect of any possible inaccuracies of the astronomical constants is assumed as unessential (precession, nutation, annual aberration, as well as inaccuracies in the proper motions of observed fundamental stars). In addition, since in the basic reduction we use the mean-latitude corrections ($\Delta\varphi_{BIH}$), it is assumed that the possible local so-called z term is relatively small. Disregarding the fact that this assumption has been substantiated in the polhody examinations carried out for many years by the Belgrade Latitude Service, the present analysis attempts to correlate the latitude corrections ($\Delta\varphi_{BIH_i}$) and the declination deviations ($\Delta\delta_{ij}$). Due to the short duration of the considered period and the relatively small number of observations of the same stars in different seasons and since relatively small values obtained with low accuracy had no essential effect on the other results of the analysis, the above assumption seems realistic. Nevertheless, in the future one should devote an adequate attention to this important question.

In accordance with all the above said in this paper, by using the least-square method, the following relation has been solved:

$$\begin{aligned}\pm\Delta\delta_{ij} =& \Delta b(t_{ij} - t_j)\sin Z_{ij} + \Delta\lambda(I_{ij} - I_j) + \\ & \kappa(\Delta\rho_{ij} - \Delta\rho_j)\end{aligned} \tag{1}$$

where, in addition to the quantities used earlier, appear also the following ones: (the sign "minus" is taken for the lower-culmination observations.)
$\pm\Delta\delta_{ij} = \pm(\delta_{ij} - \delta_j)$; δ_{ij} – declination from i-th observation ($i = 1, 2, ..., n_j$; n_j – total number of observations for j-th star, n_j is not less than 2 and is not greater than 34); δ_j – mean declination from n_j observations of j-th star ($j = 1, 2, ..., 264$); Δb – temperature flexure coefficient here searched for; t_{ij} – temperature inside the pavilion during i-th observation of j-th star; t_j – mean temperature for n_j observations of j-th star; Z_{ij} – measured zenith distance and its $\sin Z_{ij}$ practically constant for a given star in all its observations (southern stars assumed to have negative zenith distances); $\Delta\lambda$ – possible common error in the adopted division values of the two BVC levels; I_{ij} – measured inclination, expressed in division values obtained from the two levels; I_j – mean inclination of BVC following from all observations of a given star; κ – unknown coefficient whose value removes the dilemma which temperature to use in the refraction calculation, outer ($\kappa = 0$), inner ($\kappa = -1$) or, proportionally to the coefficient, intermediate one; $\Delta\rho_{ij} = (\rho(t_{in}) - \rho(t_{ex}))_{ij}$ – the difference of the refraction values calculated with the inner temperature (t_{in}) and the outer one (t_{ex}), i. e. this is a correction to be introduced in order to calculate the refraction with the inner temperature since at deriving the preliminary declinations use has been made of the outer one; $\Delta\rho_j$ – mean value of the refraction corrections applied to the j-th star.

4. ANALYSIS

The 2143 equations of condition of the form (1) are solved by the least-square method in two versions – once with (version(a)), and once without (version (b)) introducing the values obtained with the collimators for the flexure of BVC. The obtained results with their errors are presented in Table V.

As evident from Table V, considering the mean errors, all the values are relatively reliably determined. In addition one can conclude the following.

1) These values of the temperature flexure coefficient (Δb) of BVC obtained independently, though somewhat higher than those following from the comparison with $FK4$, seem to us fully realistic and applicable in the final reductions. With regard to the amount of the temperature fluctuation (column 3 of Table II) both at different observations of the same star and of the mean temperatures (column 4, the same Table) from star to star, the first term in relation (1) is dominant and largely to it a significant improvement in the accuracy is due both in respect to the random (for more details see below) and the systematic effects. Besides, though the slightly higher coefficient in the version (a) ($0\overset{''}{.}057$ compared to $0\overset{''}{.}054$), when the flexure obtained with collimators was included in the measurement on the same evenings of the observations, questions the justifiability of determining the flexure regularly conjointly with the observation (generally poor efficiency of visual pbservations becomes even poorer). The probable reason why the flexure determined with collimators is not realistic is in an inadequate construction of the BVC pavilion and in the disposition of the prime-vertical niches. The difficult question concerning the real physical cause (refraction inside the tube of the instrument, flexure of the declination circle, etc) of the presence of such a high tempperature flexure coefficient in the observed declinations, which practically cannot be determined using laboratory technique with the collimators according to Bessel's method, will remain unanswered again.
2) Relatively small values ($-0\overset{''}{.}034$ and $-0\overset{''}{.}028$) are obtained for the total error of the divisions ($\Delta\lambda$) of both BVC levels. These values indicate, above all, that the procedure of the level- division determination applied earlier, with BVC in working state and with mercury mirrors at nadir yields very realistic results (Bozhichkovich, 1986). Here it should be mentioned that in order to verify if a part of the temperature dependence of the results is perhaps also due to the levels, in (1) is introduced an additional term, corresponding to the supposed influence. Since the obtained values are very small, without any essential effect on the other results, or on the determination errors (ε_{ij}), they will not be given here, nor their term. In view of the statistics of the measured inclinations given in the second column of Table IV, the obtained results have a significant effect (maximum values about $\pm 0\overset{''}{.}20$) on a relatively small number of observations. Although in some observational nights a higher inclination was deliberately tolerated, a continuous effort was done to achieve a) mean inclinations related to individual stars and b) throughout catalogue, to be close to zero. The second striving was realised completely, unlike the first where we were successful only partly. Namely, due to the existence of a relatively slight instability of BVC (whose source and nature are not completely clear), the measured inclinations (consequently also the mean ones) are always somewhat higher for northern stars (approximately up to $+1''$) than for southern ones (approximately down to $-1''$). As evident, such a BVC characteristic suggests that, because of removing of the systematic effect within limits of $\pm 0\overset{''}{.}04$ (true, for a few stars the mean inclination attains values of $+1\overset{''}{.}5$ or $-1\overset{''}{.}5$), the values obtained above, though relatively small, nevertheless should be applied in the final treatment.
3) A difference appears in the temperature measured inside the pavilion and outside it, in the meteorological shelter. It is a relatively frequent occurence and can be significant (up to 2°C and more). For this reason, and also generally, a question arises which temperature yields the best representation of the conditions of observation. According to Table V, $\kappa = -0.40; -0.30$, it follows that this is a temperature between the inner and the outer ones, but, still, somewhat closer to the outer one. It is curious to note that the applied flexure, obtained inside the pavilion with the collimators, shifts this result slightly closer to the inner temperature ($\kappa = -0.40$).

