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Oblique Propagation and Dissipation of Alfvén Waves in Coronal Holes

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Abstract. We investigate the effect of viscosity and magnetic diffusivity on the oblique propagation and dissipation of Alfvén waves with respect to the normal outward direction, making use of MHD equations, density, temperature and magnetic field structure in coronal holes and underlying magnetic funnels. We find reduction in the damping length scale, group velocity and energy flux density as the propagation angle of Alfvén waves increases inside the coronal holes. For any propagation angle, the energy flux density and damping length scale also show a decrement in the source region of the solar wind ($< 1.05\,R_\odot$) where these may be one of the primary energy sources, which can convert the inflow of the solar wind into the outflow. In the outer region ($> 1.21\,R_\odot$), for any propagation angle, the energy flux density peaks match with the peaks of MgX 609.78 Å and 624.78 Å linewidths observed from the Coronal Diagnostic Spectrometer (CDS) on SOHO and the non-thermal velocity derived from these observations, justify the observed spectroscopic signature of the Alfvén wave dissipation.

Key words. Sun: Alfvén waves—coronal holes—solar wind.

1. Introduction

MHD waves are the prominent candidates for the heating of coronal holes and acceleration of the solar wind since the pioneering work of Parker (1965). Many spectroscopic studies show the importance of the damped and undamped Alfvén waves in the solar corona (Hassler *et al.* 1990; Harrison *et al.* 2002; O'Shea *et al.* 2005). The microscopic properties of the coronal hole plasma, e.g., viscosity and resistivity, etc., are important for conversion of the mechanical energy of Alfvén waves into thermal energy. Dwivedi & Srivastava (2006) have studied the importance of radially outward propagating Alfvén waves and their dissipation in coronal holes, which support the solar wind outflow starting at about 20 Mm as reported by Tu *et al.* (2005), and also the spectroscopic signature of Alfvén wave dissipation in the off-limb coronal hole plasma as reported by O'Shea *et al.* (2005).

In this paper, we consider the viscous and resistive dissipation of Alfvén waves propagating at various angles inside the inner part of polar coronal holes ($R < 1.35\,R_\odot$) and underlying magnetic funnels. At different propagation angles, we investigate the combined effect of viscosity and magnetic diffusivity on the spatial variation of damping length scale, group velocity and mechanical energy flux density of Alfvén waves. In section 2, we describe the Alfvén wave propagation and dissipation. Results and discussion are given in the last section.

2. Oblique propagation and dissipation of Alfvén wave

We study the Alfvén wave propagation and dissipation in the viscous, magnetic diffusive, and incompressible coronal hole plasma. The MHD equations for viscous and resistive plasma are:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} = \frac{1}{\mu}\,(\nabla\times\mathbf{B})\;\times\mathbf{B} + \rho\nu\nabla^2\mathbf{v} \qquad \text{(momentum equation)},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\nabla^2\mathbf{B} \qquad \text{(induction equation)},$$

and

$$\nabla\cdot\mathbf{B} = 0 \qquad \text{(magnetic flux conservation)},$$

where ρ, μ, η and ν respectively are mass density, magnetic permeability, magnetic diffusivity and coefficient of kinematic viscosity. When we linearize these equations, we get

$$\rho_{\mathrm{o}} \frac{\partial \mathbf{v}_1}{\partial t} = \frac{1}{\mu}\,(\mathbf{B}_{\mathrm{o}}\cdot\nabla)\,\mathbf{B}_1 + \rho_{\mathrm{o}}\nu\nabla^2\mathbf{v}_1,$$

and

$$\frac{\partial \mathbf{B}_1}{\partial t} = (\mathbf{B}_{\mathrm{o}}\cdot\nabla)\,\mathbf{v}_1 + \eta\nabla^2\mathbf{B}_1.$$

