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Credit Default Swap Valuation with Counterparty Risk*

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Using the reduced form framework with inter-dependent default correlation, we perform valuation of credit default swap with counterparty risk. The inter-dependent default risk structure between the protection buyer, protection seller and the reference entity in a credit default swap are characterized by their correlated default intensities, where the default intensity of one party increases when the default of another party occurs. We explore how settlement risk and replacement cost affect the swap rate in credit default swaps.

Keywords: Counterparty risk, contagious defaults, intensity model, credit default swap

JEL Classification Numbers: G12, G13

1. Introduction

According to the Credit Derivatives Report of British Bankers' Association (2002), nearly half of the market share of credit derivatives trading is captured by single-named credit swap contracts. A credit default swap (CDS) is a contract agreement which allows the transfer of credit risk of a risky asset/basket of risky assets from one party to the other. A financial institution may use a CDS to transfer credit risk of a risky asset while continues to retain the legal ownership of the asset[1)]. The rapid growth of the credit default swap market has reached to the stage where credit default swaps on reference entities are more actively traded than bonds issued by the reference entities. The choice of credit sensitive instruments

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* The opinions expressed in this paper are exclusively the personal views of the author and should not be cited as opinion and interpretation of HSBC, Hong Kong, China.

1) Apart from hedging purpose, CDSs are recently often used by synthetic deal managers to tailor credit risk and create arbitrage opportunity not available in the cash markets.

for empirical studies on default risk has slowly moved from risky corporate bonds to credit default swaps on the bonds (Longstaff et al. 2003; Ericsson et al., 2004). Default swap premia are believed to reflect changes in credit risk more accurately and quickly than corporate bond yield spreads.

Assume that party A holds a corporate bond and faces the credit risk arising from default of the bond issuer (reference party C). To seek protection against such default risk, party A enters a CDS contract in which he agrees to make a stream of periodic premium payments, known as the swap premium to party B (CDS protection seller). In exchange, party B promises to compensate A (CDS protection buyer) for its loss in the event of default of the bond (reference asset). A CDS involves three parties: protection buyer, protection seller, and issuer of the reference bond. Unlike interest rate swaps and currency swaps, where cash flows are exchanged between the two counterparties periodically, the protection seller pays only when default of the reference bond occurs.

As remarked by Jarrow and Yu (2001), "an investigation of counterparty risk is incomplete without studying its impact on the pricing of credit derivatives." We would like to address the following queries in this paper. How does the inter-dependent default risk structure between the protection seller and the reference bond affect the swap rate? Should we go to a European bank or a Korean bank as the protection seller for a Korean bond? Can the protection seller fulfill its obligation to make the compensation payment at the end of the settlement period, given that its credit quality may have deteriorated due to contagious effect arising from the default of the reference bond? What would be the impact on the swap rate due to potential replacement cost of entering a new CDS contract when the protection seller defaults prior to the reference bond? To determine a fair swap rate of a CDS in the presence of counterparty risks, the inter-dependent default risk structures between these parties must be considered simultaneously.

For interest rate swaps, theoretical analyzes show that the difference in swap rates between two counterparties of different credit ratings is much less than the difference in their debt rates. For example, for a 5-year interest rate swap between a given party paying LIBOR and another party paying a fixed rate, Duffie and Huang (1996) find that the replacement of the given fixed-rate counterparty with a lower quality counterparty whose bond yields are 100 basis points higher would only increase the swap rate by roughly 1 basis point. However, the very nature of contingent compensation payment upon default in a CDS may lead to a higher counterparty risk exposure compared to that of an interest rate swap. Our results show that CDS dealers should not quote the same rates to all counterparties irrespective of their credit ratings, like the usual practice in interest rate swaps market.

There have been numerous works on credit default swap valuation. Duffie (1999) proposes a non-model based pricing approach where a credit default swap is priced by reference to spreads over the riskfree rate of par floating rate bonds of the same quality. He also discusses the estimation of the hazard rate from defaultable bond prices. Based on the reduced form approach with correlated market and credit risks, Jarrow and Yildirim (2002) obtain closed form valuation formula for the swap rate of a CDS. In their model, the default intensity is assumed to be "almost" linear in the short interest rate. To examine the impact of counterparty risk on the pricing of a CDS, Jarrow and Yu (2001) assume an inter-dependent default structure that avoids looping default and simplifies the payoff structure where the protection seller's compensation is made only at the maturity of the swap. They discover that a CDS may be significantly overpriced if the default correlation between the protection seller and reference entity is ignored. Hull and White (2001) apply the credit index model for valuing CDS with counterparty risk. They argue that if the default correlation between the protection seller and the reference entity is positive, then the default of the counterparty will result in a positive replacement cost for the protection buyer. Their results show that the CDS swap rates increase with credit index correlation and the rates may differ by more than 10% when the protection seller's credit rating decreases from AAA to BBB and the value of the credit index correlation is 0.6 or higher. Using a structural default correlation model, Kim and Kim (2003) conclude that the pricing error in a CDS can be quite substantial if the correlation between the default risks of the counterparty and reference bond is ignored. Chen and Filpovic (2003) develop a generalized affine model to price credit default swaps under default correlations and counterparty risk. By the specification analysis of the affine process, they manage to incorporate market-credit risk correlation, joint credit migrations and firm specific default risk into their pricing model. Yu (2004) uses the "total hazard" approach to construct the default processes from independent and identically distributed exponential random variables. He obtains an analytic expression of the joint distribution of default times in his two-firm and three-firm contagion models. Under the framework of contagious defaults, the default risk is modeled by the reduced form approach, where the probability of default is determined by an exogenously specified instantaneous default intensity. The contagious defaults are effected by inter-dependent default risk structure between the parties, where the default intensity of one party increases when the default of another party occurs.