Table V. Results of the $\Delta\delta_{ij}$ analysis. With (version (a)), and without (version (b)),the flexure values obtained with collimators.

(1) case	(2) Δb	(3) $\Delta\lambda$	(4) κ	(5) ε_{ij}
a	$(0\overset{''}{.}057 \pm 0\overset{''}{.}0028)/°C$	$-0\overset{''}{.}034 \pm 0\overset{''}{.}0051$	-0.40 ± 0.055	$\pm 0\overset{''}{.}45$
b	$(0\overset{''}{.}054 \pm 0\overset{''}{.}0026)/°C$	$-0\overset{''}{.}028 \pm 0\overset{''}{.}0047$	-0.30 ± 0.051	$\pm 0\overset{''}{.}42$

5. CONCLUSION

In order to illustrate the effect of the quantities considered above on the mean random error of a single declination determination with BVC (ε_j) without introducing weights, by the least-square method the usual relation (2) has been solved for both cases (with (a) and without (b) flexure values) using 264 ε_{ij} values before and after (a' and b') introducing the above results.

$$(\varepsilon_j)^2 = (\varepsilon_0)^2 + (\varepsilon_1)^2(\tan Z_j)^2. \qquad (2)$$

All the four solutions of (2) with their r.m.s. errors are given in Table VI.

Table VI. The solutions ε_j by (2) applying flexure (a) and the values from Table V (a') and solutions ε_j without applying flexure (b), but using the values from Table V (b').

(1) case	(2) ε_0	(3) ε_1	(4) ε'
a	$0''.41 \pm 0''.13$	$0''.22 \pm 0''.07$	$\pm 0''.48$
a'	0.36 ± 0.11	0.19 ± 0.06	± 0.41
b	0.39 ± 0.13	0.20 ± 0.07	± 0.47
b'	0.34 ± 0.11	0.18 ± 0.06	± 0.40

As evident from Table VI (columns 2 and 4), the calculated solutions of (1) improve the accuracy of BVC observations significantly. From the third coluumn we see that the improvement in the second coefficient of (2), characterising the decrease in the accuracy of the observations with the increasing zenith distance, exists, but it is insignificant. This is understandable because the amount of this term is mostly dependent on atmospheric scintillation.

It should be emphasized here that in the present investigation no observation is rejected though about 15 at them it fully deserved. This becomes especially clear after applying the results of this analysis. However, since their rejecting had practically no effect on the results of (1), this time we abstained from doing this, though in the final treatment after applying these results, as well as a possible $E-W$ effect, this certainly should be done.

On the basis of all said above it is real to expect the final mean accuracy of the future catalogue to be even somewhat better, though it is already now very satisfactory for this instrument type with visual observing. Perhaps even more important is the fact that the situation from the systematic point of view too will significantly be improved.

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ТЕМПЕРАТУРСКА ЗАВИСНОСТ САВИЈАЊА БЕОГРАДСКОГ ВЕРТИКАЛНОГ КРУГА

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У овом раду се показује да знатан део случајне грешке једног одређивања деклинације (ε_δ), као и сезонске грешке у деклинацијама ($\Delta\delta_\alpha$), потиче од температурске променљивости необрачунатог савијања код Београдског вертикалног круга. Температурски коефицијент савијања износи $0''.054/°$C.