Assuming the plane wave solutions with the variables having the phase-factor $\exp\,[i\,(k\cdot r\,|\cos\theta| - \omega t)]$, we get the following dispersion relation in linear MHD context,

$$Pk^4 + Qk^2 - \omega^2 = 0, \tag{1}$$

where $P = \nu\eta\cos^4\theta$, $Q = \lfloor v_A^2 - i\omega\,(\nu+\eta)\rfloor\cos^2\theta$, and $v_A = \mathbf{B}/\sqrt{\mu\rho}$ is the Alfvén velocity. The angle of propagation with respect to normal outward direction is θ. We take $\rho\nu = 1.0045\times10^{-16}T_R^{2.5}\,\mathrm{g\,cm^{-1}\,s^{-1}}$ (Spitzer 1962) and magnetic diffusivity $\eta = 1.144\times10^{13}T_R^{-1.5}\,\mathrm{cm^2\,s^{-1}}$ (Priest 1982). We take the density profile in a polar coronal hole as a function of radial height from the empirical relation of Doyle *et al.* (1999):

$$N_e(R) = \frac{1\times10^8}{R^8} + \frac{2.5\times10^3}{R^4} + \frac{2.9\times10^5}{R^2}\;(\mathrm{cm^{-3}}). \tag{2}$$

The mass density is $\rho_{\mathrm{o}}(R) = 0.6\,m_p\,N_e(R)$, where m_p is the proton mass. Using David *et al.* (1998) temperature measurements in coronal holes, the temperature profile fit is given by Pekünlü *et al.* (2002),

$$T_R = -2\times10^7R^2 + 5\times10^7R - 3\times10^7\;(\mathrm{K}). \tag{3}$$

The temperature below $1.05\,\mathrm{R}_\odot$ and above $1.30\,\mathrm{R}_\odot$ is obtained by the extrapolated points of the fit. The energy flux density of Alfvén wave is $W = \rho v_{\mathrm{NT}}^2(R)(\partial\omega/\partial k)$, where $(\partial\omega/\partial k)$ is group velocity and $v_{\mathrm{NT}}(R)$ is velocity equivalent of the non-thermal component of relevant spectral line at FWHM. Using Banerjee *et al.* (1998) measurements for Si VIII ion, the non-thermal velocity profile fit is given by Pekünlü *et al.* (2002),

$$\begin{aligned} v_{\mathrm{NT}}(R) &= -1522.3R^4 + 8638.2R^3 - 18191R^2 \\ &\quad + 16882R - 5786.5 \quad (\mathrm{km\ s^{-1}}). \end{aligned} \tag{4}$$

The measurements start from 27 arcsec ($\sim 1.03\,\mathrm{R}_\odot$). The values below $\sim 1.03\,\mathrm{R}_\odot$ and above $\sim 1.26\,\mathrm{R}_\odot$ are obtained from the extrapolated points of the fit.

Using the measurements of Tu *et al.* (2005); Banaszkiewicz *et al.* (1998) and Hackenberg *et al.* (2000), the empirical relation for the magnetic field in the inner coronal hole is given by Dwivedi & Srivastava (2006),

$$B = \frac{a}{1 + \exp\left[(R-b)/c\right]} + d, \quad \text{for } 1.00\,\mathrm{R}_\odot < R < 1.05\,\mathrm{R}_\odot, \ (\mathrm{Gauss}), \tag{5a}$$

where $a = 113.1348$, $b = 1.0035$, $c = 0.0024$, and $d = 8.7852$, and

$$\begin{aligned} B &= 95.20344 - 129.39079\,R + 45.49857\,R^2, \\ &\text{for } 1.05\,\mathrm{R}_\odot < R < 1.35\,\mathrm{R}_\odot. \end{aligned} \tag{5b}$$