In this paper, we would like to analyze the impact of correlated risks between the three parties in a CDS using similar contagion models. Instead of following Yu's approach, we employ the change of measure introduced by Collins-Dufresne et al. (2002) in our valuation procedures. Using this change of measure, our counterparty risk model reduces to the standard reduced form model. Specifically, the probability measure of firm i is defined by its default intensity which is absolutely continuous with respect to the risk neutral measure and zero probability is assigned to firm i if default occurs before maturity. Compared to the total hazard construction (Yu, 2004), the analytic derivation procedures using the change of measure approach become less tedious. In his CDS pricing model, Yu (2004) places several assumptions on the payment structures in order to simplify his calculations. Firstly, the protection buyer is assumed to make continuous premium payment at the swap rate till expiration, provided that the buyer does not default prior to expiry. However, the swap payment terminates upon default of either one of the three parties in market practice. Secondly, the protection seller is assumed to make the contingent compensation payment on the expiration date, provided that the protection seller survives beyond the expiration date of the swap.

Distinctive from others' work on CDS valuation with counterparty risk, we consider the more realistic scenario in which the compensation payment upon default of the reference party is made at the end of the settlement period after default. If the protection seller defaults prior to the reference entity, then the protection buyer renews the CDS with a new counterparty. Supposing that the default risks of the protection seller and reference entity are positively correlated, we would like to estimate the expected replacement cost due to an increase in the swap rate in the new CDS. The change of measure technique provides an effective tool for CDS valuation in our more refined pricing model. Furthermore, we extend our counterparty risk framework to the three-firm contagion model by including the possibility of default of the protection buyer. This represents an extension from unilateral default to bilateral defaults among the counterparties.

The paper is organized as follows. In Section 2, we present the setup of the two-firm contagion model. We employ the inter-dependent default model to analyze the effects of *settlement risk* and *replacement cost* on the fair swap rate of a CDS. In Section 3, the analysis of correlated default risks in CDS valuation is extended to the three-firm model where all three parties have inter-dependent default structure. The paper is ended with conclusive remarks in the last section.

2. Two-firm Model

We consider an uncertain economy with a time horizon of T described by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t=0}^T, P)$ satisfying $\mathcal{F} = \mathcal{F}_T$, where P is the risk-neutral (equivalent martingale) measure in the sense of Harrison and Kreps (1979), that is, all security prices discounted by the risk-free interest rate process r_t are martingale under P. We use the Cox framework to specify the random default times. We denote the default time of firm i by

$$\tau^i = \inf\left\{t : \int_0^t \lambda_s^i \, ds \geq E^i\right\}, \tag{1a}$$

where $\{E^i\}_{i\in I}$ is a set of independent unit exponential random variables. We further assume that τ^i possesses a strictly positive $\mathcal{F}_t$-predictable intensity λ_t^i with right-continuous sample paths such that

$$M_t^i = N_t - \int_0^{t\wedge\tau^i} \lambda_s^i \, ds \tag{1b}$$

is a $(P, \mathcal{F}_t)$-martingale. Under the above characterization, given the $\mathcal{F}_t^X$-adapted intensity λ_t^i, the conditional survival probability of firm i is given by

$$P[\tau^i > T|\mathcal{F}_t] = E\left[\exp\left(-\int_t^T \lambda_s^i \, ds\right)\middle|\mathcal{F}_t\right]. \tag{2}$$

In this section, we perform credit default swap valuation using the two-firm contagion model. The likelihood of default of the protection seller (firm B) with random default time τ^B and the reference entity (firm C) with random default time τ^C are modeled by their correlated default intensities while the protection buyer (firm A) is assumed to be default-free. Under the CDS contract, a periodic stream of swap premium payments will be paid to the protection seller until the occurrence of a contractually defined credit event (either protection seller defaults or the reference entity defaults) or the expiration of the contract, whichever comes earlier. If the reference entity defaults prior to the expiration of the contract, then the protection buyer receives the compensation from the protection seller on the settlement date (at the end of the settlement period). The compensation is given by the difference between the face value and the recovery value of the reference entity, less the swap premium that has accrued since the last payment date. The accrued premium is calculated on a time-proportional basis. If the protection seller defaults prior to the default of the reference entity, the contract terminates. The protection buyer enters a new contract with another counterparty for the remaining life of the original CDS.

