J. Astrophys. Astr. (2003) **24,** 11–21

The Hubble Relation for a Comprehensive Sample of QSOs

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Received 2002 December 11; accepted 2003 May 31

Abstract. A correlation between redshifts (z) and apparent magnitudes (V) (Hubble relation) of Quasi Stellar Objects (QSOs) has long been sought. Such a correlation exists for galaxies whose redshifts are of cosmological origin. However, a plot of the two quantities representing the Hubble diagram for QSOs exhibits, in general, a wild scatter. This raises the question whether redshifts of QSOs are cosmological. On the other hand, most luminous QSOs in groups, and subsamples with particular properties, have been reported to show the Hubble relation. In the present paper, we analyse all optically non-variable QSOs in a comprehensive sample. In our analysis we grouped the objects into certain intervals of apparent magnitudes. Correlations obtained between redshifts and magnitudes are all statistically robust. Also, the Hubble relation in the usual form $V = 5 \log z + C$ is obeyed very convincingly for QSOs with $V < 19.5$.

Key words. Cosmology—Hubble relation—quasi stellar objects or QSOs—redshifts.

The debate over the origin of redshifts of Quasi Stellar Objects (QSOs) still continues as to whether it is cosmological, intrinsic or both. One of the tests to check the origin is to look for the correlation between their apparent magnitudes and redshifts exhibited by the linearity of a plot of the two quantities. It is known that such plots for galaxies show linearity and a significant correlation. As such a correlation between redshifts (z) and apparent magnitudes (V, m_B, M_v, m_v, B_{AV}, different symbols used by different authors) of QSOs has been sought since a long time. Sandage (1965) was probably the first to attempt a Hubble diagram for 10 QSOs then known between $17.32 < V < 18.21$, that showed "a relatively small scatter about a mean $[m, z]$ line whose equation is $m_B = 5 \log z + 18.186$". However, a plot of $(1 + \log z)$ against M_v, between $15 < M_v < 19.5$, for "all quasi-stellar objects and N-type galaxies for which data are available" by Hoyle & Burbidge (1966) demonstrated a large scatter and was inconclusive with no correlation. On the other hand, the analysis by Longair & Scheuer (1967) showed a significant correlation between $\log z$ and m_v for a sample of 75 QSOs ($15.5 < m_v < 19.5$, except 3C 273). Again, a plot of $\log z$ against $\log f(2500)$, $f(2500)$ being the flux density at the emitted frequency 2500 Å, by Schmidt (1968) for 40 3CR QSS exhibited a large scatter with no correlation.

Based on a suggestion by McCrea (1972) "that the brightest quasar at each redshift might have a significant Hubble relation", Bahcall & Hills (1973) analysed 105 QSOs

($m_v < 19.5$). A plot of $\log z$ against m_{2500} for seven of the brightest objects yielded a slope of 5.0. However, a somewhat different analysis of "all QSOs from the list of De Veny *et al.* (1971) for which at least certain photoelectric magnitude is given" ($V < 19.25$), led Burbidge & O'Dell (1973) to obtain a slope of 4.3 for the most luminous, and 3.1 and 3.2 for second and third most luminous QSOs respectively. On the other hand, in a sample of 130 QSOs ($15 < B_{AV} < 19$), brightest objects at each redshift were found by Pica & Smith (1983) "to obey the Hubble's relation quite well when variability is accounted for".

It is known that, in general, QSOs exhibit a wild scatter in plots of apparent magnitudes against redshifts. Nevertheless, subsamples with particular properties of QSOs have exhibited much less scatters and significant correlations. Thus, Setti & Woltjer (1971) demonstrated that the scatter is appreciably reduced when objects with steep radio spectra and extended structures are only plotted. Such an analysis was repeated by Stannard (1973) for 166 QSOs ($15.5 < V < 19$) who found a significant correlation between $\log z$ and V for flat radio spectra QSOs and no correlation for concave radio spectra QSOs. On the other hand, two separate subsamples of QSOs, one exhibiting radio polarization and the other exhibiting optical polarization, have been shown by Basu (1993) to obey the Hubble diagram at high levels of significance.

Any statistically significant correlation between redshifts and optical magnitudes of QSOs has thus been exhibited only by limited samples, by brightest sources, or by particular subsamples. The two main reasons for the lack of correlation in the general sample are attributed to either that the redshifts of these objects are non-cosmological, or that the luminosity functions of QSOs evolve with epochs. Possibility of at least a certain fraction of QSOs being actually blueshifting (Basu & Haque-Copilah 2001) should also be borne in mind in this respect.