We have taken $1.05\,\mathrm{R}_\odot$ as a reference height and made all the physical quantities (e.g., T, η, ν, v_A, ρ, B and v_{NT}) dimensionless, by dividing the respective values at $R = 1.05\,\mathrm{R}_\odot$, for example, $T_R \rightarrow (T_R/T_{R=1.05\,\mathrm{R}_\odot})$, etc. We analyze the propagation and dissipation of high frequency Alfvén wave in the magnetic funnel for the time periods $\tau_A = 0.0001$ s in the short-wavelength assumption. We also consider the shortest period $\tau_{A_{\mathrm{ref}}} = 0.01$ s as a reference time to make the time periods dimensionless, i.e., $\tau_A = \tau_A/\tau_{A_{\mathrm{ref}}}$. We have used these dimensionless physical quantities in the dispersion relation (equation 1) and calculated the wave number (which would be actually $k_{\mathrm{mod}} \approx k_{\tau_A,R}/k_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}}$) in our model. Using values of k_{mod}, we have calculated damping length scale (D), energy flux density (W), and group velocity (V_g). The dimensionless quantities can be expressed as follows:

$$\begin{aligned} D &\approx \frac{2\pi}{k_{\mathrm{mod}}} \approx \frac{2\pi}{k_{\tau_A,R}/k_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}}} \\ &\approx \frac{2\pi}{k_{\tau_A,R}} \times \frac{k_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}}}{2\pi} \times 2\pi \\ &\approx 2\pi \times \frac{D_{\tau_A,R}}{D_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}}}. \end{aligned}$$

Hence, we can write

$$D_{\tau_A,R} \approx \frac{1}{2\pi} \times D \times D_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}} \quad (\mathrm{cm}), \tag{6a}$$

$$W_{\tau_A,R} \approx \frac{1}{2\pi} \times W \times W_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}} \quad (\mathrm{ergs\ cm^{-2}\ s^{-1}}), \tag{6b}$$

and

$$V_{g_{\tau_A},R} \approx \frac{1}{2\pi} \times V_g \times V_{g_{\tau_{A\,\mathrm{ref}}},R_{\mathrm{ref}}} \quad (\mathrm{cm\ s^{-1}}), \tag{6c}$$

where $D_{\tau_A,R}$, $W_{\tau_A,R}$, $V_{g_{\tau_A},R}$ are damping length scale, energy flux density and group velocity of the Alfvén wave of time period τ_A at any radial height R respectively in c.g.s. units and D, W, V_g are their corresponding dimensionless values. $D_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}}$, $W_{\tau_{A\,\mathrm{ref}},R_{\mathrm{ref}}}$, $V_{g_{\tau_{A\,\mathrm{ref}}},R_{\mathrm{ref}}}$ are damping length scale, energy flux density and group velocity for the Alfvén wave with reference time period 0.01 s at the reference height 1.05 $R_\odot$ in c.g.s units.

3. Results and discussion

Equation (1) is used to study the combined effect of viscosity and magnetic diffusivity on Alfvén waves propagating at different angles with respect to the normal direction outwards to the Sun. We have also calculated the damping length scale, group velocity and mechanical energy flux density. The solution of the equation gives four roots, in which two are the complex conjugates of the other two. These calculations, using linear and incompressible MHD approximation, exhibit two wave modes. The one mode corresponds to slow-wave, which has lower wave velocity and damping length scale. These waves cannot reach into the corona. Only the fast-wave mode is considered here which propagates into the corona. The calculations and details for the radially propagating high-frequency Alfvén waves of different time periods are given in our companion paper Dwivedi & Srivastava (2006). In the present work, we analyze the propagation and dissipation of the high frequency Alfvén waves, which are propagating at a different angle.

Figure 1 shows the combined effect of viscosity and magnetic diffusivity on the spatial variation of damping length scale of high frequency Alfvén waves. The sharp decrement of damping length scale in this figure indicates an efficient viscous and resistive damping of Alfvén waves below 25 Mm in the magnetic funnel region. It is clear that damping length scale decreases as the propagation angle increases at a particular height. This implies that Alfvén waves, propagating at a larger angle with respect to outward normal direction, dissipate more efficiently.

