Measurements of the Velocity-Pressure Correlation in a Circular Turbulent Jet *1

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Measurements of the velocity-pressure correlations, double- and triple-velocity correlations called the pressure-diffusion, Reynolds stress and the turbulent transport, respectively, in the self-preserving circular turbulent jet were carried out in the water tank in order to evaluate the magnitude and the structure of the turbulent diffusion terms. The velocity and the pressure in the jet were simultaneously measured by an X-type hot film sensor and a pressure transducer with a fine static pressure tube respectively. Measured data show that the structure of the turbulent transport in the circular turbulent jet is similar to the structure modeled by differentiating the Reynolds stress called the gradient diffusion model when the pressure-diffusion is unrelated, however, the structure of the pressure-diffusion and the turbulent transport are different from the gradient diffusion model when these terms should be modeled together. The problems and possibility of the modeling of the pressure-diffusion term are discussed using measured data.

Keywords : *Velocity-Pressure Correlation, Pressure Diffusion, Circular Turbulent Jet, Pressure Fluctuation, Hot Film Sensor, Turbulence Modeling*

1. Introduction

In the present time, the numerical simulation using the time-averaged turbulence transport equations is a powerful and practical tool to study a very high Reynolds number flow, which is like a flow past a ship. The study of the turbulence modeling, which is an indispensable problem to solve the basic equations of the turbulent flow numerically, has been evolving with the development of the computational fluid dynamics. It is particularly important to study the Reynolds stress transport equation, due to the fact that the turbulent kinetic energy equation, known as more practical turbulence transport equation, is also derived from the Reynolds stress transport equation.

The pressure-diffusion term, one of the terms of the Reynolds stress transport equation, has been disregarded in the process of the modeling because the term has been considered small compared to the turbulent transport term or the term has been considered a part of the turbulent diffusion terms. Recently, we consider that the pressure-diffusion is an important function in a high Reynolds number turbulent shear flow and it has to be studied.

Two methods to confirm a turbulence modeling are known. The first one is the direct numerical simulation (DNS), but the DNS has no capability of being used for a high Reynolds number flow. The second one is a measurement. Measurements for the Reynolds stress and the turbulent transport have been carried out by many researchers [1)2)3)], however, it has been considered that measurements for the pressure-diffusion, the redistribution, and the dissipation are impossible. Shirahama and Toyoda [4)] introduced a measurement method of the pressure-diffusion in the wind tunnel. They used an X-type hot film sensor for the measurement of fluctuating velocities and a microphone for the measurement of the fluctuating pressure

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in order to measure the pressure-diffusion in the jets.

In accordance with Shirahama and Toyoda method, we measured the pressure-diffusion in a circular turbulent jet. An X-type hot film sensor for the measurement of the velocities was also used. Meanwhile, we carried out experiments in a water tank because lower frequency was expected, as compared to an experiment in a wind tunnel, and we adopted a pressure transducer for the purpose of a measurement of a pressure fluctuation. Our total system had a sufficient performance, since the predicted frequency range of the flow was up to 1kHz.

The flow which we investigated is a self-preserving circular turbulent jet. It is known that the jet is a self-preserving flow for the time-averaged velocity, when $x/D > 8$, where D is the inside diameter of the nozzle, and x is the distance from the nozzle. It is also known that the jet, where $x/D > 8$, is not a self-preserving flow for the Reynolds stress which is still developing even when $x/D = 40$ [5]. Görtler's theory and Tollmien's theory [6] are known as principal theories of jets. According to Görtler's theory, the eddy viscosity ν_t is considered a constant value, and its time-averaged velocity distributions agree with experiments in the central part of the jet. Nevertheless, the distributions are shifted from experiments in the peripheral part of the jet. On the contrary, the time-averaged velocity distributions of Tollmien's theory are shifted from experiments in the central part of the jet; however, the distributions agree with experiments in the peripheral part of the jet. A gradient diffusion model of the Reynolds stress under the Görtler's assumption ($\nu_t = const.$) will be discussed, and in addition the Görtler's assumption modified by an intermittency factor will be studied.