To simplify our CDS valuation, we assume a flat term structure of riskless interest rate r[2)]. In our two-firm contagion model, the inter-dependent default risk structure between firm B and firm C is characterized by the correlated default intensities:

$$\lambda_t^B = b_0 + b_2 1_{\{\tau^C \le t\}} \tag{3a}$$

$$\lambda_t^C = c_0 + c_2 1_{\{\tau^B \le t\}}, \tag{3b}$$

where the default intensity λ_t^B (λ_t^C) jumps by the amount b_2 (c_2) when firm C (B) defaults. The parameters b_0, b_2, c_0 and c_2 are assumed to be constant and distinct. Without loss of generality, we take the notional to be \$1 and assume zero recovery upon default. Since it takes no cost to enter a CDS, the value of the swap rate $S_2(T)$ under this two-firm model is determined by

$$\begin{aligned} &\sum_{i=1}^{n} E\left[e^{-rT_i}\, S_2(T)\, 1_{\{\tau^B \wedge \tau^C > T_i\}}\right] + S_2(T) A_2(T) \\ = \quad & E\left[e^{-r(\tau^C+\delta)} 1_{\{\tau^C \le T\}} 1_{\{\tau^B > \tau^C + \delta\}}\right], \end{aligned} \tag{4}$$

where $\{T_1, \cdots, T_n\}$ are the swap payment dates with $0 = T_0 < T_1 < \cdots < T_n = T$ and δ is the length of the settlement period. Here, $\tau^C + \delta$ represents the settlement date at the end of the settlement period. We assume that the payment dates are uniformly distributed, that is, $T_{i+1} - T_i = \Delta T$ for $1 \le i \le n-1$. The first term in Eq. (4) gives the present value of the sum of periodic swap payments (terminated when either B or C defaults or at maturity) and $S_2(T)A_2(T)$ is the present value of the

[2)] Without loss of analytical tractability, our framework can be extended to stochastic interest rate within the class of affine structure.

accrued swap premium for the fraction of period between τ^C and the last payment date. The present value of accrued swap premium is given by

$$S_2(T)A_2(T) = S_2(T)\sum_{i=1}^{n} E\left[e^{-r\tau^C}\left(\frac{\tau^C - T_{i-1}}{\Delta T}\right)1_{\{T_{i-1}<\tau^C<T_i\}}1_{\{\tau^B>\tau^C\}}\right], \tag{5}$$

where the accrued premium is paid at τ^C and $\dfrac{\tau^C - T_{i-1}}{\Delta T}$ represents the fraction of the time interval between successive payment dates. To compute $S_2(T)$, we set the present value of protection buyer's payment equal to the present value of the compensation payment made at $\tau^C + \delta$, conditional on default of C prior to T and no default of B prior to $\tau^C + \delta$. The buyer may face potential replacement cost when $\tau_B < \min(\tau_C, T)$. However, since $S_2(T)$ represents the fair swap rate charged by the seller party B, the replacement cost should not be included in the calculation of the swap premium.

Compared to other CDS valuation models in the literature, our pricing framework models the payoff structures closer to reality, in particular, the compensation is payable in the end of settlement period after C's default, and periodic discrete payments are made at $T_1, \ldots, T_n$.

2.1. Change of Measure

We adopt the change of measure introduced by Collins-Dufresne et al. (2002) in our valuation procedure of the swap rate. Accordingly, we define a firm-specific probability measure P^i which puts zero probability on the paths where default occurs prior to the maturity T. Specifically, the change of measure is defined by

$$Z_T \triangleq \left.\frac{dP^i}{dP}\right|_{\mathcal{F}_T} = 1_{\{\tau^i>T\}}\exp\left(\int_0^T \lambda_s^i\, ds\right), \tag{6}$$

where P^i is a firm-specific (firm i) probability measure which is absolutely continuous with respect to P on the stochastic interval $[0, \tau^i)$. One can show that Z_T is a uniformly integrable P-martingale with respect to $\mathcal{F}_T$ and is almost surely strictly positive on $[0, \tau^i)$ and almost surely equal to zero on $[\tau^i, \infty)$ [see Collins-Dufresne et al. (2004)]. To proceed the calculations under the measure P^i, we enlarge the filtration to $\mathcal{F}^i = (\mathcal{F}_t^i)_{t\geq 0}$ as the completion of $\mathcal{F} = (\mathcal{F}_t)_{t\geq 0}$ by the null sets of the probability measure P^i.

