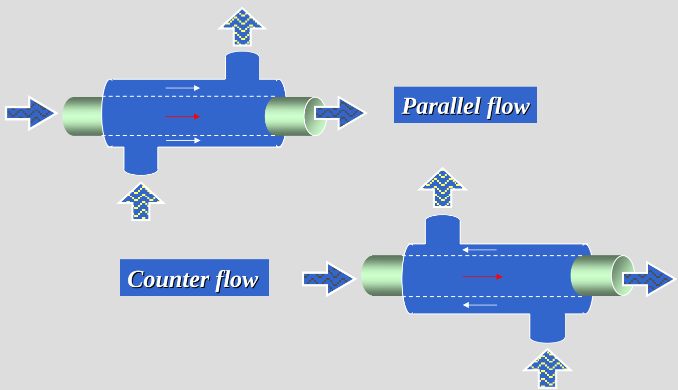
CHAPTER 10

Heat Exchanger

A **heat exchanger** allows heat to be transferred between two fluids without the fluids contacting each other.

§ 12-1 TYPES OF HEAT EXCHANGERS

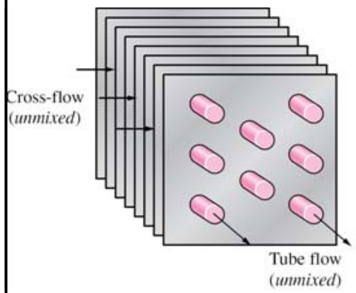
1. **double-pipe** heat exchanger



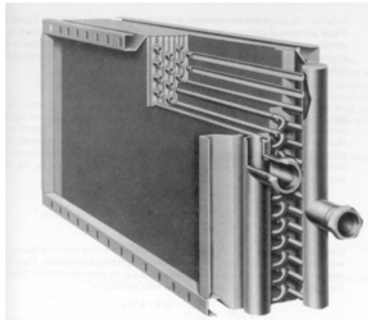
2. **Compact** heat exchanger

area density $\beta > 700\ m^2/m^3$

The ratio of the heat transfer surface area to its volume



Very large heat transfer surface area per unit volume !!!



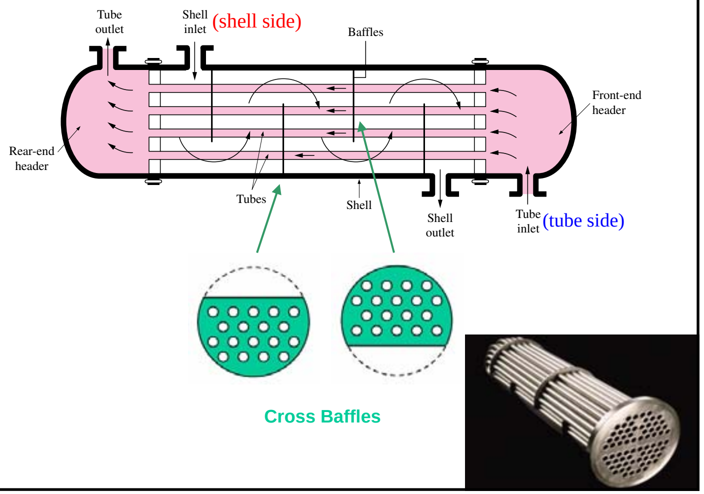
e.g. Car radiator

Plate and Fin / Compact heat exchanger

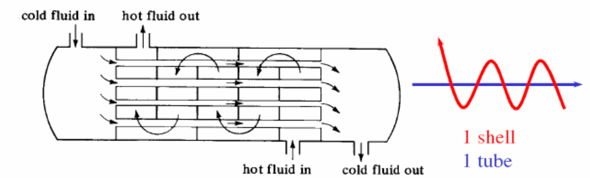
- Dense array of plates that guide alternating channels of fluids (typically air)
- Series of fins connect the plates and greatly increase the heat transfer area
- Advantage: very large heat transfer surface area per unit volume .
- One common application: Aircraft environmental control systems

3. Shell-and-tube heat exchanger

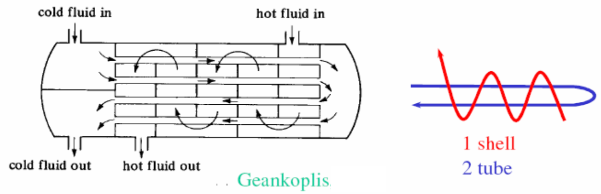
- Account for 60% of heat exchangers in use today
- Can handle large flows, low temperatures and pressures, high temperatures and pressures
- e.g.
  - Basco Type 500 U-tube Water Heater
  - 1 Shell Pass
  - 16 Tubes