However, analyses of possible evolution of luminosity functions of QSOs have so far been performed with non-Λ cosmology. As such, the question remains open as to whether the QSO redshift is of cosmological origin. We were therefore prompted to analyse QSO data with a view to find out any correlation between their redshifts and apparent magnitudes which may throw light on the cosmological origin of redshifts of QSOs. The purpose of the present paper is to report the outcome of an analysis that yields a highly significant correlation between redshifts and apparent magnitudes of QSOs, the sample used being that of Hewitt & Burbidge (1993, HB).

The HB sample is one of the largest collection of QSOs with available information of a wide range of properties. While HB by no means, presents a complete sample, it certainly represents a comprehensive sample. The most important advantage of the HB sample for the present investigation is that it provides information of the optical variability of the objects. For an analysis where the quantity apparent magnitude of the object is one of the only two parameters used, optical variability is an essential property to be taken into account. The significance of this measure is further revealed by the findings in Pica & Smith (1983) mentioned earlier, which clearly shows that optical variability is a significant criterion for studies in the redshift-magnitude relation of QSOs. The other samples that are currently available, e.g., the 2dF or SDSS, may have a relatively larger number of objects, but lack in information on variability, and, as such, are unsuitable for such studies.

The HB sample lists 7315 objects. We eliminated all BL Lac objects, all optically variable QSOs, all QSOs with doubtful redshift values, and were left with 6146 non-variable QSOs with confirmed redshifts. Although somewhat smaller than the 2dF sample or SDSS, this is certainly a huge and statistically meaningful data set. No K-correction or galactic extinction correction has been applied, since these corrections are known to be small for QSOs (Sandage 1966).

Figure 1 shows the apparent magnitude (V) distribution of the sample, at intervals of 0.5 in V, viz., $14.0 \leq V < 14.5$, $14.5 \leq V < 15.0$, etc. Figures 2(a) to 2(c)