Under the default risk structure specified in Eq. (3a, b), the survival probabilities of firm B and firm C are defined recursively through each other and this leads to the phenomenon of "looping default". Under the new measure P^B defined by Eq. (6), $\lambda_t^C = c_0$ for $t < T$, so this effectively neglect the impact of firm B's default on the intensity of firm C, so looping default no longer exists. An analogous argument also holds under the measure P^C.

Using the change of measure, the joint density of default times (τ^B, τ^C) is found to be (see Appendix A)

$$f(t_1,t_2) = \begin{cases} c_0(b_0+b_2)e^{-(b_0+b_2)t_1-(c_0-b_2)t_2}, & t_2 \le t_1, \\ b_0(c_0+c_2)e^{-(c_0+c_2)t_2-(b_0-c_2)t_1}, & t_2 > t_1. \end{cases} \tag{7}$$

The marginal density of the default times τ^B and τ^C can be obtained by integrating the joint density $f(t_1,t_2)$. This gives

$$\frac{P[\tau^B \in dt_1]}{dt_1} = \frac{(b_0+b_2)c_0}{c_0-b_2}\left[e^{-(b_0+b_2)t_1} - e^{-(b_0+c_0)t_1}\right] + b_0e^{-(b_0+c_0)t_1} \tag{8a}$$

and

$$\frac{P[\tau^C \in dt_2]}{dt_2} = \frac{(c_0+c_2)b_0}{b_0-c_2}\left[e^{-(c_0+c_2)t_2} - e^{-(b_0+c_0)t_2}\right] + c_0e^{-(b_0+c_0)t_2}. \tag{8b}$$

Consequently, the marginal survival probabilities are given by

$$P[\tau^B > t_1] = \frac{c_0e^{-(b_0+b_2)t_1} - b_2e^{-(b_0+c_0)t_1}}{c_0-b_2}, \tag{9a}$$

and

$$P[\tau^C > t_2] = \frac{b_0e^{-(c_0+c_2)t_2} - c_2e^{-(b_0+c_0)t_2}}{b_0-c_2}. \tag{9b}$$

2.2. Swap Premium in the Two-firm Model

Using the joint density $f(t_1,t_2)$ given in Eq. (7) and performing the expectation calculations in Eq. (4), one can show that the swap premium is given by (see Appendix B)

$$S_2(T) = \frac{c_0e^{-(b_0+b_2+r)\delta}(1-e^{-\beta T})}{\beta}\left[\frac{e^{-\beta\Delta T}(1-e^{-\beta n\Delta T})}{1-e^{-\beta\Delta T}} + A_2(T)\right]^{-1}, \tag{10}$$

where $\beta = b_0 + c_0 + r$ and the expression for $A_2(T)$ is given in Appendix B. It is interesting to observe that $S_2(T)$ is independent of c_2, though the calculation of $S_2(T)$ involves $E[e^{-rT_i}1_{\{\tau^B\wedge\tau^C>T_i\}}]$. A more careful consideration reveals that an increase of the default intensity of C by c_2 due to B's default would have impact only on the replacement cost. Since the calculation of $S_2(T)$ does not include the effect of replacement cost, the independence of $S_2(T)$ on c_2 seems logically.

The impact of contagious default structure between the protection seller B and the reference asset C on the swap premium is illustrated in Figure 1. Consistent with our intuition, the swap premium decreases with b_0 as the protection buyer is willing to pay a lower premium when dealing with a more risky protection seller. The swap premium becomes smaller as b_2 assumes a higher value because the default of C increases the default probability of B. Similar to other credit risk factors, the swap premium is highly sensitive to the underlying default risk of C proxied by c_0.

From Eq. (10), one deduces that the swap premium is not quite sensitive to the length of the protection period. This is consistent with the empirical studies by Aunon-Nerin et al. (2002). They have tested several specifications for the maturity effect, but none of them appear to be significant.