1-1 Shell and Tube Heat Exchanger

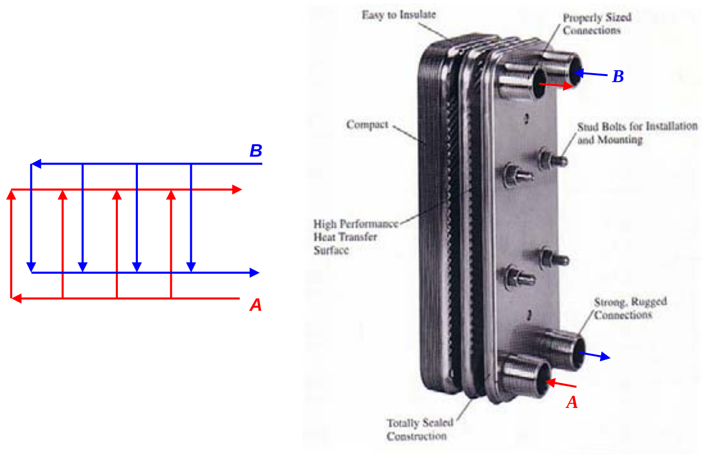


1-2 Shell and Tube Heat Exchanger

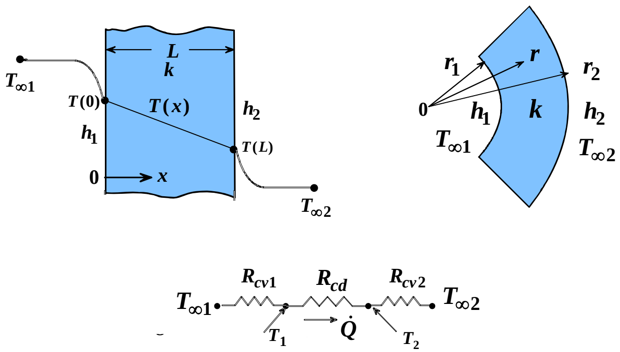


4. Plate and Frame

- Series of plates with flow channels embossed in them.
- The two fluids are guided through alternating rows of the plates
- Be well suited for liquid-to-liquid heat exchange application



§ 12-2、3 HEAT EXCHANGER MECHANISM



1. Heat Transfer Equation

Overall Heat Transfer Coefficient U

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = UA\Delta T = U_iA_i\Delta T = U_oA_o\Delta T \qquad \frac{1}{UA} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R_{\text{total}}$$

(1) $R_{\text{total}} = \frac{1}{h_iA_i} + R_{\text{wall}} + \frac{1}{h_oA_o}$ Where $R_{\text{wall}} = \begin{cases} R_{cd,\text{wall}} = \frac{L}{Ak} \\ R_{cd,\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi kL} \end{cases}$

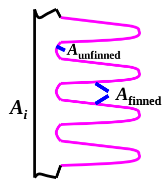
(2) **Large k, small thickness** → $R_{\text{wall}} \approx 0$ → $R_{\text{total}} \approx \frac{1}{h_iA_i} + \frac{1}{h_oA_o}$

Fin efficiency

(3) If finned, $A = \eta_{\text{fin}}A_{\text{fin}} + A_{\text{unfinned}}$

(4) $R_{\text{total}} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$

Fouling factor
（污垢因子）



2. Heat Balance Equation

(1) $\dot{Q} = \dot{m}_c C_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = \dot{m}_h C_{ph}(T_{h,\text{in}} - T_{h,\text{out}})$

Heat capacity rate $\dot{C} = \dot{m}C_p$ (W/K)

$$\dot{Q} = \dot{C}_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = \dot{C}_{ph}(T_{h,\text{in}} - T_{h,\text{out}})$$

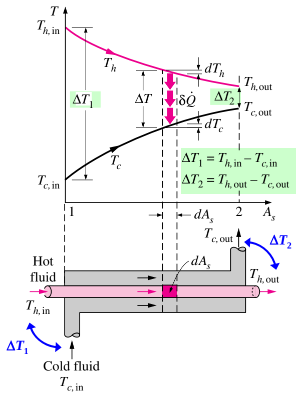
(2) If Boiling or condensation at constant temperature happens to one fluid

$$\dot{Q} = \dot{m}h_{fg}$$

h_{fg} is the vaporization enthalpy of the fluid

§ 12-4 THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

1. Parallel flow (PF)



$$d(\Delta T) = d(T_h - T_c) = dT_h - dT_c$$

$$\delta\dot{Q} = -\dot{m}_h C_{ph} dT_h = \dot{m}_c C_{pc} dT_c$$

$$d(\Delta T) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

$$\delta\dot{Q} = U\Delta T dA_s$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -\int_0^{A} U\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) dA_s$$

