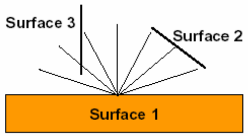
Chapter 9 Radiation Heat Transfer

9-6 THE VIEW FACTOR

1. Definition of View Factor

Radiation heat exchange between surfaces depends on the *orientation* of the surfaces relative to each other, which is accounted for by the *view factor* (*shape factor, configuration factor, and angle factor*) .



Assumption:

- Surfaces are diffuse emitters and reflectors.
- Surfaces are isothermal.
- Surfaces are separated by a nonparticipating medium such as a vacuum or air.

View factor from a surface *i* to a surface *j* ($F_{i\to j}$ or F_{ij}):

The fraction of the radiation leaving surface i that strikes surface j directly

$$F_{i\to j} = \frac{\dot{Q}_{i\to j}}{\dot{Q}_i}$$

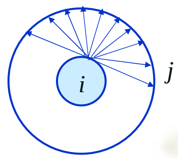
Similarly, **View factor** from a surface *j* to a surface *i* ($F_{j\to i}$ or F_{ji}):

$$F_{i\to j} = \frac{\dot{Q}_{i\to j}}{\dot{Q}_i}$$

View factor is a purely geometric quantity and it is independent on surfaces' temperature and radiation properties.

$F_{i\to j} = 0$ means that two surfaces do not see each other.

$F_{i\to j} = 1$ means that surface *j* completely surrounds surface *i*



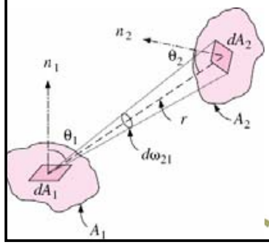
2. View Factor Integral

(1) Differential view factor from dA_1 to dA_2

$$\dot{Q}_{dA_1 \to dA_2} = I_1 \cos\theta_1 dA_1 d\omega_{21} = I_1 \cos\theta_1 dA_1 \frac{dA_2 \cdot \cos\theta_2}{r^2}$$

$$\dot{Q}_{dA_1} = J_1 dA_1 = \pi I_1 dA_1$$

$$\Rightarrow \quad dF_{dA_1 \to dA_2} = \frac{\dot{Q}_{dA_1 \to dA_2}}{\dot{Q}_{dA_1}} = \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2$$



Similarly,

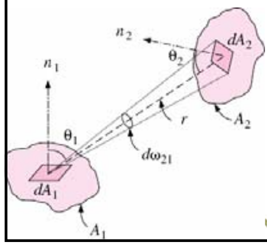
$$dF_{dA_2 \to dA_1} = \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1$$

(2) View factor from dA_1 to A_2

$$F_{dA_1 \to A_2} = \int_{A_2} dF_{dA_1 \to dA_2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2$$

Similarly,

$$F_{dA_2 \to A_1} = \int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1$$

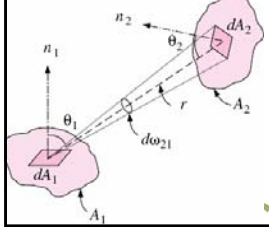


(3) View factor from A_1 to A_2

$$F_{12} = \frac{\dot{Q}_{A_1 \to A_2}}{\dot{Q}_{A_1}} = \frac{\int_{A_1}\int_{A_2} \dot{Q}_{dA_1 \to dA_2}}{\int_{A_1} \dot{Q}_{dA_1}} = \frac{\int_{A_1}\int_{A_2} \frac{I_1 \cos\theta_1 \cos\theta_2}{r^2} dA_1 dA_2}{\int_{A_1} \pi I_1 dA_1}$$

$$= \frac{\int_{A_1}\int_{A_2} I_1 \cos\theta_1 \cos\theta_2 dA_1 dA_2}{A_1 \pi I_1 r^2} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_2}{\pi r^2} dA_1$$

$$= \frac{1}{A}\int_{A_1}\int_{A_2} F_{dA_1 \to dA_2} dA_1$$



So,

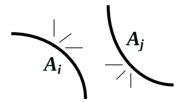
$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$