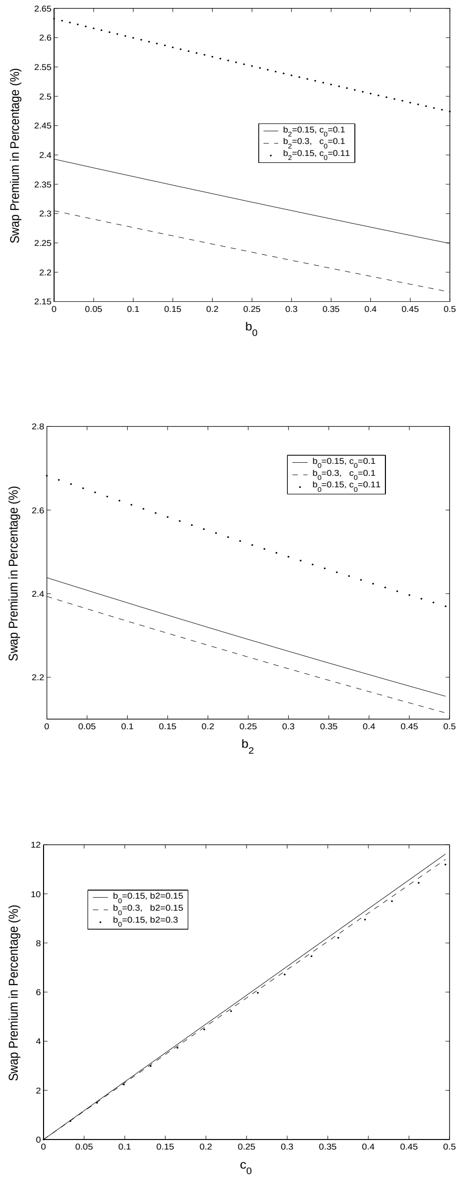

Figure 1 Swap premium in a two-firm model. The protection buyer (firm A) is assumed to be default-free. The swap premium $S_2(T)$ is plotted against various parameters in the contagion risk model, illustrating the impact of the intrinsic and correlated risks of the reference entity and protection seller on the swap premium. The base parameter values are: $r = 0.05$, $\delta = 0.25$, $\Delta T = 0.25$, $b_0 = 0.15$, $b_2 = 0.15$, $c_0 = 0.1$, $c_2 = 0.1, T = 10$.

2.3. Settlement Risk and Replacement Cost

We now turn our attention to the settlement risk and replacement cost in a CDS. Suppose a financial institution enters a CDS to protect its underlying asset. But this does not mean that default risk can be fully hedged due to the possibility of swap seller's default occurred before the settlement date. Observe that if firm B is default-free, the swap premium is then given by

$$\sum_{i=1}^{n} E\left[e^{-rT_i}\bar{S}_2(T)\ 1_{\{\tau^C>T_i\}}\right] + \bar{S}_2(T)\bar{A}_2(T) = E\left[e^{-r(\tau^C+\delta)}1_{\{\tau^C\leq T\}}\right], \tag{11}$$

where

$$\bar{A}_2(T) = \sum_{i=1}^{n} E\left[e^{-r\tau^C}\left(\frac{\tau^C - T_{i-1}}{\Delta T}\right)1_{\{T_{i-1}<\tau^C<T_i\}}\right].$$

To examine the effect of settlement risk on the swap premium, we define the swap premium spread $V(T)$ to be the difference of the swap premium with and without settlement risk, that is,

$$V(T) = \bar{S}_2(T) - S_2(T). \tag{12}$$

Intuitively speaking, it is not clear that whether $V(T)$ is strictly positive. In a CDS, the protection buyer inevitably faces a trade-off between a higher present value of compensation for its loss in the event of C's default, that is,

$$E\left[e^{-r(\tau^C+\delta)}1_{\{\tau^C\leq T\}}\right] \geq E\left[e^{-r(\tau^C+\delta)}1_{\{\tau^C\leq T\}}1_{\{\tau^B>\tau^C+\delta\}}\right]$$

and a higher present value of total swap payments due to an obligation to make compensation to swap buyer upon the default of underlying asset, that is,

$$E\left[e^{-rT_i}1_{\{\tau^C>T_i\}}\right] + \bar{A}_2(T) \geq E\left[e^{-rT_i}1_{\{\tau^B\wedge\tau^C>T_i\}}\right] + A_2(T).$$

It is quite straightforward to derive $\overline{S}_2(T)$, which can be obtained by setting $b_0 = b_2 = 0$ in $S_2(T)$.

The change on settlement risk premium with respect to the settlement period δ is illustrated in Figure 2. We observe that the settlement risk premium increases as δ becomes larger, and its sensitivity is very significant. Doubling the value of c_0 from 0.1 to 0.2 leads to a significant increase of 60 basis points in the settlement risk premium. The effect of b_0 and b_2 have relatively less influence on the settlement risk premium. We find that the default correlation between the protection seller and the reference asset, proxied by b_2, is slightly more important than the underlying risk of the protection seller when settlement risk is analyzed. This suggests that the protection buyer should be aware of the credit rating of its counterparty B, its correlated default risk with the reference asset as well as the settlement period in order to determine a fair swap premium.