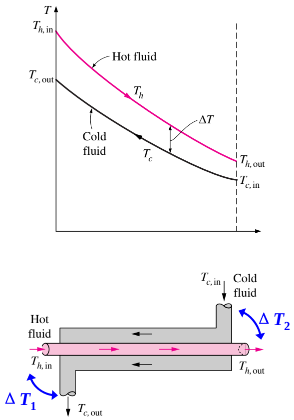
$$\ln\frac{\Delta T_1}{\Delta T_2} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in}) = \dot{m}_h C_{ph}(T_{h,in} - T_{h,out})$$

$$\dot{Q} = U A_s \Delta T_{\text{lm}}$$

LMTD $\Delta T_{\text{lm}} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

2. Counter flow (CF)



Heat transfer rate

$$\dot{Q} = U A_s \Delta T_{\text{lm}}$$

LMTD $\Delta T_{\text{lm}} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$
$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$

3. Multipass & Cross flow heat exchangers

Heat transfer rate $\dot{Q} = U A_s \Delta T_{\text{lm}}$

where $\Delta T_{\text{lm}} = F \Delta T_{\text{lm,CF}}$

- $\Delta T_{\text{lm,CF}} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$ $\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$, $\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$
- F — correction factor, obtained from the chart

$\Delta T_{\text{lm,PF}} < \Delta T_{\text{lm}} \le \Delta T_{\text{lm,CF}} \Rightarrow F \le 1$

F — *correction factor*

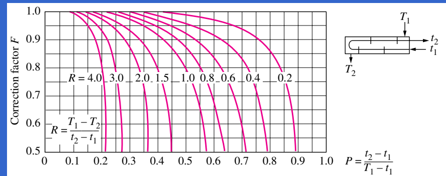
Shell-and-tube heat exchanger

$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{\text{tube side}}}{(\dot{m}C_p)_{\text{shell side}}}$$

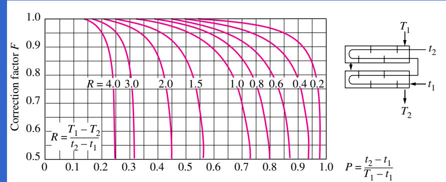
1 — *inlet*
2 — *outlet*
T — *shell-side* temperatures
t — *tube-side* temperatures

1-2,4,6,…



(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

2-4,6,8,…



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

4. Application of LMTD method to heat exchanger analysis

Heat exchanger analysis

1) Select a heat exchanger (*calculate size*) (设计)

Know: $m_h C_{ph}$, $m_c C_{pc}$, 3'T'

Predict: heat tansfer surface area A_s ?

2) Evaluate the performance of a specified heat exchanger (*calculate heat transfer rate and outlet temperature*) (校核)

Know: $m_h C_{ph}$, $m_c C_{pc}$, UA_s, only 2'T'($i.e.\ T_{c,\text{in}}, T_{h,\text{in}}$)

Calculate: $\dot{Q}$, $T_{c,\text{out}}$, $T_{h,\text{out}}$

1) Select a heat exchanger (*calculate size*)

Know: $m_h C_{ph}$, $m_c C_{pc}$, 3'T'

e.g. 10-3

Predict: heat tansfer surface area A_s ?

Procedure: 1) Calculate the unknown T and Q from heat balance equation

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = \dot{m}_h C_{ph}(T_{h,\text{in}} - T_{h,\text{out}})$$

2) Calculate $\Delta T_{\text{lm}} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$ and F if necessary

3) Select or calculate U e.g. $U = \dfrac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}}$

4) Calculate A_s from heat transfer equation

$$A_s = \frac{\dot{Q}}{U\ \Delta T_{\text{lm}}}$$

2) Evaluate the performance of a specified heat exchanger (*calculate heat transfer rate and outlet temperature*)

Know: $m_h C_{ph}$, $m_c C_{pc}$, UA_s, only 2'T'($i.e.\ T_{c,\text{in}}, T_{h,\text{in}}$)

Calculate: $\dot{Q}$, $T_{c,\text{out}}$, $T_{h,\text{out}}$

LMTD method needs Iteration !