2. Pressure-Diffusion

The following equation is referred to as the Reynolds stress transport equation :

$$
\begin{aligned}
\partial_t R_{ij} &+ \overline{v}_k \partial_k R_{ij} \\
&= P_{ij} + T_{ij} + \Sigma_{ij} + \pi_{ij} + \Phi_{ij} - \varepsilon_{ij}, \quad (1) \\
P_{ij} &= -R_{jk}\partial_k \overline{v}_i - R_{ik}\partial_k \overline{v}_j, \quad (2) \\
T_{ij} &= -\partial_k(\overline{v_i' v_j' v_k'}), \quad (3) \\
\Sigma_{ij} &= \nu \partial_k \partial_k R_{ij}, \quad (4) \\
\pi_{ij} &= -\frac{1}{\rho}\partial_k(\overline{p' v_i'}\delta_{jk} + \overline{p' v_j'}\delta_{ik}), \quad (5) \\
\Phi_{ij} &= \frac{1}{\rho}\overline{p'(\partial_i v_j' + \partial_j v_i')}, \quad (6) \\
\varepsilon_{ij} &= 2\nu \overline{\partial_k v_i' \partial_k v_j'}, \quad (7)
\end{aligned}
$$

where $R_{ij} = \overline{v_i v_j}$ is the Reynolds stress, P_{ij} is the production, T_{ij} is the turbulent transport, Σ_{ij} is the molecular diffusion, π_{ij} is the pressure-diffusion, Φ_{ij} is the redistribution, ε_{ij} is the dissipation and δ_{ij} is the Kronecker delta. P_{ij} , T_{ij} and Σ_{ij} are measurable but Φ_{ij} and ε_{ij} are unmeasured terms.

It is rare to use this equation itself for application problems, because eleven basic equations are required for the numerical simulation. Accordingly six equations of Eq.(1) in eleven basic equations are often reduced to one equation called the turbulent kinetic energy equation. In the present study, we could not investigate the turbulent kinetic energy k, because we do not have a triple hot film sensor which can be used in the water, therefore, we go back to the Reynolds stress transport equation, and study the pressure-diffusion of the equation.

The Reynolds stress transport equation (Eq.(1)) cannot be solved without the modeling for T_{ij}, Σ_{ij}, π_{ij} and ϕ_{ij}, since the equation is not a closure. In this paper, we focus on the diffusion terms ($D_{ij} = T_{ij} + \Sigma_{ij} + \pi_{ij}$) of the equation. It is well-known that Σ_{ij} is small, compared with T_{ij} and π_{ij}, in view of $\nu \ll \nu_t$, consequently hereafter we ignore Σ_{ij}. The most fundamental modeling of D_{ij} is a gradient diffusion model of the Reynolds stress :

$$\overline{v_i' v_j' v_k'} + \frac{1}{\rho}(\overline{p' v_i'}\delta_{jk} + \overline{p' v_j'}\delta_{ik}) \approx -\frac{\nu_t}{\sigma_k}\partial_k(\overline{v_i' v_j'}), \quad (8)$$

where σ_k is the turbulent Prandtl number considered a constant of 1. From now, the velocity fluctuations (v_1', v_2') in the x- and y-direction are expressed as u' and v' respectively. In this paper, the number of cases in Eq.(8) will be measured and discussed.

3. Experimental Setup

Experiments were performed in the circulating water channel at the National Defense Academy. The principal dimensions of the observation part of the circulating water channel are a length of 5.0m, a width of 1.8m and a depth of 1.0m. The circulating water channel was activated only in the course of calibrating a hot film sensor, and the circulating water channel was used only as a static water tank while the turbulence in the circular turbulent jet was measured.