From protection buyer's perspective, it is uncertain to pay a stream of fixed payments for credit protection throughout the whole period due to the default of

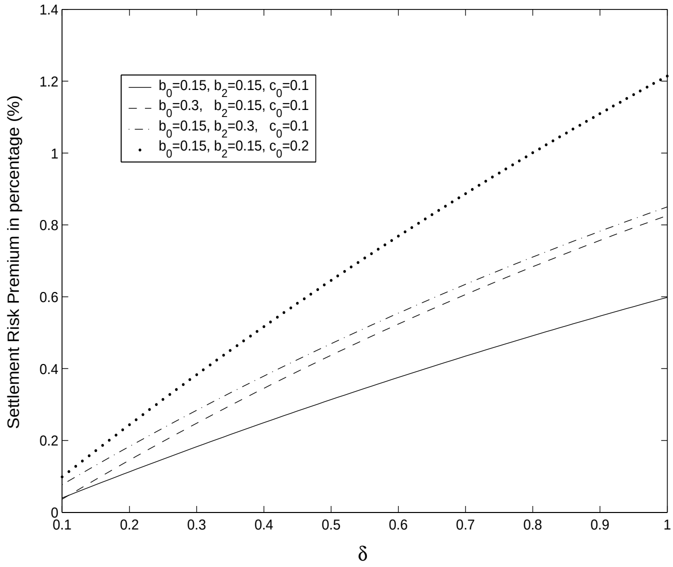

Figure 2 Change of settlement risk premium on δ. The base parameter values are: $r = 0.05$, $\Delta T = 0.25$, $b_0 = 0.15$, $b_2 = 0.15$, $c_0 = 0.1$, $c_2 = 0.1$.

the protection seller. In the presence of counterparty risk, the credit rating of the reference entity varies over time. This results a change in swap premium when entering a CDS at a different time. Specifically, the protection buyer can benefit or lose from a new contact, depending upon the credit rating of the reference entity at the default time of the original protection seller. In our model, the rating movement depends on the sign of b_2 and c_2. We define the replacement cost as the excess premium required to enter a new contract upon the default of the original protection seller. Mathematically, the default intensity of the reference entity becomes a constant after the default of the protection seller, that is,

$$\lambda_t^C = c_0 + c_2. \tag{13}$$

Assume that the effect of swap maturity is very insensitive to the premium so that we can ignore this factor into consideration. Let $\widetilde{S}_2$ denote the price of a CDS with the above default intensity. Conditional on the default of the protection seller before maturity, the replacement cost is $\widetilde{S}_2 - S_2$. To magnify the effect of counterparty risk on the replacement cost, assuming all other factors being fixed, the expected replacement cost[3)] is given by

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[3)] Another possible way to compute the present value of replacement cost is to use the *LIBOR risky measure* introduced by Schönbucher (2000). One can take the spot swap rate S_0 observed in the market and take the expectation $e^{-rT}E[(S_T - S_0)]$ where S_T is the forward swap rate.

Table 1 The entries illustrate the effect of the underlying default risk (b_0 and c_0) and the counterparty risk (b_2 and c_2) on the swap premium (first row), settlement premium (second row) and replacement cost (third row). We take the notional to be $1, risk-free interest rate r to be 5%, maturity T of 10 years, and settlement period δ of 0.25 year.

	(b_0, c_0)		
(b_2, c_2)	$(0.05, 0.05)$	$(0.1, 0.05)$	$(0.05, 0.1)$
	1.21%	1.21%	2.44%
$(0.05, 0.05)$	0.02%	0.03%	0.05%
	0.59%	0.85%	0.63%
	1.20%	1.20%	2.41%
$(0.1, 0.05)$	0.04%	0.05%	0.08%
	0.62%	0.89%	0.73%
	1.22%	1.21%	2.44%
$(0.05, 0.1)$	0.02%	0.03%	0.05%
	1.12%	1.67%	1.23%

$$P[\tau^B < T](\widetilde{S}_2 - S_2).$$

The relation between the default intensity parameters and swap premium, settlement premium and expected replacement cost is illustrated in Table 1. The results illustrate the quantitative insight of how the inter-dependent default structure affects swap premium as well as settlement risk and replacement cost. We observe that the expected replacement cost increases with the level of counterparty risk, i.e, b_2 and c_2. Our finding indicates that the effect of counterparty risk on the reference entity has a much stronger influence on the replacement cost than that on the protection seller. Also, the swap settlement premium increases with b_2. This means that the protection buyer faces a higher settlement risk when the protection seller has a stronger correlation with the reference entity.