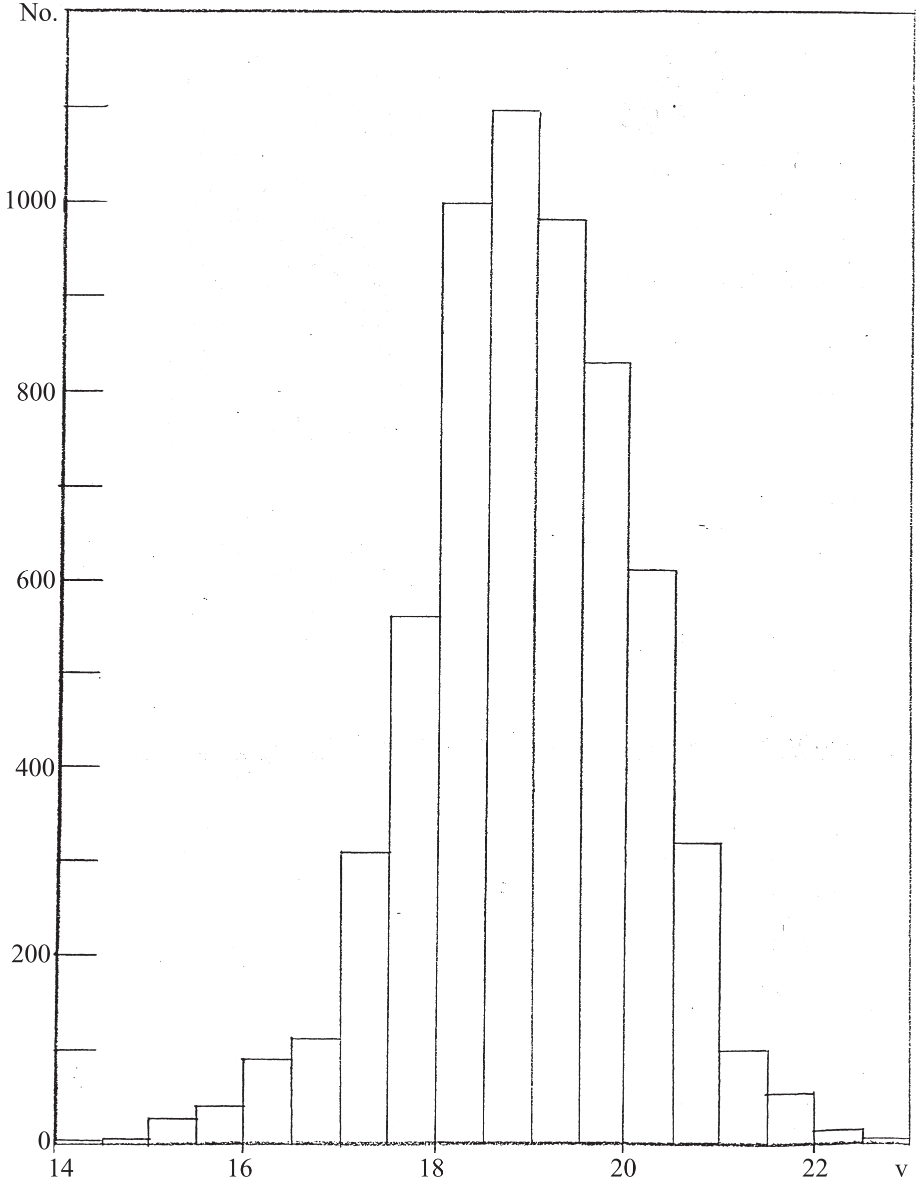

Figure 1. Distribution of apparent magnitudes (V) of non-variable QSOs with confirmed redshifts from Hewitt & Burbidge (1993, HB).

