J. Astrophys. Astr. (2007) **28,** 133–138

On Surface Tension for Compact Stars

R. Sharma* & S. D. Maharaj†
Astrophysics and Cosmology Research Unit, School of Mathematical Sciences, University of KwaZulu–Natal, Private Bag X54001, Durban 4000, South Africa.
**e-mail: 206526115@ukzn.ac.za*
†*e-mail: maharaj@ukzn.ac.za*

Received 2007 February 22; accepted 2007 May 15

Abstract. In an earlier analysis it was demonstrated that general relativity gives higher values of surface tension in strange stars with quark matter than neutron stars. We generate the modified Tolman–Oppenheimer–Volkoff equation to incorporate anisotropic matter and use this to show that pressure anisotropy provides for a wide range of behaviour in the surface tension than is the case with isotropic pressures. In particular, it is possible that anisotropy drastically decreases the value of the surface tension.

Key words. Relativity—pulsars—equation of state.

1. Introduction

Stars that are more compact than neutron stars, at present, have become a subject of considerable interest as they provide us natural laboratories for testing QCD. Over the last couple of decades, various models have been proposed to explain the compactness and properties of some of the observed compact objects. Pioneering works in this field have put forward new concepts of compact matter, namely strange stars (Witten 1984; Farhi & Jaffe 1984) and boson stars (Kaup 1968; Ruffini & Bonazzola 1969; Colpi *et al.* 1986). Due to the high matter densities within such stars one expects pressure to be anisotropic in general, *i.e.*, in the interior of such stars the radial pressure and tangential pressure are different. An anisotropic energy momentum is a topic which is often ignored in the calculations of compact stars. However, since the pioneering work of Bowers & Liang (1974) there has been extensive research in the study of anisotropic relativistic matter in general relativity. The analysis of static spherically symmetric anisotropic fluid spheres is important in relativistic astrophysics. Ruderman (1972) showed that nuclear matter may be anisotropic in the high density ranges of order 10^{15} gm cm^{-3} where nuclear interactions have to be treated relativistically. Anisotropy in compact objects may occur due to the existence of a solid core or the presence of type 3A superfluid (Kippenhahm & Weigert 1990), phase transition (Sokolov 1980), pion condensation (Sawyer 1972), slow rotation (Herrera & Santos 1997), mixture of two gases (Letelier 1980) or strong magnetic fields (Weber 1999). Also objects made up of self-interacting scalar particles known as boson stars are naturally anisotropic in their configurations. Anisotropic models for compact self gravitating objects have been studied by Herrera & Santos (1997); Rao *et al.* (2000); Corchero (2001); Mak & Harko

(2003); Ivanov (2002); Dev & Gleiser (2003); Hernández & Núñez (2004); Chaisi & Maharaj (2005), and many others. Anisotropic models for compact objects have been shown to achieve high red-shift values (Bowers & Liang 1974; Herrera & Santos 1997; Ivanov 2002; Mak & Harko 2003), and they are stable (Herrera & Santos 1997; Dev & Gleiser 2003). In this article, we show that pressure anisotropy may also affect the surface tension of compact stars. We believe that this aspect has not been considered yet in the context of anisotropic stellar models.

2. Surface tension of strange stars

In a recent paper by Bagchi *et al.* (2005), it has been shown that objects composed of u, d and s quarks popularly known as 'strange stars' give higher values of surface tension than neutron stars, a necessary criterion for the existence of stable strange stars in the Universe. This calculation is based on equations of state (EOS) for strange matter formulated by Dey *et al.* (1998). In an approximated linearized form, the EOS may be written as (Zdunik 2000; Gondek-Rosińska *et al.* 2000)

$$p = a(\rho - \rho_b), \tag{1}$$

where ρ is the energy density, ρ_b is the density at the surface, p is the isotropic pressure, and a is a parameter related to the velocity of sound ($a = dp/d\rho$).

To calculate the surface tension, one assumes that the star is a huge spherical ball composed of strange matter which is self-bound and non-rotating. The excess pressure on the surface of the star can be expressed as

$$|\Delta p|_{r=R} = \frac{2S}{R}, \tag{2}$$

where S is the surface tension of the star and R is the radius of curvature. At the surface

$$|\Delta p|_{r=R} = r_n \frac{dp}{dr}|_{r=R}, \tag{3}$$

where r_n is the radius of the quark particle given by $r_n = (1/\pi n)^{1/3}$ where n is the baryon number density. As strange stars are very compact, a relativistic treatment is necessary to find their configurations and other physical parameters. Thus for a given EOS, one uses the Tolman–Oppenheimer–Volkoff (TOV) equation (Oppenheimer & Volkoff 1939)

$$\frac{dp}{dr} = -\frac{G(\rho + p)\left[\frac{m(r)}{c^2 r} + \frac{4\pi r^2 p}{c^4}\right]}{c^2 r\left(1 - \frac{2Gm(r)}{r}\right)} \tag{4}$$

to find the surface tension of the star, making use of equations (2) and (3). This method helps to yield higher values of surface tension as compared to neutron stars including the possible explanation for the existence of strange stars in the Universe and other related phenomena like delayed γ-ray bursts (Bagchi *et al.* 2005).