3. Credit Default Swap with Defaultable Buyer

To study the effect of correlated default risk between all parties in a CDS on the swap premium, we extend our counterparty risk model to the three-firm model. The default risk structure is specified by the inter-dependent default intensities

$$\lambda_t^A = a_0 + a_1 1_{\{\tau^B \le t\}} + a_2 1_{\{\tau^C \le t\}}, \tag{14a}$$

$$\lambda_t^B = b_0 + b_1 1_{\{\tau^A \le t\}} + b_2 1_{\{\tau^C \le t\}}, \tag{14b}$$

$$\lambda_t^C = c_0 + c_1 1_{\{\tau^A \le t\}} + c_2 1_{\{\tau^B \le t\}}. \tag{14c}$$

From this setting, it is evident that the default probability of each party in the CDS depends on the default status of other firms. This model nests a number of simpler models. For instance, it reduces to the two-firm model in Section 2 if we take $a_0 = a_1 = a_2 = 0$. If we take $c_1 = c_2 = 0$, the default status of both counterparties in the CDS do not affect the credit rating of the reference asset. One may provide the financial interpretation as follows: the reference asset, say a risky bond, is issued by a large firm C whose default has an economy-wide impact. A small firm A holds this bond and wants to enter a CDS for the protection of the bond upon C's default. Suppose that A finds a swap seller, say B, in the same sector, so A and B have correlated default risk.

3.1. Swap Premium in the Three-firm Model

We employ the three-firm model specified by Eqs. (6a, 6b, 6c) to price a CDS. Under this framework, we study the effect of each party's default on the swap premium. Suppose that the protection buyer (firm A) holds a defaultable asset issued by firm C, and enter a CDS contract from the protection seller (firm B). Distinct from the two-firm model, the protection buyer is obligated to pay the periodic swap premium until the expiration of the contract, or the occurrence of the default either by the protection seller, the reference asset or itself, whichever is earlier. As before, upon the default of the reference asset, the protection buyer receives from the protection seller the difference between the face value and the recovery value of the reference entity. Moreover, if the protection buyer defaults prior to the default of the reference asset, the protection seller can simply walk away from the contract and has no obligation to pay the compensation to the protection seller.

In the presence of defaultable swap buyer, the swap premium is determined by

$$\begin{aligned} &\sum_{i=1}^{n} E\left[e^{-rT_i} S_3(T)\, 1_{\{\tau^A\wedge\tau^B\wedge\tau^C>T_i\}}\right] + S_3(T)A_3(T) \\ = \;& E\left[e^{-r(\tau^C+\delta)} 1_{\{\tau^C\le T\}} 1_{\{\tau^A>\tau^C\}} 1_{\{\tau^B>\tau^C+\delta\}}\right]. \end{aligned} \tag{15}$$

To determine the swap premium $S_3(T)$, it requires the knowledge of the joint density $f(t_1,t_2,t_3)$ of (τ^A,τ^B,τ^C). By following similar calculation procedures as those for the two-firm model, the joint density $f(t_1,t_2,t_3)$ is found to be

$$f(t_1,t_2,t_3) = \begin{cases} a_0(b_0+b_1)(c_0+c_1+c_2) & \\ \times e^{-(a_0-b_1-c_1)t_1-(b_0+b_1-c_2)t_2-(c_0+c_1+c_2)t_3}, & t_1<t_2<t_3, \\ a_0(c_0+c_1)(b_0+b_1+b_2) & \\ \times e^{-(a_0-b_1-c_1)t_1-(c_0+c_1-b_2)t_3-(b_0+b_1+b_2)t_2}, & t_1<t_3<t_2, \\ b_0(a_0+a_1)(c_0+c_1+c_2) & \\ \times e^{-(b_0-a_1-c_2)t_2-(a_0+a_1-c_1)t_1-(c_0+c_1+c_2)t_3}, & t_2<t_1<t_3, \\ c_0(a_0+a_2)(b_0+b_1+b_2) & \\ \times e^{-(c_0-a_2-b_2)t_3-(a_0+a_2-b_1)t_1-(b_0+b_1+b_2)t_2}, & t_3<t_1<t_2, \\ b_0(c_0+c_2)(a_0+a_1+a_2) & \\ \times e^{-(b_0-c_2-a_1)t_2-(c_0+c_2-a_2)t_3-(a_0+a_1+a_2)t_1}, & t_2<t_3<t_1, \\ c_0(b_0+b_2)(a_0+a_1+a_2) & \\ \times e^{-(c_0-b_2-a_2)t_3-(b_0+b_2-a_1)t_2-(a_0+a_1+a_2)t_1}, & t_3<t_2<t_1. \end{cases} \tag{16}$$

With the aid of $f(t_1, t_2, t_3)$, the swap premium $S_3(T)$ is given by

$$S_3(T) = L_\chi(T)\left[\frac{e^{-\alpha\Delta T}(1 - e^{-\alpha n\Delta T})}{1 - e^{-\alpha\Delta T}} + A_3(T)\right]^{-1} e^{-r\delta}, \tag{17}$$