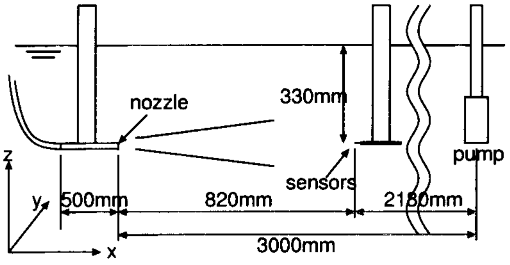

Fig. 1 Experimental setup.

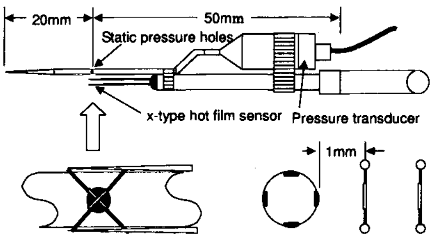

Fig. 2 Static pressure tube and X-type hot film sensor.

A plastic pipe used as a nozzle ($20mm\phi$), a water pump and sensors were installed as illustrated in Fig.1. The nozzle and the water pump were connected by a hose ($23mm\phi$), and a circular turbulent jet was generated by operating the pump. The nozzle was perpendicularly cut off. The straight length of the pipe was 500mm.

The coordinate system in this paper is also shown in Fig.1.

An X-type hot film sensor (TSI 1241-20W), which is able to measure 2-components (x-y) fluctuating velocities simultaneously, was used. The length of sensing area of the sensor was 1.02mm, and the diameter of the support of the sensor was 3.2mm.

As shown in Fig.2, the fluctuating pressure was measured by a very fine static pressure tube (outside diameter 1mm, inside diameter 0.8mm) with the pressure transducer (SSK P204-02, rated capacity:0.2kg/cm^2). Four static pressure holes, whose diameter is 0.4mm, were in the position of 20mm from the tip of the static pressure tube. The closest static pressure hole was set 1mm apart from the hot film sensor probe.

The fluctuating velocity and pressure distributions

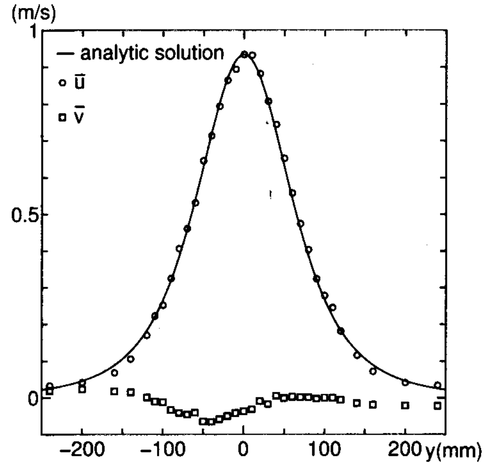

Fig. 3 Time-averaged velocity distributions and Görtler's theory ($z = 0$).

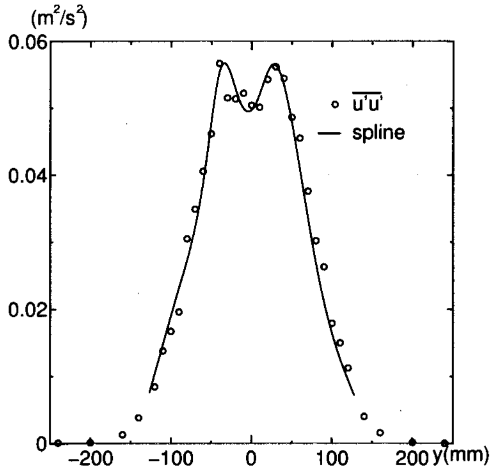

Fig. 4 Reynolds stress $\overline{u'u'}$ and hand-traced spline curve ($z = 0$).

were measured by a digitizer (IFA300) simultaneously, and the measured data were transferred to a computer.