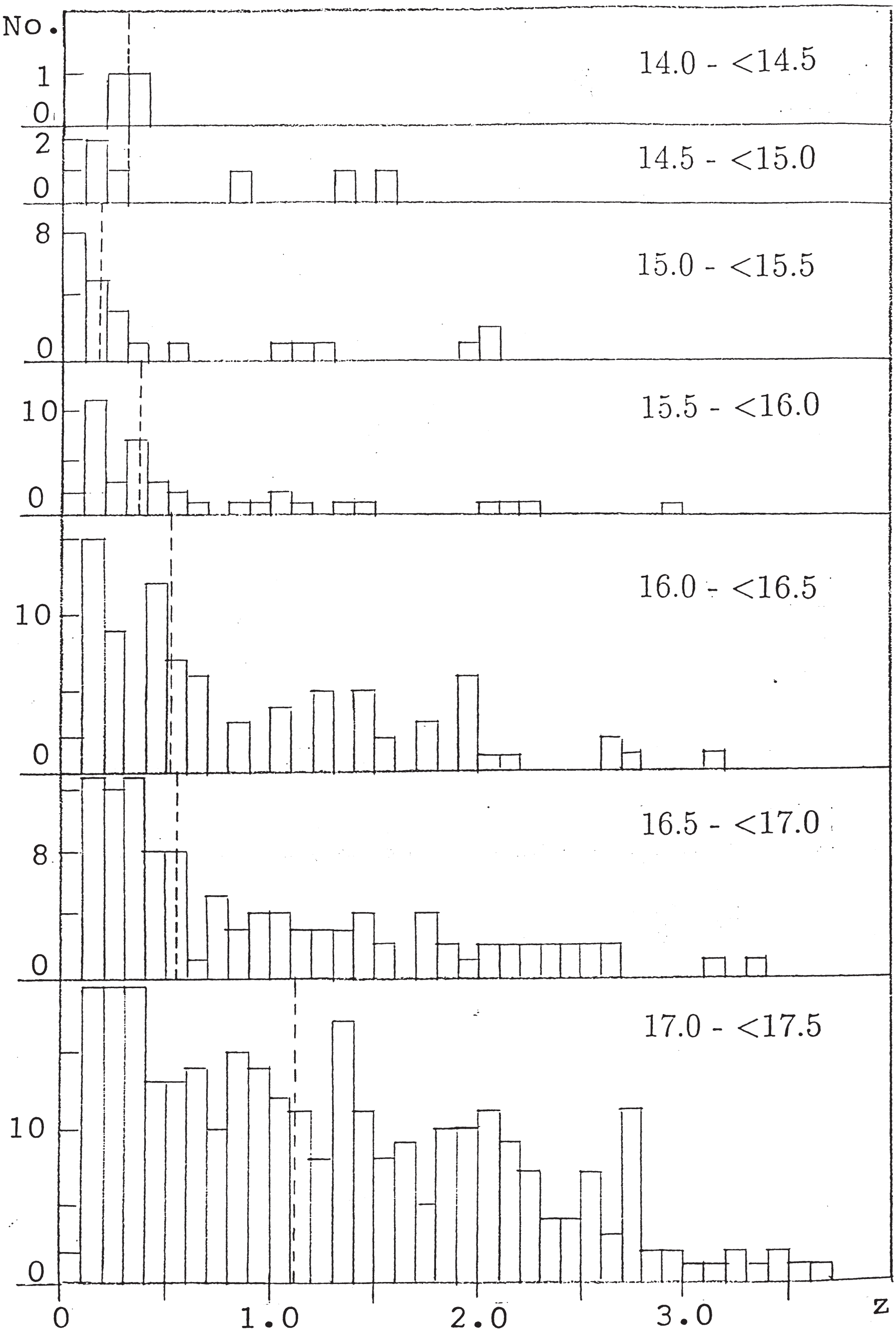

Figure 2(a). Redshift (z) distribution of our sample of QSOs (as in Fig. 1) for the apparent magnitude (V) intervals indicated in each panel. Note that each redshift bin denotes values $0 \leq z < 0.1$, $0.1 \leq z < 0.2$, etc. The dotted line indicates the position of the median ($\tilde{z}$) of the distribution.

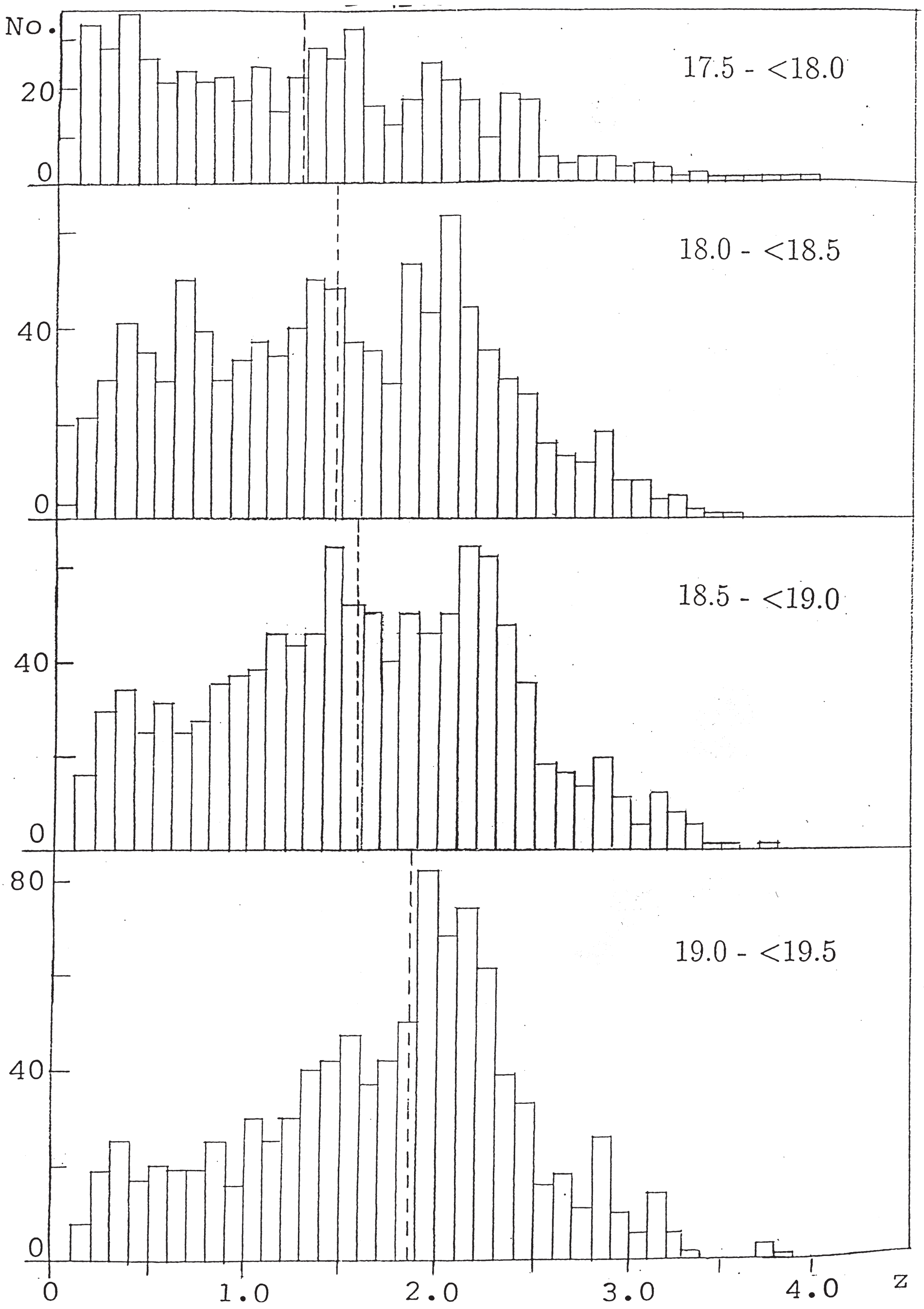

Figure 2(b). Same as in Figure 2(a).