However, at very high densities, anisotropy may be significant in such stars which may contribute to the surface tension. If we assume that pressure within such a star is anisotropic in general then the TOV equation (4) gets modified yielding different results as obtained by Bagchi *et al.* (2005). In the following sections, we derive the modified TOV equation with anisotropic pressure and perform some numerical calculations to show the effects of pressure anisotropy on the surface tension of compact stars.

3. Anisotropic TOV equation

We first formulate the modified TOV equation with anisotropic pressure. We assume the line element for a static spherical object in the standard form

$$ds^2 = -e^{\gamma(r)}c^2dt^2 + e^{\mu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{5}$$

where $\gamma(r)$ and $\mu(r)$ are the two unknown metric functions. Without any loss of generality, the energy momentum tensor for an anisotropic star may be written as

$$T_{ij} = (\rho c^2 + p_r)u_i u_j + p_r g_{ij} + (p_r - p_\perp)n_i n_j, \tag{6}$$

where u_i is the fluid four-velocity, n_i is a radially directed unit space-like vector. We assume that $p_r \neq p_\perp$ and $p_\perp - p_r = \Delta$ gives the measure of pressure anisotropy in this model.

The Einstein's field equations are then given by

$$\frac{8\pi G}{c^4}\rho = \frac{\left(1 - e^{-\mu}\right)}{r^2} + \frac{\mu' e^{-\mu}}{r}, \tag{7}$$

$$\frac{8\pi G}{c^4}p_r = \frac{\gamma' e^{-\mu}}{r} - \frac{\left(1 - e^{-\mu}\right)}{r^2}, \tag{8}$$

$$\frac{8\pi G}{c^4}p_\perp = \frac{e^{-\mu}}{4}\left(2\gamma'' + \gamma'^2 - \gamma'\mu' + \frac{2\gamma'}{r} - \frac{2\mu'}{r}\right), \tag{9}$$

where primes denote differentiation with respect to the radial coordinate r. Equations (7)–(9) may be combined together to yield

$$(\rho + p_r)\gamma' + 2p_r' + \frac{4}{r}(p_r - p_\perp) = 0 \tag{10}$$

which is a conservation equation.

If we write the metric function μ in terms of mass function $m(r)$ as

$$e^{-\mu} = 1 - \frac{Gm(r)}{c^2 r} \tag{11}$$

then equation (10) becomes

$$\frac{dp_r}{dr} = -(\rho + p_r)\frac{\left(\frac{Gm(r)}{c^2 r} + \frac{4\pi G r^2 p_r}{c^4}\right)}{r\left(1 - \frac{2Gm(r)}{c^2 r}\right)} + \frac{2}{r}(p_\perp - p_r). \tag{12}$$

Equation (12) is the the modified TOV equation in the presence of pressure anisotropy. For a given central density ρ_c or central pressure p_r^c and anisotropic parameter Δ, equation (12) may be integrated to find the mass $M = m(R)$ and radius R of the star provided the EOS $p_r = p_r(\rho)$ is known. Local anisotropy thus effects the geometry of the star.

At the surface of the star $r = R$, the radial pressure p_r vanishes. However, the tangential pressure $p_\perp$ is not necessarily zero at the surface. The two pressure profiles within the star should satisfy the following conditions: $p_r > 0$ and $p_\perp > 0$. The maximum value of the anisotropic parameter Δ *vis-a-vis* the tangential pressure $p_\perp$ is constrained by the physical requirement that the radial pressure gradient dp_r/dr should be negative in the stellar interior; other physical requirements may, however, put a more stringent restriction on the values of Δ. Thus for finite values of $p_\perp$ at the boundary $\Delta(r = R) = p_\perp^b$, equation (12) becomes

$$\left.\frac{dp_r}{dr}\right|_{r=R} = -\frac{\rho_b \frac{GM}{c^2 R}}{R\left(1 - \frac{2GM}{c^2 R}\right)} + \frac{2p_\perp^b}{R}. \tag{13}$$

If $p_\perp^b$ is not negligible at the boundary, equation (13) shows that it is possible to get different sets of values of surface tension as obtained by Bagchi *et al.* (2005) for isotropic matter. Thus it is possible to generate a wide range of behaviour in the surface tension for anisotropic matter than is the case for isotropic pressures.

