CHAPTER 3
STEADY HEAT CONDUCTION

§ 3-1 § 3-3 § 3-4

Steady Heat Conduction and Thermal Resistance Network

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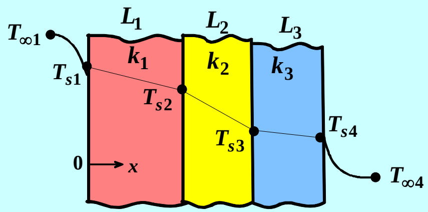
1) Conduction in a Multi-layer Plane Wall



Series Layers

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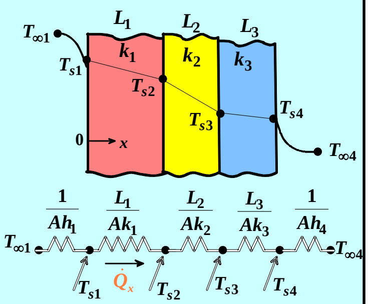
❑ Thermal Resistance Network (Series Layers)

Alternate approach to determine $\dot{Q}_x$:

$$\dot{Q}_x = \frac{\Delta T}{\sum R} = \frac{T_{\infty1} - T_{\infty2}}{R_{\text{total}}}$$

ΔT = overall temperature difference across all thermal resistances

ΣR = sum of all thermal resistances



$$\dot{Q}_x = \frac{T_{\infty1} - T_{\infty4}}{\frac{1}{A\,h_1} + \frac{L_1}{A\,k_1} + \frac{L_2}{A\,k_2} + \frac{L_3}{A\,k_3} + \frac{1}{A\,h_4}} = \frac{T_{\infty1} - T_{\infty4}}{\frac{1}{A\,h_1} + \sum_{i=1}^{3}\frac{L_i}{A\,k_i} + \frac{1}{A\,h_4}}$$

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Determining temperature at any point:

e.g. **Interface temperature T_{s2}**

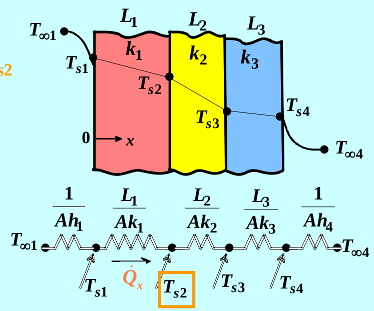
$T_1(L_1)$ **at** $x = L_1$ **:**

$$\dot{Q}_x = \frac{T_{\infty1} - T_1(L_1)}{\frac{1}{A\,h_1} + \frac{L_1}{A\,k_1}}$$

or

$$\dot{Q}_x = \frac{T_1(L_1) - T_{\infty4}}{\frac{L_2}{A\,k_2} + \frac{L_3}{A\,k_3} + \frac{1}{A\,h_4}}$$

where $\dot{Q}_x$ is already known.



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❑ Overall Heat Transfer Coefficient for a Plane Wall - U (综合传热系数)

Define: ***Overall heat transfer coefficient U*** **for plane walls**

$$\dot{Q}_x = \frac{\Delta T}{\sum R} = U A \Delta T$$

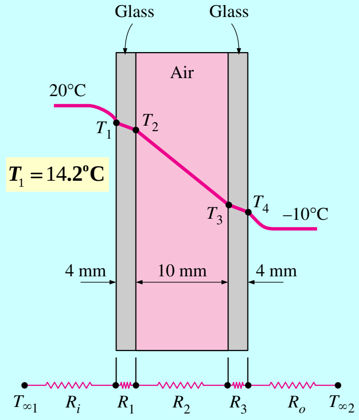
ΔT = overall temperature drop across all resistances

$$U = \frac{1}{A \sum R} \qquad (\mathrm{W/m^2 \cdot K})$$

e.g. ***U*** **for the multi-layer wall of last problem is**

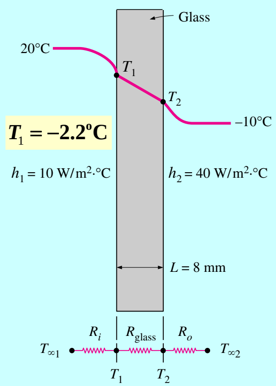
$$U = \frac{1}{\frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_4}}$$

Double-pane window



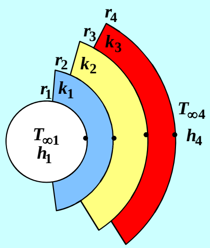
$\dot{Q} = 69.2$ W

Single-pane window



$\dot{Q} = 266$ W

2) Radial Conduction in a Multi-layer Cylindrical Wall



Assume:
(1) One-dimensional
(2) Steady state
(3) Constant conductivity
(4) No heat generation
(5) Perfect interface contact