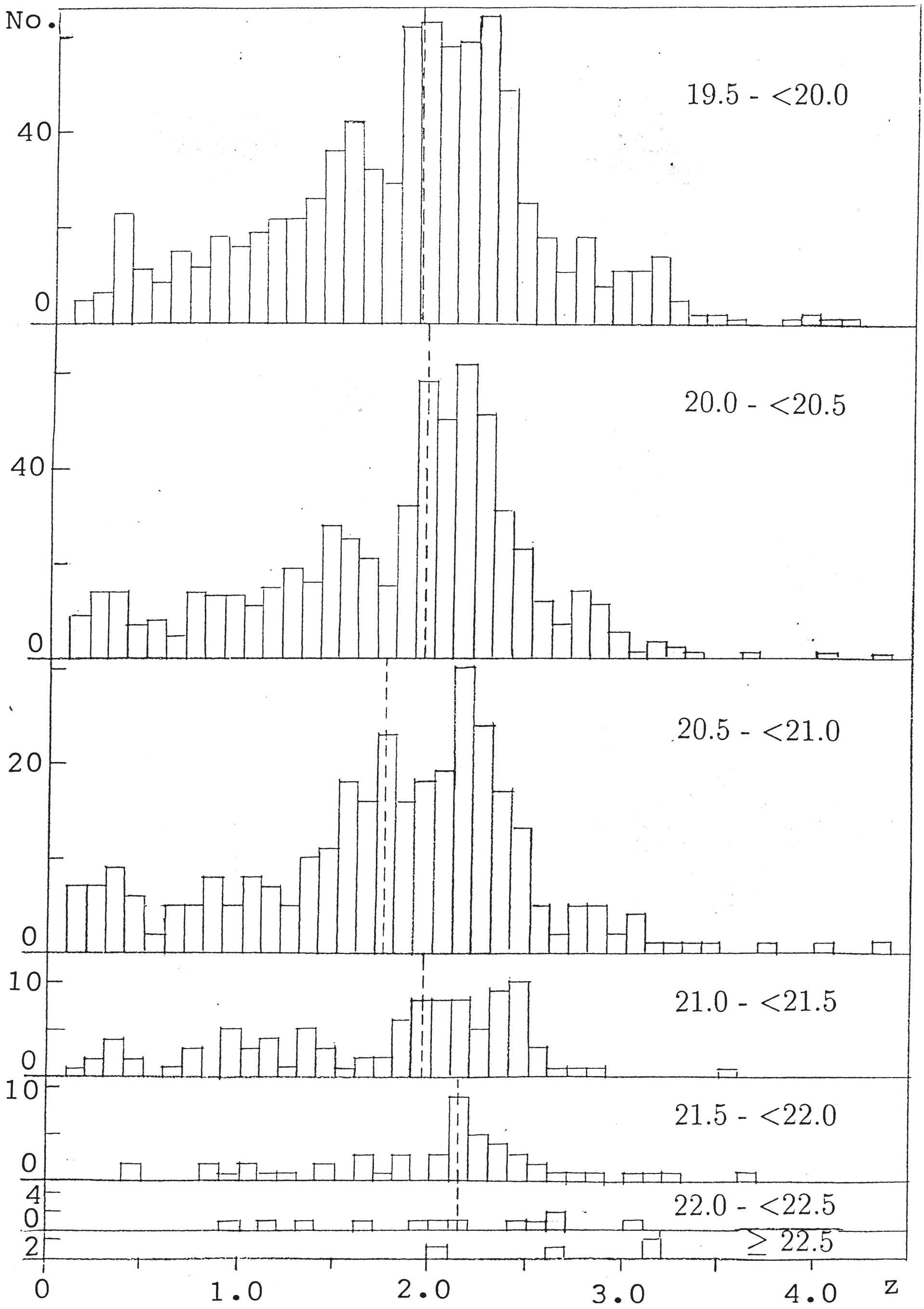

Figure 2(c). Same as in Figure 2(a). One QSO in the bin $4.7 \leq z < 4.8$, V-interval 20-<20.5, is out of scale and not shown.