Procedure: 1) Guess one unknown T and calculate Q from heat balance equation

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = \dot{m}_h C_{ph}(T_{h,\text{in}} - T_{h,\text{out}})$$

2) Calculate $\Delta T_{\text{lm}} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$ and F if necessary

3) Calculate Q' from heat transfer equation

$$\dot{Q}' = U A_s\ \Delta T_{\text{lm}}$$

4) Compare Q and Q' , adjust and repeat

Simplified method: Effectiveness-NTU method

§ 12-4 THE EFFECTIVENESS-NTU METHOD (效能 - 传热单元数法)

Number of transfer unit

Heat transfer effectiveness $\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}}$ $(\varepsilon \le 1)$

$T_{h,\text{in}}$

Where $\dot{Q} = \dot{m}_c C_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = \dot{m}_h C_{ph}(T_{h,\text{in}} - T_{h,\text{out}})$

$\dot{Q}_{\max} = (\dot{m}C_p)_{\min}(T_{h,\text{in}} - T_{c,\text{in}})$ $T_{c,\text{in}}$

So $\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon \dot{C}_{\min}(T_{h,\text{in}} - T_{c,\text{in}})$

1. Parallel flow (PF)

Assume $\dot{m}_h C_{ph} \le \dot{m}_c C_{pc}$ then $\dot{C}_{\min} = \dot{C}_h$

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\dot{C}_h(T_{h,\text{in}} - T_{h,\text{out}})}{\dot{C}_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}}$$

$$\Rightarrow T_{h,\text{in}} - T_{h,\text{out}} = \varepsilon\left(T_{h,\text{in}} - T_{c,\text{in}}\right) \quad (1)$$

$$\dot{Q} = \dot{C}_c(T_{c,\text{out}} - T_{c,\text{in}}) = \dot{C}_h(T_{h,\text{in}} - T_{h,\text{out}})$$

Capacity ratio (热容比) $C = \frac{\dot{C}_{\min}}{\dot{C}_{\max}}$

$$\Rightarrow T_{c,\text{out}} - T_{c,\text{in}} = \frac{\dot{C}_h}{\dot{C}_c}(T_{h,\text{in}} - T_{h,\text{out}}) = C(T_{h,\text{in}} - T_{h,\text{out}}) \quad (2)$$

(1) + (2)

$$T_{h,\text{in}} - T_{c,\text{in}} - \left(T_{h,\text{out}} - T_{c,\text{out}}\right) = \varepsilon\left(T_{h,\text{in}} - T_{c,\text{in}}\right) + C(T_{h,\text{in}} - T_{h,\text{out}})$$

Substitute (1)

$$= \varepsilon\left(T_{h,\text{in}} - T_{c,\text{in}}\right) + C\varepsilon\left(T_{h,\text{in}} - T_{c,\text{in}}\right)$$

$$= \varepsilon(1+C)\left(T_{h,\text{in}} - T_{c,\text{in}}\right)$$

$$T_{h,\text{in}} - T_{c,\text{in}} - \left(T_{h,\text{out}} - T_{c,\text{out}}\right) = \varepsilon(1+C)\left(T_{h,\text{in}} - T_{c,\text{in}}\right)$$

$$1 - \frac{T_{h,\text{out}} - T_{c,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \varepsilon(1+C)$$

$$1 - \frac{\Delta T_2}{\Delta T_1} = \varepsilon(1+C)$$

$$\Rightarrow \quad \varepsilon = \left(1 - \frac{\Delta T_2}{\Delta T_1}\right)/(1+C)$$

Substitute $\ln\frac{\Delta T_1}{\Delta T_2} = -UA_s\left(\frac{1}{\dot{C}_h} + \frac{1}{\dot{C}_c}\right)$

So
$$\varepsilon = \frac{1 - \exp\left[-\frac{UA_s}{\dot{C}_h}\left(1 + \frac{\dot{C}_h}{\dot{C}_c}\right)\right]}{1+C} = \frac{1 - \exp\left[-\frac{UA_s}{\dot{C}_h}(1+C)\right]}{1+C}$$

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1) $\dot{m}_h C_{ph} \le \dot{m}_c C_{pc}$

$$C = \frac{\dot{C}_{\min}}{\dot{C}_{\max}} = \frac{\dot{C}_h}{\dot{C}_c} \qquad \varepsilon = \frac{1 - \exp\left[-\frac{UA_s}{\dot{C}_h}(1+C)\right]}{1+C}$$