4. Experimental Results and Discussions

Calibration for the X-type hot film sensor was performed using velocities in a uniform flow generated by activating the circulating water channel. The voltage of the uniform flow was measured, and the measured voltage was calibrated to the velocities measured by a pitot tube. During calibrations, the nozzle and the water pump in Fig.1 were removed. The calibration

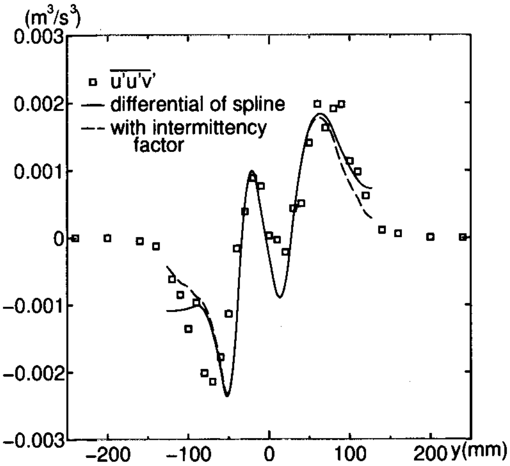

Fig. 5 Turbulent transport $\overline{u'u'v'}$ and differential of Reynolds stress $\overline{u'u'}$ ($z = 0$).

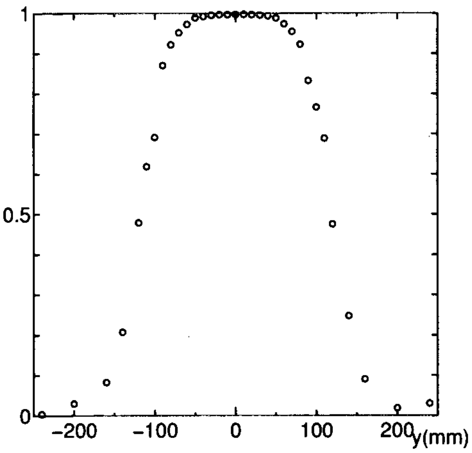

Fig. 6 Measured intermittency factor in jet ($z = 0$).

data for velocity were fitted to a fourth dimensional function, as was usually used. The measuring time for every measuring point in this study was 52 seconds. The sampling rate was 5kHz.

Calibration for the static pressure was conducted by the pressure transducer with the static pressure tube going up and down in the z-direction in the still water. The calibration data for the static pressure were fitted to a linear function.

In Fig.3, the time-averaged velocity distributions at 820mm distance from the nozzle are shown. Comparing Görtler's theory [6] based on the maximum velocity and the half velocity, measured velocity distributions of the jet in $-100mm < y < 100mm$ region agree with the theory. Nevertheless, the velocity distributions outside of $y = \pm 150mm$ are slightly smaller than the theory , as mentioned before.

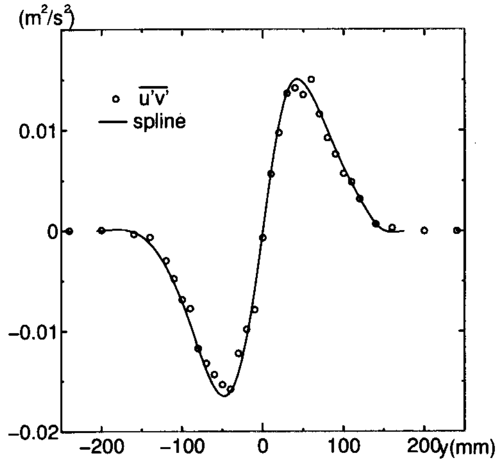

Fig. 7 Reynolds stress $\overline{u'v'}$ and hand-traced spline curve ($z = 0$).

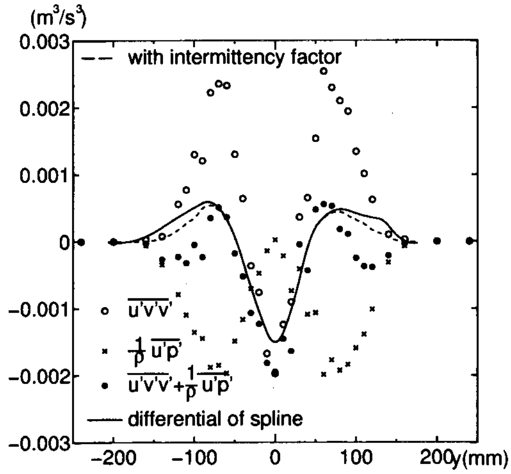

Fig. 8 Pressure-diffusion $\overline{u'p'}$, turbulent transport $\overline{u'v'v'}$ and differential of Reynolds stress $\overline{u'v'}$ ($z = 0$).