show the redshift distribution of QSOs in each V-interval at bins $0 < z < 0.1$, $0.1 \leq z < 0.2$, etc. The most striking feature exhibited by Figs. 2(a) to 2(c) is that the distributions are largely asymmetrical or skewed. Moreover, the asymmetry or the skewness changes from positive to negative in the sense that the number of objects increases quite remarkably from lower redshifts to higher redshifts as one proceeds from V-intervals with smaller values of V to V-intervals with larger values of V. This led us to examine the matter further.

The central tendency of a distribution is given by the average value. Among the various measures of the average, the median is a better index of the central tendency than the arithmatic mean in the distributions of Figs. 2 that are largely asymmetrical or skewed. Additional special advantage is that the median is known not to depend on extreme values of the distribution, while the mean cannot be calculated in case of a distribution with open ends. The position of the corresponding median value ($\tilde{z}$) in each distribution is shown in Figs. 2(a) to 2(c). Furthermore, the 95% confidence limits ($\pm c_{95}$), which is a measure of the probability that 95% of the data are collected within these limits, have been calculated by using the formula (Sachs 1984)

$$c_{95} = 1.58(Q_3 - Q_1)/\sqrt{n},$$

where Q_1 and Q_3 are the first and the third quartiles respectively and n is the total number of objects in the distribution.

Table 1 shows the median ($\tilde{z}$) and the confidence limits ($\pm c_{95}$) of the distribution for each V-interval. Table 1 and Figs. 2(a) to 2(c) clearly demonstrate the trend that

Table 1. Median ($\tilde{z}$) and confidence limits ($\pm c_{95}$).

V-interval	$\tilde{z}$	$\pm c_{95}$
14.0-< 14.5	0.3000	0.1117
14.5-< 15.0	0.3000	0.8869
15.0-< 15.5	0.1800	0.1693
15.5-< 16.0	0.3571	0.0287
16.0-< 16.5	0.5214	0.1556
16.5-< 17.0	0.5625	0.1731
17.0-< 17.5	1.1364	0.1244
17.5-< 18.0	1.2795	0.0833
18.0-< 18.5	1.4615	0.0623
18.5-< 19.0	1.5923	0.0759
19.0-< 19.5	1.8590	0.2693
19.5-< 20.0	1.9135	0.0467
20.0-< 20.5	1.9466	0.0545
20.5-< 21.0	1.7469	0.0761
21.0-< 21.5	1.9524	0.1914
21.5-< 22.0	2.1500	0.1600
22.0-< 22.5	2.1500	0.4218
≥ 22.5	3.2000	0.7505

Table 2. Correlation coefficient (r), slope (s) and constant (A).

Range $14.0 \leq V$	r	s	A
< 19.5	0.9429	4.9826	17.6393
< 20.0	0.8432	6.2112	18.1261
< 20.5	0.9428	5.6338	17.8569
< 21.0	0.9303	6.1728	18.0111
< 21.5	0.9232	6.6534	18.1324
< 22.0	0.9196	7.0922	18.2348
< 22.5	0.9194	7.5873	18.3460
≥ 22.5	0.9274	7.8186	18.3776

the quantity $\tilde{z}$ increases with the quantity V in a systematic manner. This led us to pursue the matter in more detail.

We plotted $\log \tilde{z}$ against V_{mp}, where V_{mp} is the mid-point of the corresponding V-interval, viz., $V_{\text{mp}} = 14.25$ for the interval $14.0 \leq V < 14.5$, $V_{\text{mp}} = 14.75$ for the interval $14.5 \leq V < 15.0$, etc. The relationship was found to be linear. We therefore computed the correlation coefficient (r) between $\log \tilde{z}$ and V_{mp}, and the equation of the best fitting least square regression line for the linear relation for all the individual V-ranges, viz., between $V = 14.0$ and $V < 19.5$ to $V \geq 22.5$ in steps of 0.5 in V. The values of r, and the Hubble slope (s) and the corresponding constant (A) for the line defining the Hubble relation

$$V_{\text{mp}} = s \log \tilde{z} + A$$

are given in Table 2.