Series Layers

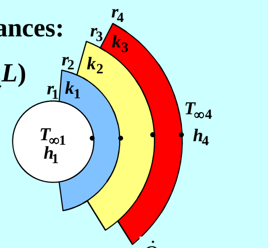
❑ Thermal Resistance Network (Series Layers)

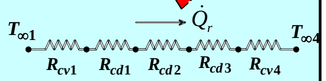
2 Conv. resistances and 3 Cond. resistances:

$R_{cv1} = 1/(2\pi r_1 h_1 L)$ $\quad R_{cv4} = 1/(2\pi r_4 h_4 L)$

$$R_{cd1} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} \qquad R_{cd2} = \frac{\ln(r_3/r_2)}{2\pi k_2 L}$$

$$R_{cd3} = \frac{\ln(r_4/r_3)}{2\pi k_3 L}$$





Heat transfer rate:

$$\dot{Q}_r = \frac{T_{\infty1} - T_{\infty4}}{\frac{1}{2\pi r_1 h_1 L} + \frac{\ln(r_2/r_1)}{2\pi k_1 L} + \frac{\ln(r_3/r_2)}{2\pi k_2 L} + \frac{\ln(r_4/r_3)}{2\pi k_3 L} + \frac{1}{2\pi r_4 h_4 L}}$$

❑ Overall Heat Transfer Coefficient for a Cylindrical Wall - *U*

Define *U* as

$$\dot{Q}_r = \frac{\Delta T}{\sum R} = U\,A(r)\,\Delta T$$

A*(*r*) = area normal to the coordinate *r

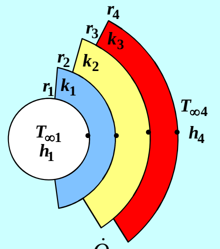
***ΔT* = the overall temperature drop**

$$U = \frac{1}{A(r)\ \sum R} \neq \text{const}$$

***A*(*r*) depends on the radius *r*. Therefore, *U* depends on *r*.**

e.g.

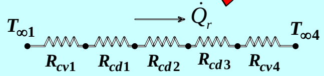
Basing *U* on the area corresponding to the inner and outer radius respectively.



$$\Delta T = (T_{\infty 1} - T_{\infty 4})$$

$$A(r_1) = 2\pi r_1 L \qquad A(r_4) = 2\pi r_4 L$$

$$U = \frac{1}{A(r)\ \sum R}$$

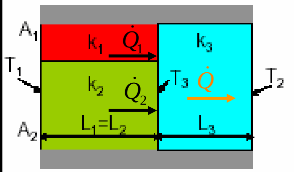


$$U_1 = \frac{1}{\dfrac{1}{h_1} + \dfrac{r_1 \ln(r_2/r_1)}{k_1} + \dfrac{r_1 \ln(r_3/r_2)}{k_2} + \dfrac{r_1 \ln(r_4/r_3)}{k_3} + \dfrac{r_1}{r_4 h_4}}$$

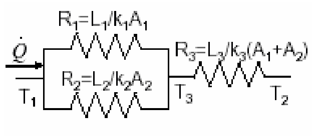
$$U_4 = \frac{1}{\dfrac{r_4}{r_1\ h_1} + \dfrac{r_4 \ln(r_2/r_1)}{k_1} + \dfrac{r_4 \ln(r_3/r_2)}{k_2} + \dfrac{r_4 \ln(r_4/r_3)}{k_3} + \dfrac{1}{h_4}}$$

3) Conduction in a composite Wall (combined parallel and series layers)

insulated



insulated



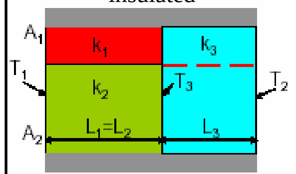
$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad R_{12} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{total} = R_{12} + R_3$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}} = \frac{T_1 - T_3}{R_{12}} = \frac{T_3 - T_2}{R_3}$$