2) $\dot{m}_c C_{pc} \le \dot{m}_h C_{ph}$

$$C = \frac{\dot{C}_{\min}}{\dot{C}_{\max}} = \frac{\dot{C}_c}{\dot{C}_h} \qquad \varepsilon = \frac{1 - \exp\left[-\frac{UA_s}{\dot{C}_c}(1+C)\right]}{1+C}$$

$$C = \frac{C_{\min}}{C_{\max}} \qquad \varepsilon = \frac{1 - \exp\left[-\frac{UA_s}{\dot{C}_{\min}}(1+C)\right]}{1+C}$$

Number of Transfer Units $\text{NTU} = \frac{UA_s}{\dot{C}_{\min}}$ *A measure of the heat transfer surface area A_s*

PF: $\varepsilon = \frac{1 - \exp\left[-\text{NTU}(1+C)\right]}{1+C}$

2. Counter flow (CF)

CF: $\varepsilon = \dfrac{1-\exp\left[-\mathrm{NTU}(1-C)\right]}{1-C\exp\left[-\mathrm{NTU}(1-C)\right]}$

PF: $\varepsilon = \dfrac{1-\exp\left[-\mathrm{NTU}(1+C)\right]}{1+C}$

CF: $\varepsilon = \dfrac{1-\exp\left[-\mathrm{NTU}(1-C)\right]}{1-C\exp\left[-\mathrm{NTU}(1-C)\right]}$

- $\dot{C}_{\max} \to \infty$ (with phase change) i.e. $C = \dfrac{\dot{C}_{\min}}{\dot{C}_{\max}} \to 0$

$$\varepsilon = \varepsilon_{\max} = 1-\exp\left[-\mathrm{NTU}\right]$$

- $C = \dfrac{C_{\min}}{C_{\max}} = 1$

PF: $\varepsilon = \dfrac{1-\exp\left[-2\cdot\mathrm{NTU}\right]}{2}$

CF: $\varepsilon = \dfrac{\mathrm{NTU}}{1+\mathrm{NTU}}$

3. Other flow

Table 10-4 or Figure 10-19

For a given NTU and C: $\varepsilon_{\mathrm{PF}} \le \varepsilon \le \varepsilon_{\mathrm{CF}}$

4. Application of ε - NTU method to heat exchanger analysis

1) Select a heat exchanger (*calculate size*)

① $\varepsilon = \dfrac{\dot{Q}}{\dot{Q}_{\max}}$

e.g. 10-6

② $\varepsilon = f(\mathrm{NTU},\ C) \Rightarrow \mathrm{NTU}$ (Table10-5 or Figure 10-19)

③ $\mathrm{NTU} = \dfrac{UA_s}{\dot{C}_{\min}} \Rightarrow A_s$

2) Evaluate the performance of a specified heat exchanger (*calculate heat transfer rate and outlet temperature*)

① Calculate A_s, U, $\dot{Q}_{\max}$ and NTU $\quad \left(\dot{Q}_{\max} = \left(\dot{m}C_p\right)_{\min}(T_{h,\mathrm{in}} - T_{c,\mathrm{in}}),\ \mathrm{NTU} = \dfrac{UA_s}{\dot{C}_{\min}}\right)$

② $\varepsilon = f(\mathrm{NTU},\ C)$ (Table 10-4 or Figure 10-19)

③ $\dot{Q} = \varepsilon\dot{Q}_{\max}$

e.g. 10-7

④ $\dot{Q} = \dot{m}_c C_{pc}(T_{c,\mathrm{out}} - T_{c,\mathrm{in}}) = \dot{m}_h C_{ph}(T_{h,\mathrm{in}} - T_{h,\mathrm{out}}) \Rightarrow T_{h,\mathrm{out}}$ and $T_{c,\mathrm{out}}$

Heat exchanger analysis

1) Select a heat exchanger (*calculate size*)

Know: $m_h C_{ph},\ m_c C_{pc},\ 3\,'T\,'$

LMTD method

Predict: heat tansfer surface area A_s ?

2) Evaluate the performance of a specified heat exchanger (*calculate heat transfer rate and outlet temperature*)

Know: $m_h C_{ph},\ m_c C_{pc},\ UA_s,$ only $2\,'T\,'\,(i.e.\,T_{c,\mathrm{in}}, T_{h.\mathrm{in}})$

ε - NTU method

Calculate: $\dot{Q},\ T_{c,\mathrm{out}},\ T_{h,\mathrm{out}}$