It is enlightening to notice in Table 2 that all the correlations yield very high values and are statistically significant at 99.99% level, the correlation coefficient having the maximum value for the V-range $14.0 \leq V < 19.5$. It is very unlikely that such statistically robust correlations have appeared by chance. The comprehensive sample of QSOs therefore demonstrate a highly significant relationship between confirmed redshift values and non-variable apparent magnitudes. If there is no other major factor involved, viz., luminosity evolution, the above findings would imply that QSO redshifts are of cosmological origin.

An additional feature noticed in Table 2 is that, for the V-range $14.0 \leq V < 19.5$ which yields the maximum value of the correlation coefficient, the slope (s) is very close to 5.0. Fig. 3 shows the plot of $\log \tilde{z}$ against V_{mp} for the V-range $14.0 \leq V < 19.5$ and the corresponding best fitting least square regression line. The line for the slope $s = 5.0$ is also shown in the figure. The closeness of the two lines is at once evident. It should be noted that $s = 5.0$ is the slope expected "from the expansion of the universe if luminosities are evaluated assuming quasars are at cosmological distances" (Bahcall & Hills 1971).

To investigate the matter further, we performed the usual t-test to test the validity of the hypothesis that the slope exhibited by the line for the V-range $14.0 \leq V < 19.5$ is *not* statistically different from 5.0 magnitude per decade, where