Figure 2 shows the combined effect of viscosity and magnetic diffusivity on the spatial variation of energy flux density (W) of high frequency Alfvén waves. The sharp decrement of the mechanical energy flux density below 25 Mm indicates the efficient viscous and resistive dissipation of Alfvén waves. At any particular height, the energy flux density decreases with the increment of propagation angle of the Alfvén waves. At any particular propagation angle, the rate of increment of W with radial height approximately agrees with that of Pekünlü *et al.* (2002) in the region between 1.05 $R_\odot$ and 1.15 $R_\odot$ However, we find the decrement of Alfvén wave energy flux density in the region beyond 1.21 $R_\odot$, which may likely indicate the Alfvén wave dissipation. O'Shea *et al.* (2005) measurements of MgX 609.78 Å and 624.78 Å lines from the CDS on SOHO, provide the variation of linewidths and line ratio in the regions

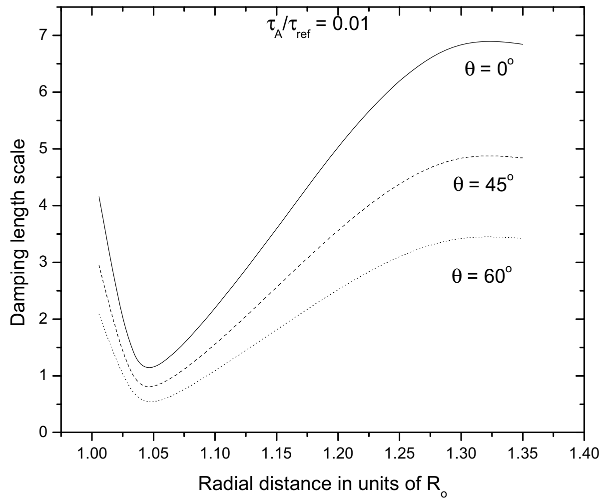

Figure 1. Spatial variation of damping length scales of Alfvén wave under combined effect of viscosity and magnetic diffusivity.

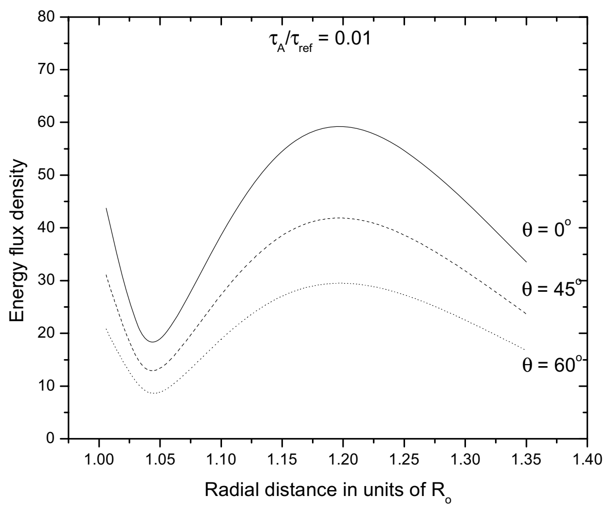

Figure 2. Spatial variation of energy flux density of Alfvén wave under combined effect of viscosity and magnetic diffusivity.

far off-limb at northern polar coronal holes. They find that linewidths show a decrement above $\sim 1.21\,R_\odot$. Dwivedi & Srivastava (2006) have calculated the non-thermal velocity from the linewidth measurements of O'Shea *et al.* (2005) at the effective ion temperature of 10^6 K (see Fig. 3). For any propagation angle, our theoretically calculated Alfvén wave energy flux density shows a decrement beyond $1.21\,R_\odot$ where non-thermal velocity, calculated by Dwivedi & Srivastava (2006) from the linewidth measurements of O'Shea *et al.* (2005), also starts reducing. These results indicate the dissipation of outwardly propagating Alfvén waves, which causes the reduction in the