4. Numerical results

To get an estimate of the effects of pressure anisotropy on the surface tension, we consider the strange matter EOS given by equation (1). We consider two particular cases as discussed by Gondek-Rosińska *et al.* (2000):

- EOS SS1: where, $a = 0.463$, $\rho_b = 1.15 \times 10^{15}$ gm cm^{-3}, $\rho_c = 4.68 \times 10^{15}$ gm cm^{-3}, $n(r = R) = 0.725\,\text{fm}^{-3}$, $n(r = 0) = 2.35\,\text{fm}^{-3}$, $M = 1.435\ M_\odot$, $R = 7.07$ km.
- EOS SS2: where, $a = 0.455$, $\rho_b = 1.33 \times 10^{15}$ gm cm^{-3}, $\rho_c = 5.5 \times 10^{15}$ gm cm^{-3}, $n(r = R) = 0.805\,\text{fm}^{-3}$, $n(r = 0) = 2.638\,\text{fm}^{-3}$, $M = 1.323\ M_\odot$, and $R = 6.55$ km.

Numerical calculations show that for a given mass and radius, if we gradually introduce anisotropy, the absolute value of the surface tension decreases as can be seen in Fig. 1. For example, it is observed that even if we consider a tangential pressure of 100 MeV fm^{-3} at the surface, the surface tension decreases drastically. It is to be noted here that the anisotropy parameter should be so chosen that all the regularity conditions (Delgaty & Lake 1998) are satisfied. Thus, although in Fig. 1, the surface tension increases beyond a certain value of the anisotropic parameter, we ignore this region as the radial pressure gradient becomes positive in this region. The results are given in Table 1.

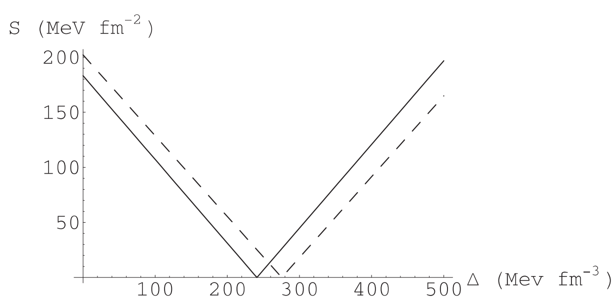

Figure 1. Surface tension S plotted against Δ. The solid line is for EOS SS1 and the dotted line is for EOS SS2.

Table 1. Anisotropic effect on the surface tension of strange stars.

		$\frac{dp_r}{dr}\|_{r=R}$ (MeV fm^{-3} km^{-1})		S (MeV fm^{-2})	
EOS	r_n (fm)	$p_\perp = 0$	$p_\perp = 100$ (MeV fm^{-3})	$p_\perp = 0$	$p_\perp = 100$ (MeV fm^{-3})
SS1	0.76	68.18	39.90	183.19	107.19
SS2	0.73	84.09	53.56	202.14	128.75

5. Discussions

We have shown that anisotropy plays an important role in the calculation of surface tension of compact stars. The origin of such anisotropies within compact objects may be different for different objects. We may, however, ask whether it is necessary at all to consider anisotropic effects on the surface tension of strange stars. The answer is in the affirmative since one possibility for the origin of anisotropies within strange stars could be the presence of charged particles at the surface. It has recently been reported that in strange stars, the electric field could be as high as 10^{19} eV/cm (Usov 2004), which indicates the possibility of a large charge distribution within such objects. Therefore we need to consider the effect of charge while deriving the the gross features of such stars. It can be shown that in the presence of charge, the TOV equation is modified to

$$\frac{dp}{dr} = -(\rho + p)\frac{\left(\frac{Gm(r)}{c^2 r} + \frac{4\pi G r^2 p}{c^4}\right)}{r\left(1 - \frac{2Gm(r)}{c^2 r}\right)} + \frac{Q(r)}{4\pi r^4}\frac{dQ(r)}{dr}, \tag{14}$$

where, $Q(r)$ is the total charge confined within a sphere of radius r. Note that the Einstein–Maxwell system is always anisotropic which is often treated as an isotropic system of field equations for mathematical simplicity (see for example, Ray *et al.* 2004). Also recent works (Schmitt 2005) suggest that a natural mechanism to explain the strong pulsar kicks in neutron stars could be the existence of asymmetric phases in quark matter.