$$t = (5 - s)/\sigma_s.$$

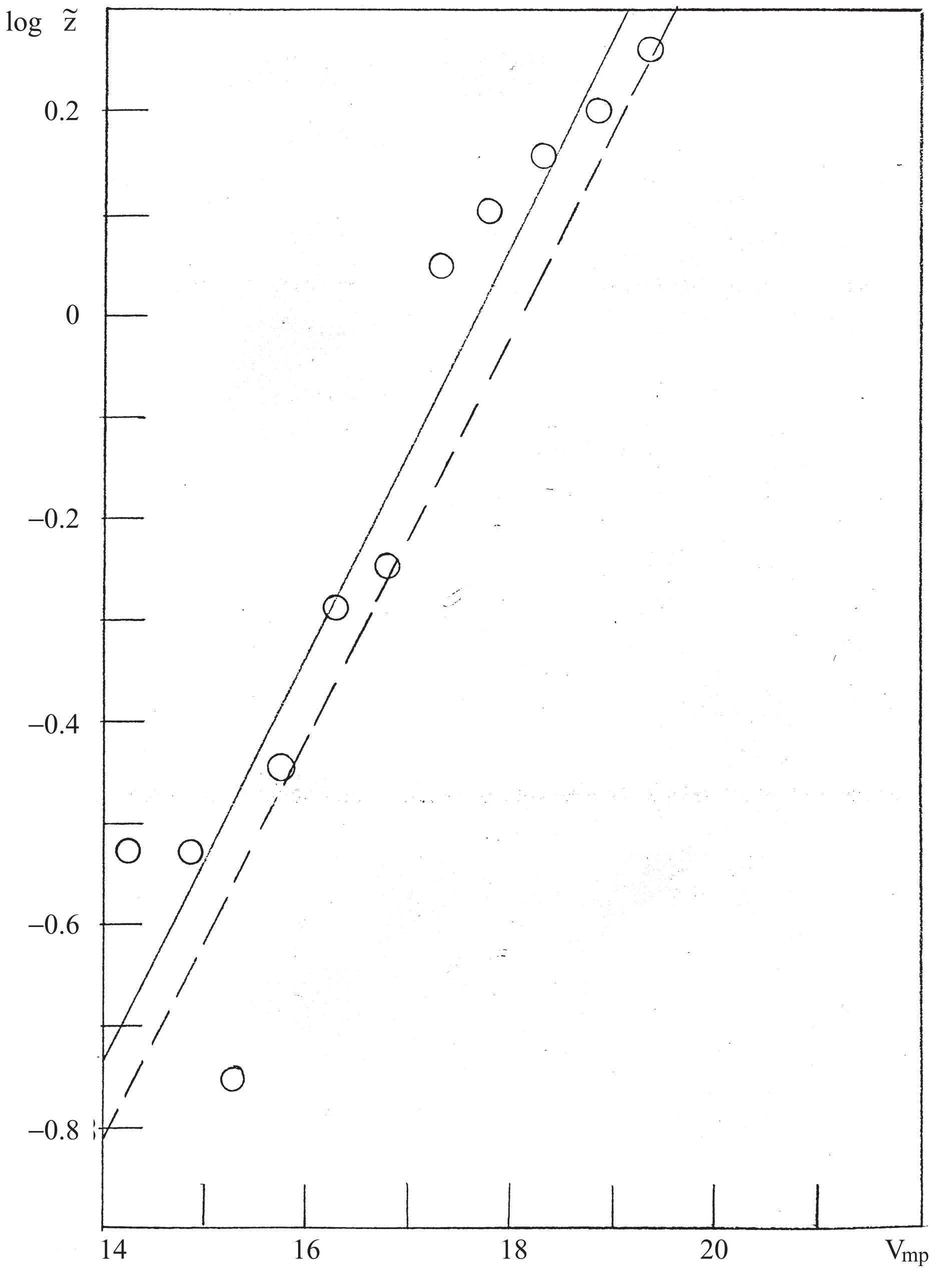

Figure 3. Plots of log $\tilde{z}$ against V_{mp} (the mid-point of each V-interval in Figures 2) for the V-range $14.0 \leq V < 19.5$. The solid line is the best fitting least square regression line $V_{mp} = 4.9826 \log \tilde{z} + 17.6393$, and the dotted line represents the Hubble relation $V = 5 \log z + 18$.

σ_s is the root mean square of the vertical deviations of the observed $\log \tilde{z}$ values from the best fitting least square line. It is estimated as

$$\sigma_s = [(D_1^2 + D_2{}^2 + \ldots D_N^2)/N]^{1/2},$$

where $D_1, D_2, \ldots D_N$ $(N = 11)$ are the differences between the values of $\log \tilde{z}$ observed and those calculated from the fitted straight line for each V_{mp} plotted in the Fig. 3. With $\sigma_s = 0.1118$ and 9 degrees of freedom, the value of $t = 0.1556$ is significant only at 55% level, indicating that the departure of the slope from the value of 5.0 is *not* statistically significant. The t-test for all other V-ranges in Table 2 yielded significance levels above 95% (not shown here), indicating that the departures are statistically significant.

It appears from Table 1 that the confidence limits of $\tilde{z}$ are rather high for the V interval $14.5 \leq V < 15.0$, and are also somewhat high for two others, viz., $14.0 \leq V < 14.5$ and $15.0 \leq V < 15.5$. Any possible error involved in the determination of $\tilde{z}$ implied by these few higher values of confidence limits may alter the corresponding correlation coefficients presumably by certain small amounts, but would have little impact on the relationship reported here, since the values of the correlation coefficient are already very high. The same is true for the closeness of the Hubble slope (s) to the value of 5. The significance level of the t-test for the interval $14.0 \leq V < 19.5$ is quite low. A small change in the value of t due to any similar error and the consequent change in the significance level would not have any appreciable effect on the result. The conclusion reached in the present analysis therefore will not change, although values of r, s, t and σ_s may change slightly. Having said that, it should be noted that the comparitively higher values of confidence limits in the few cases mentioned above are due to the nature of the distribution produced by a rather small number of objects at those V-intervals as can be seen in Fig. 2(a). It is expected that additional objects will be discovered in future, and the matter should be followed up.

The investigation presented above demonstrates that redshifts of QSOs are correlated with their apparent magnitudes. Further, such correlation exists over the entire range of apparent magnitudes studied here, viz., $14.0 \leq V \geq 22.5$. Results of the analysis suggest that redshifts of QSOs are of cosmological origin, similar to redshifts of galaxies. The importance of the present study is that it has been done for a comprehensive sample, representing the general population of QSOs, as opposed to studies in subsamples, limited samples, or specially selected samples mentioned earlier.

Furthermore, the study also finds that QSOs with $14.0 \leq V < 19.5$ obey the Hubble relation of the form $V = 5 \log z + C$ very convincingly, while significant departures from this relation is noticed as more and more weaker sources are added to the analysis. It may be noted that $V = 19.5$ may be the “selection limit” (Longair & Scheur 1967). As for the sample used in the present analysis, an inspection of Fig. 1 would show that the number of QSOs fall off very rapidly at $V \geq 19.5$. There appears to be a relative paucity of data for weaker QSOs. It will be of great interest to see the outcome of such analysis as future observations yield data of more and more weaker sources.

Finally, in view of the findings of the present analysis, evolution of the luminosity function should be reexamined in the light of the currently developed Λ-cosmology. This is of course out of scope of this work.

Acknowledgement

The author thanks an anonymous referee for helpful suggestions.

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