From now, the gradient diffusion model of the Reynolds stress shown in Eq.(8) will be compared with the measured data, and accuracy of the modeling will be investigated.

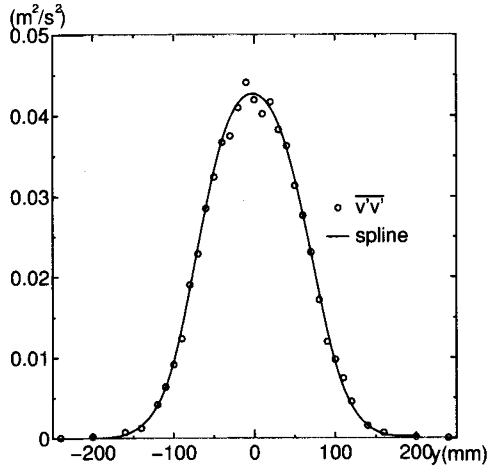

Fig. 9 Reynolds stress $\overline{v'v'}$ and hand-traced spline curve ($z = 0$).

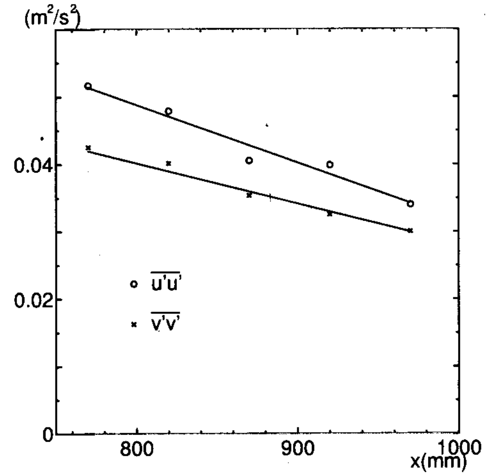

Fig. 11 Reynolds stress in x-direction ($y = 0$).

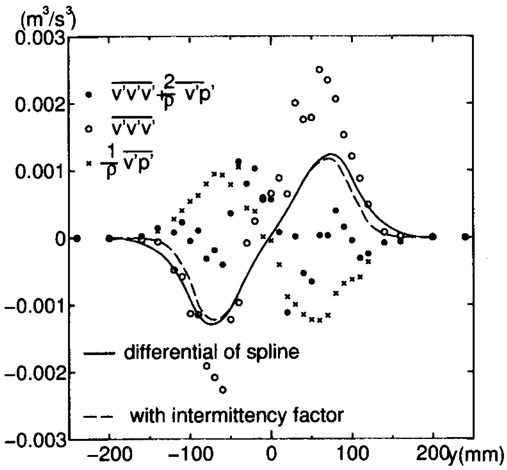

Fig. 10 Pressure-diffusion $\overline{v'p'}$, turbulent transport $\overline{v'v'v'}$ and differential of Reynolds stress $\overline{v'v'}$ ($z = 0$).

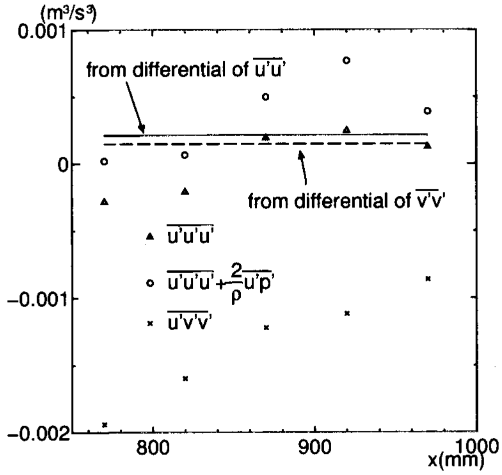

Fig. 12 Pressure-diffusion and turbulent transport in x-direction ($y = 0$).