Similarly,

$$F_{21} = \frac{1}{A_2}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$

3. View Factor Relations

(1) Reciprocity Rule



$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$

$$F_{21} = \frac{1}{A_2}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$

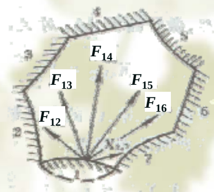
$$\Rightarrow \quad A_i F_{i \to j} = A_j F_{j \to i}$$

(2) Summation Rule

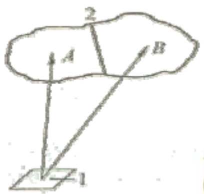
e.g. For a *n*-surface enclosure, *energy balance* gives

$$\dot{Q}_{1\to 1} + \dot{Q}_{1\to 2} + \dot{Q}_{1\to 3} + \cdots + \dot{Q}_{1\to n} = \sum_{i=1}^{n} \dot{Q}_{1\to i} = \dot{Q}_1$$

$$F_{1\to 1} + F_{1\to 2} + F_{1\to 3} + \cdots + F_{1\to n} = \sum_{i=1}^{n} F_{1\to i} = 1$$



(3) Superposition Rule



The view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j .

$$F_{1\to 2} = \sum_{i=1}^{n} F_{1\to 2i}$$

$$\dot{Q}_{1\to 2} = \dot{Q}_{1\to 2A} + \dot{Q}_{1\to 2B}$$

$$\Rightarrow A_1 E_{b1} F_{1\to 2} = A_1 E_{b1} F_{1\to 2A} + A_1 E_{b1} F_{1\to 2B}$$

$$\Rightarrow F_{1\to 2} = F_{1\to 2A} + F_{1\to 2B} \quad \text{or} \quad A_1 F_{1\to 2} = A_1 F_{1\to 2A} + A_1 F_{1\to 2B}$$

Note: $F_{2\to 1} = F_{2A\to 1} + F_{2B\to 1}$?

$$\dot{Q}_{2\to 1} = \dot{Q}_{2A\to 1} + \dot{Q}_{2B\to 1}$$

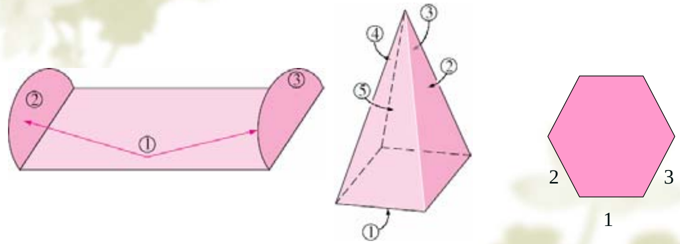
$$\Rightarrow A_2 E_{b2} F_{2\to 1} = A_{2A} E_{b2} F_{2A\to 1} + A_{2B} E_{b2} F_{2B\to 1}$$

$$\Rightarrow A_2 F_{2\to 1} = A_{2A} F_{2A\to 1} + A_{2B} F_{2B\to 1}$$

Reciprocity Rule

(4) Symmetry Rule

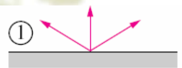
e.g. *Surfaces are symmetric about the surface* 1.



$F_{1\to 2} = F_{1\to 3}$	$F_{1\to 2} = F_{1\to 3} = F_{1\to 4} = F_{1\to 5}$	$F_{1\to 2} = F_{1\to 3}$
$F_{2\to 1} = F_{3\to 1}$	$F_{2\to 1} = F_{3\to 1} = F_{4\to 1} = F_{5\to 1}$	$F_{2\to 1} = F_{3\to 1}$

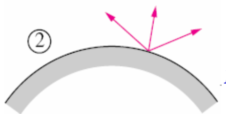
➢ **Special case:** **View factor from a surface to itself** $F_{i\to i}$

Surface cannot "sees" itself.



$F_{1\to 1} = 0$

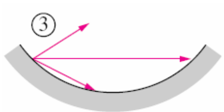
(*a*) Plane surface



$F_{2\to 2} = 0$

(*b*) Convex surface

Surface "sees" itself.