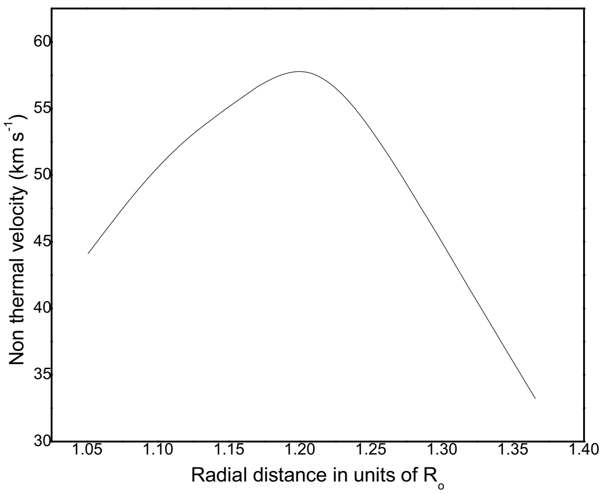

Figure 3. Spatial variation of non-thermal velocity, calculated by using the linewidth observations of O'Shea *et al.* (2005).

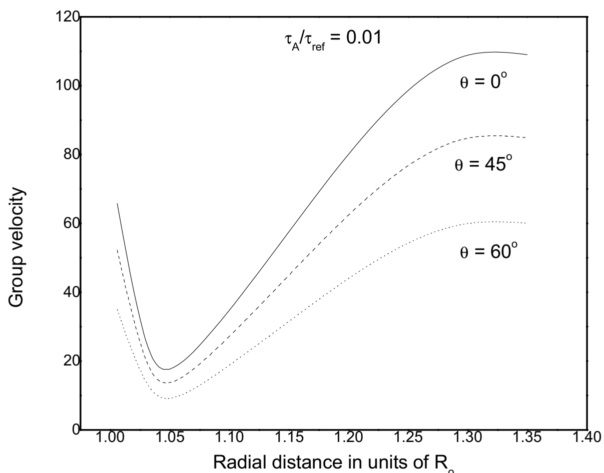

Figure 4. Spatial variation of group velocity of Alfvén wave under combined effect of viscosity and magnetic diffusivity.

non-thermal component of the observed linewidths. This result is also supported by the reduction of our theoretically calculated Alfvén energy flux density for all propagation angles and their peaks match with the peaks in the radial profiles of observed linewidths and non-thermal velocity.

Figure 4 shows the combined effect of viscosity and magnetic diffusivity on the spatial variation of group velocity of high frequency Alfvén waves. We see that the group velocity of Alfvén waves decreases in the region below 25 Mm. As the propagation angle increases, the group velocity of Alfvén wave frequency decreases at any particular height.

Tu *et al.* (2005) made the correlation of the Doppler-velocity and radiance maps of spectral lines emitted by various ions (e.g., Si II, C IV and Ne VIII) with the force-free magnetic field as extrapolated from photospheric magnetograms to different altitudes. They have reported that the solar wind outflow starts between 5 Mm and 20 Mm above the photosphere. Our results show that the region where the damping length scale and energy flux density of Alfvén waves propagating at different angles reduce sharply, coincide with the region where the solar wind outflow starts in the magnetic funnel. Therefore, we suggest that viscous and resistive dissipation of Alfvén waves below 25 Mm may be one of the primary energy sources for the solar wind outflow. As the propagation angle increases, energy flux density reduces at any particular height. At the higher propagation angle, damping length scale also reduces. This shows that although the energy flux density carried over by Alfvén waves which is propagating more steeply, is less at any particular height yet it dissipates quickly via viscous and resistive coronal hole plasma and has a significant role for the conversion of inflow of the solar wind into outflow, below 25 Mm in the underlying magnetic funnels inside the coronal holes.

In conclusion, we have emphasized the importance of the Alfvén waves propagating at different angles with respect to the normal outward direction, and their dissipation in coronal holes, supporting that solar wind outflow starts between 4 Mm and 20 Mm as reported by Tu *et al.* (2005), and also the spectroscopic signature of Alfvén wave dissipation in the off-limb coronal hole plasma as reported by O'Shea *et al.* (2005).

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