Eq.(8) when $i = j = 1, k = 2$ is expressed as $\overline{u'u'v'} \approx -\nu_t/\sigma_k \cdot \partial_2(\overline{u'u'})$, which does not include the pressure-diffusion. Fig.4 shows the measured Reynolds stress $\overline{u'u'}$. A well-known two peak structure can be found. The solid line in Fig.4 is a hand-traced spline curve of $\overline{u'u'}$, and differential of the curve in the y-direction is drawn in Fig.5 as a solid line. It obviously can be said that the turbulent transport $\overline{u'u'v'}$ corresponds to the differential of the $\overline{u'u'}$ curve under the condition : $\nu_t/\sigma_k = 2.5 \times 10^{-3}$, which is determined in order to equalize these two quantities. According to Görtler's theory, the eddy viscisity ν_t in the jet is considered a constant, and σ_k is usually defined as a constant of 1. It is evident from the above consideration that the gradient diffusion model of the Reynolds stress $\overline{u'u'}$ for the turbulent transport $\overline{u'u'v'}$ without the pressure-diffusion is an accurate modeling.

The time-averaged velocity distributions disagree with experimental results in the peripheral part of the jet, as mentioned before, because ν_t of Görtler's theory is considered a constant. There are two states in jet, a laminar flow and a turbulent flow. The rate in time when the state is a turbulent flow is called the intermittency factor. The eddy viscosity ν_t is improved by the intermittency factor γ as $\nu_t = \gamma\nu_{tc}$, where ν_{tc} is the eddy viscosity at the center of the jet. In Fig.6, the intermittency factor obtained from the measured velocity distributions is shown. The flow is considered as a turbulent flow, when the absolute value of the time derivative of the velocity in the x-direction is larger than 3/2, and when the above condition is found for two consecutive times. The constant value 3/2 is determined as the intermittency factor becomes 1 in the central part of the jet, and 0 in the peripheral part. The dash line in Fig.5 is the improved curve of the solid line by the intermittency factor. Evident difference is not found between both lines.

Second, we discuss the gradient diffusion model of the Reynolds stress $\overline{u'v'}$ for the turbulent transport $\overline{u'v'v'}$ and the pressure-diffusion $1/\rho \cdot \overline{u'p'}$. The turbulence modeling is expressed as $\overline{u'v'v'} + 1/\rho \cdot \overline{u'p'} \approx -\nu_t/\sigma_k \cdot \partial_2(\overline{u'v'})$, where $\nu_t/\sigma_k = 2.5 \times 10^{-3}$, which is defined above. Measured $\overline{u'v'}$ distributions are shown in Fig.7. The solid line in the figure is the hand-traced spline curve of $\overline{u'v'}$. The differential of the spline curve is shown in Fig.8 as a solid line. It obviously can be found that to ignore the pressure-diffusion is inadequate, and the turbulent transport $\overline{u'v'v'}$ should be modeled with the pressure-diffusion $1/\rho \cdot \overline{u'p'}$. From Fig.8, disagreement between the solid line and the black round marks outside of $\pm 50mm$ can be found, although the magnitude of the solid line agrees with the black round marks inside of $\pm 50mm$. The dash line is the improved line of the solid line by the intermittency factor shown in Fig.6. The dash line is better than the solid line, however, it can be said that the gradient diffusion model of the Reynolds stress $\overline{u'v'}$ cannot sufficiently explain the structure of the turbulent transport $\overline{u'v'v'}$ and the pressure-diffusion $1/\rho \cdot \overline{u'p'}$. The fact suggests that an individual modeling for each term is necessary.