It is to be noted here that, for simplicity, we ignored the effect of rotation in the present work although pulsars are magnetized rotators and a strong magnetic field ($\sim 10^{12}$ G) is observed at the surface of such stars. Pulsars known as magnetars may even have a magnetic field as strong as $\sim 10^{14-15}$ G. Though we do not have an established theory for the microscopic origin of such a strong magnetic field, it is agreed that Ferro-magnetization may occur in the high density quark matter which, in turn, may modify the EOS for strange matter. The derivation and the form of the modified EOS in the presence of strong magnetic field or superfluidity (responsible for anisotropy) is a complex issue and a more detailed analysis is required to see the effect of the modified EOS on the overall configuration *vis-a-vis* surface tension of compact stars.

To conclude, without going into the microscopic details of a star, it can be shown that surface tension is affected in the presence of anisotropy. For the very existence of strange stars in our Universe a crucial condition put forward was a large value of S by Alcock & Olinto (1989) which according to Bagchi *et al.* (2005) can be achieved by a general relativistic treatment of strange stars. However, in this article we have shown that a wide range of values of S are possible if we consider anisotropy in the energy momentum tensor; an issue ignored in the previous calculation (Bagchi *et al.* 2005).

Therefore, on the basis of surface tension for compact stars, no conclusive remarks at this moment can perhaps be made on the possible existence of strange stars. There could be, however, some other means to justify the existence of such stars which will be taken up elsewhere.

Acknowledgements

RS acknowledges the financial support (grant no. SFP2005070600007) from the National Research Foundation (NRF), South Africa. SDM acknowledges that this work is based upon research supported by the South African Research Chair Initiative of the Department of Science and Technology and the National Research Foundation.

References

Alcock, C., Olinto, A. 1989, *Phys. Rev. D*, **39**, 1233.
Bagchi, M., Sinha, M., Dey, M., Dey, J., Bhowmick, S. 2005, *Astron. & Astrophys.*, **440**, L33.
Bowers, R. L., Liang, E. P. T. 1974, *J. Astrophys.*, **188**, 657.
Chaisi, M., Maharaj, S. D. 2005, *Gen. Relat. Grav.*, **37**, 1177.
Colpi, M., Shapiro, S. L., Wasserman, I. 1986, *Phys. Rev. Lett.*, **57**, 2485.
Corchero, E. S. 2001, *Astrophysics and Space Science*, **275**, 259.
Delgaty, M. S. R., Lake, K. 1998, *Comput. Phys. Commun.*, **115**, 395.
Dev, K., Gleiser, M. 2003, *Gen. Relat. Grav.*, **35**, 1435.
Dey, M., Bombaci, I., Dey, J., Ray, S., Samanta, B. C. 1998, *Phys. Lett. B*, **438**, 123.
Farhi, E., Jaffe, R. L. 1984, *Phys. Rev. D*, **30**, 2379.
Gondek-Rosińska, D., Bulik, T., Zdunik, J. L., Gourgoulhon, E., Ray, S., Dey, J., Dey, M. 2000, *Astron. & Astrophys.*, **363**, 1005.
Hernández, H., Núñez, L. A. 2004, *Can. J. Phys.*, **82**, 29.
Herrera, L., Santos, N. O. 1997, *Phys. Rep.*, **286**, 53.
Ivanov, B. V. 2002, *Phys. Rev. D*, **65**, 104011.
Kaup, D. J. 1968, *Phys. Rev.*, **172**, 1331.
Kippenhahm, R., Weigert, A. 1990, *Stellar Structure and Evolution*, Springer, Berlin.
Letelier, P. 1980, *Phys. Rev. D*, **22**, 807.
Mak, M. K., Harko, T. 2003, *Proc. Roy. Soc. London A*, **459**, 393.
Oppenheimer, J. R., Volkoff, G. M. 1939, *Phys. Rev.*, **55**, 374.
Rao, J. K., Annapurna, M., Trivedi, M. M. 2000, *Pramana – J. Phys.*, **54**, 215.
Ray, S., Malheiro, M., Lemos, J. P. S., Zanchin, V. T. 2004, *Braz. J. Phys.*, **34**, 310.
Ruderman, R. 1972, *Ann. Rev. Astron. Astrophys.*, **10**, 427.
Ruffini, R., Bonazzola, S. 1969, *Phys. Rev.*, **187**, 1767.
Sawyer, R. F. 1972, *Phys. Rev. Lett.*, **29**, 382.
Schmitt, A., Shovkovy, I. A., Wang, Q. 2005, *Phys. Rev. Lett.*, **94**, 211101.
Sokolov, A. I. 1980, *JETP*, **79**, 1137.
Usov, V. V. 2004, *Phys. Rev. D*, **70**, 067301.
Weber, F. 1999, *Pulsars as astrophysical observatories for nuclear and particle physics*, Institute of Physics, Bristol.
Witten, E. 1984, *Phys. Rev. D*, **30**, 272.
Zdunik, J. L. 2000, *Astron. & Astrophys.*, **359**, 311.