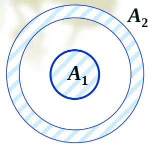
$F_{3\to 3} \neq 0$

(*c*) Concave surface

4. View Factors for Enclosures

❑ **2-surface enclosure: (total 4 view factors)**

i)

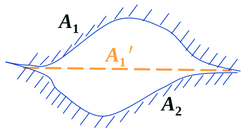


$F_{11} + F_{12} = 1$
$F_{21} + F_{22} = 1$
$A_1 F_{12} = A_2 F_{21}$
$F_{11} = 0$

⇒

$F_{11} = 0$
$F_{12} = 1$
$F_{21} = A_1/A_2$
$F_{22} = 1 - A_1/A_2$

ii)



Area A_1' is known.

$F_{11} \neq 0$
$F_{1'1'} = 0$
$F_{12} = F_{11'}$
$F_{21} = F_{21'}$

⇒

$F_{11} = 1 - A_1'/A_1$
$F_{12} = A_1'/A_1$
$F_{21} = A_1'/A_2$
$F_{22} = 1 - A_1'/A_2$

❑ 3-surface enclosure: (total 9 view factors)

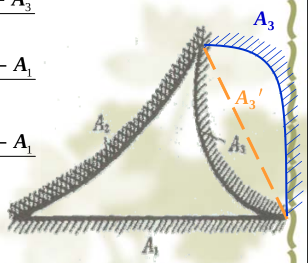
$$F_{11}+F_{12}+F_{13}=1 \qquad A_1F_{12}=A_2F_{21} \qquad F_{11}=0$$
$$F_{21}+F_{22}+F_{23}=1 \qquad A_1F_{13}=A_3F_{31} \qquad F_{22}=0$$
$$F_{31}+F_{32}+F_{33}=1 \qquad A_2F_{23}=A_3F_{32} \qquad F_{33}=0$$

⇩

$$F_{12}=\frac{A_1+A_2-A_3'}{2A_1} \qquad F_{21}=\frac{A_2+A_1-A_3'}{2A_2}$$
$$F_{13}=\frac{A_1+A_3'-A_2}{2A_1} \qquad F_{23}=\frac{A_2+A_3'-A_1}{2A_2}$$
$$F_{31}=\frac{A_3'+A_1-A_2}{2A_3} \qquad F_{32}=\frac{A_3'+A_2-A_1}{2A_3}$$

Corresponding widths L are known, then

$$F_{12}=\frac{L_1+L_2-L_3}{2L_1} \quad \ldots\ldots$$

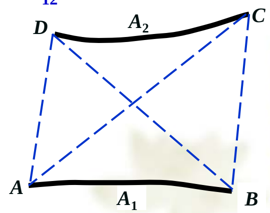


5. View Factors for Open Systems

❑ Between 2 Infinitely Long Surfaces F_{12}

$$F_{12}=1-F_{11}-F_{AB,AD}-F_{AB,BC}$$

$$\begin{cases} F_{11}=0 \\ F_{AB,AD}=\dfrac{AB+AD-BD}{2AB} \\ F_{AB,BC}=\dfrac{AB+BC-AC}{2AB} \end{cases}$$



Crossed-Strings Method

$$F_{12}=\frac{(AC+BD)-(AD+BC)}{2AB}$$

$$F_{ij}=\frac{\sum(\text{Crossed strings})-\sum(\text{Uncrossed strings})}{2\times(\text{String on surface } i)}$$

❑ Between 2 finite Surfaces F_{12}

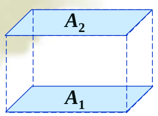


FIGURE 9-31

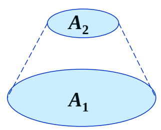


FIGURE 9-33

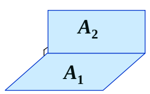


FIGURE 9-32

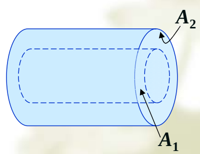


FIGURE 9-34

e.g. Calculate view factor F_{13}

$$F_{1,3}=F_{1,(2+3)}-F_{1,2}$$

FIGURE 9-32