Next, we discuss the gradient diffusion model of Reynolds stress $\overline{v'v'}$ for the turbulent transport $\overline{v'v'v'}$ and the pressure-diffusion $1/\rho \cdot \overline{v'p'}$. Measured Reynolds stress $\overline{v'v'}$ is shown in Fig.9, and the solid line is the hand-traced spline curve of $\overline{v'v'}$. The solid line in Fig.10 is the differential of the spline curve, when $\nu_t/\sigma_k = 2.5 \times 10^{-3}$. The form of the curve is similar to $\overline{v'v'v'}$, however, the magnitude is different. Comparing between the solid line and the black round marks, it can be said that the turbulent transport $\overline{v'v'v'}$ may be modeled by the gradient diffusion model of the Reynolds stress $\overline{v'v'}$, but an independent modeling for the pressure-diffusion $1/\rho \cdot \overline{v'p'}$ may be needed. From the dash line of Fig.10, it is found that the problem of the modeling is not resolved although the intermittency factor is adopted.

So far, we have been discussing gradient diffusion models in the y-direction. Finally, we discuss a gradient diffusion model in the x-direction. In Fig.11, the Reynolds stress distributions in the x-direction are shown. It is seemed that the distributions are decreasing linearly. The gradient diffusion model demands that $\overline{u'u'u'} + 2/\rho \cdot \overline{u'p'}$ and $\overline{u'v'v'}$ be constants, however, as shown in Fig.12, they are not constants. It seems that the gradient diffusion model of the Reynolds stress in the x-direction for the turbulent transport and the pressure-diffusion collapses.

The time-averaged experimental data introduced in this paper and some supplementary explanations in Japanese are opened to the public domain: http://www.nda.ac.jp/cc/users/yabu [8].

5. Conclusions

The pressure-velocity correlation, double- and triple-velocity correlations in a self-preserving circular turbulent jet were measured in the water tank by means of an X-type hot film sensor and a pressure transducer with a very fine static pressure tube in order to confirm an accuracy of a turbulent modeling for the turbulent transport and the pressure-diffusion.

The magnitude of the pressure-diffusion is sufficiently so large in a circular turbulent jet that it may not be disregarded.

The gradient diffusion model of the Reynolds stress $\overline{u'u'}$ in the y-direction agrees with the measured turbulent transport $\overline{u'u'v'}$, when the pressure-diffusion is unrelated to the modeling.

Nevertheless, the gradient diffusion model of the Reynolds stress $\overline{u'v'}, \overline{v'v'}$ in the y-direction disagrees with adding the measured turbulent transport $\overline{u'v'v'}$, $\overline{v'v'v'}$ and pressure-diffusion $1/\rho \cdot \overline{u'p'}$, $2/\rho \cdot \overline{v'p'}$ respectively, when the pressure-diffusion is related to the modeling.

The gradient diffusion model modified by an intermittency factor is better than the modeling of $\nu_t = const.$, however, the accuracy of the modified modeling is insufficient.

The gradient diffusion model of the Reynolds stress $\overline{u'u'}$, $\overline{v'v'}$ in the x-direction has no capability of illustrating the turbutent transport and the pressure-diffusion.

The limitation of the gradient diffusion model for adding the turbulent transport and the pressure-diffusion is found.

6. Acknowledgements

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Discussion

[Discussion] (Kobe University of Merchant Marine) Shigeru Nishio

I am pleased if you show me the characteristics of your system in frequency domain. Do you find any large difficulty with the phase delay or the damping of measured pressure, which would be caused by the very fine static tube, the limited capability of pressure sensor and the characteristics of signal processing system?

[Author's Reply]

Thank you for your discussion. We used an A/D converter (digitizer) which can acquire multiple channel voltages simultaneously. According to the details of the specification in catalog data for the pressure transducer and the hot film sensor, our measuring instruments have enough frequency capability for the measurement targets. We also considered the frequency response characteristic of the pressure transducer using sonar in the range of 5kHz and 10kHz. From the measurement results, we found that the pressure transducer responds to the sound of the sonar very well, and it means that the transducer can be used for the measurement of the turbulent flow whose predicted frequency is under 1kHz in the water. We did not investigate the damping; however, the static pressure tube we used is the same as the tube introduced and investigated by Shirahama et al.[4] We thought that the dampling is small.