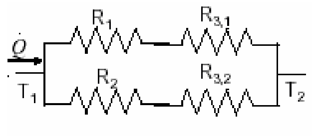
3) Conduction in a composite Wall (combined parallel and series layers)

insulated



insulated

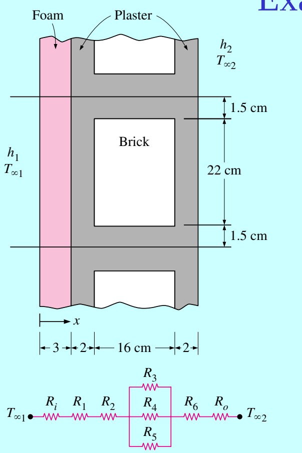
Alternatively,



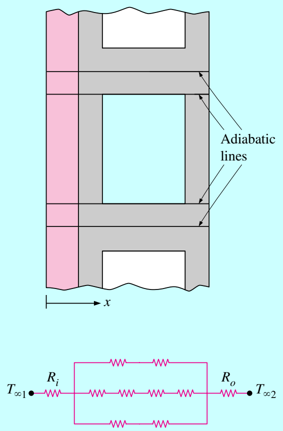
$$R_{total} = \frac{(R_1 + R_{3,1})(R_2 + R_{3,2})}{R_1 + R_{3,1} + R_2 + R_{3,2}}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

Example



or



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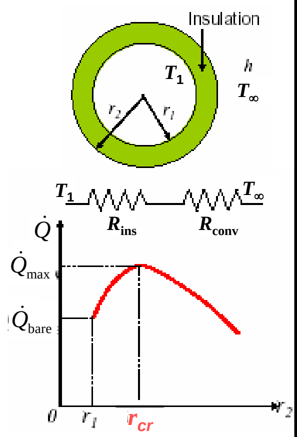
§ 3.5 Critical Radius of Insulation

- Consider a cylindrical pipe with length L and radius r_1 which is insulated by a material with thermal conductivity k.
- Total heat transfer rate through the insulation is

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}}$$

- Note that increasing r_2 increases R_{ins}, but decreases R_{conv}.
- The heat transfer rate may increase or decrease by increasing r_2.



- The value of r_2 at which heat transfer reaches its maximum value is called **critical radius of insulation**, r_{cr} .
- It is obtained by solving

$$\frac{d\dot{Q}}{dr_2} = 0$$

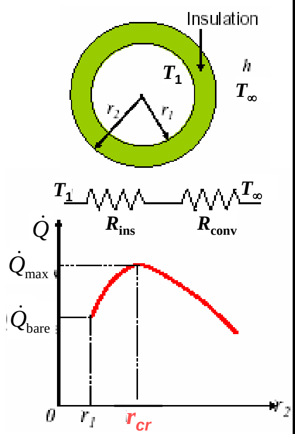
- For a cylindrical pipe

$$r_{cr,\text{cylinder}} = \frac{k}{h}$$

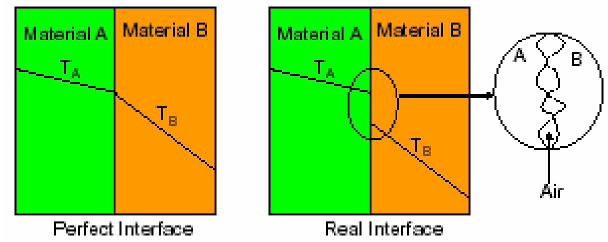
- For a spherical shell

$$r_{cr,\text{sphere}} = \frac{2k}{h}$$

e.g. If k=0.05 W/m ·°C and h=5 W/m² · °C, $r_{cr,\text{cyl}}$ =1 cm and $r_{cr,\text{sph}}$ = 2 cm.



§ 3.2 Thermal Contact Resistance



- **Perfect interface contact vs. actual contact**
- **Air gaps act as a resistance to heat flow**
- **The temperature drop depends on the *thermal contact resistance* R_c**

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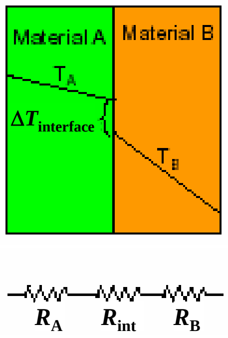
- ***Thermal contact resistance*** R_c **is determined experimentally**

Fourier's law:

$$\dot{Q} = \frac{\Delta T_{\text{interface}}}{R_{\text{int}}}$$

where ***thermal interface resistance***

$$R_{\text{int}} = \frac{R_c}{A} \qquad (^{\circ}\text{C/W})$$



- **If write $R_{\text{int}} = 1/h_c A$**

  h_c , *thermal contact conductance*

  Note that $R_c = 1/h_c$ ($m^{2}\cdot{}^{\circ}C/W$)

- **Thermal contact resistance or conductance depends on *surface conditions, roughness, pressure and temperature***

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Thermal interface materials

- **Thermal Interface Materials** are *soft, compliant, and high thermal conductivity* materials which are used between two surfaces to reduce the interface thermal resistance.

  Examples: thermal grease/paste, thermal enhanced rubbers

- They are always used between the die and heat spreader, as well as between the die or package and heat sink.
- Thermal conductivity of typical interface materials are about 1 ~ 10 W/m·°C.

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