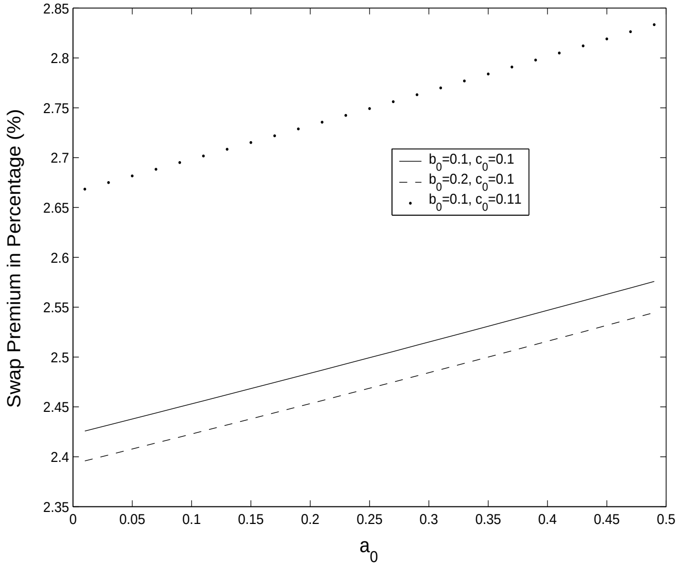
where $\alpha = a_0 + b_0 + c_0 + r$, $L_\chi(T)$ and $A_3(T)$ are presented in Appendix D. Note that when we set $a_0 = a_1 = a_2 = 0$, $S_3(T)$ reduces to the swap premium $S_2(T)$ in Eq. (10).

In Figures 3 and 4, we plot the swap premium against varying values of default intensity parameters in the three-form model. Figure 3 illustrates that the reference asset's default risk proxied by c_0 gives the most significant impact on swap premium, and an increasing higher value of c_0 gives rise to a higher swap premium. On the other hand, the default risk of the protection buyer has little impact on the swap premium, i.e., an increase in the likelihood of default of the protection buyer (a higher value of a_0) only increases the swap premium marginally. It is because when the financial health of the protection seller affects the default status of the underlying asset, this contagion effect makes the underlying asset more risky, in turn the protection seller would take a higher swap premium. Unlike the effect of the protection seller and the underlying asset, this effect is of third-order, so the impact is much less significant.

The expression for the swap premium in Eq. (17) shows no dependence on a_1, c_1, and c_2. Financially speaking, prior to the default of the underlying asset, the default event of the protection buyer or the protection seller will terminate the contract. This explains why a_1, c_1 and c_2 have no influence on the swap premium. Though $S_3(T)$ in Eq. (17) has dependence on a_2, one can show that the change of the swap premium is insensitive to a_2, and this can be explained by a similar argument. Figure 4 illustrates the change of swap premium with respect to varying levels of the default risk of the protection seller. The swap premium declines with the credit quality of the protection seller proxied by b_0. However, the degree of magnitude in the change is relatively low compared with that of c_0. In addition, a stronger correlated default risk with other counterparties on the protection seller, a lower swap premium the protection buyer is willing to pay, as seen by the increases in b_1 and b_2 lead to lower swap premium, while the swap premium is more sensitive to b_2. The order of sensitivity to the protection seller is b_2, b_0 and b_1. Similar to the two-firm model, the swap premium is also insensitive to maturity.

4. Conclusion

In this paper, using both the two-firm and three-firm contagion risk models, we provide the insight on how counterparty risks influence the swap rate in a credit default swap. It may be possible for a financial institution to have good estimates of marginal distributions (or default intensities), yet end up with the wrong evaluation of its credit exposure. For example, firm A could estimate a parametric model using the bond prices issued by firm B. However, if the counterparty risk for firm B is incorrectly identified by another party whose default is independent of firm



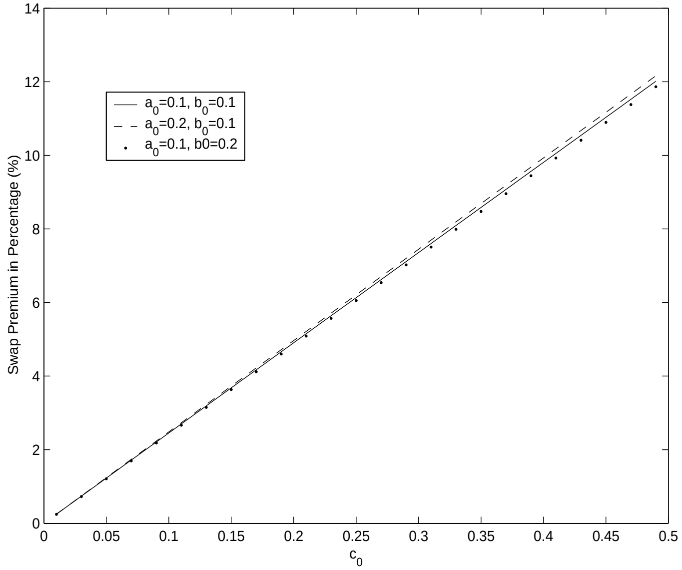


Figure 3 Impact of risk of default of the protection buyer and the reference entity on swap premium. The base parameter values are: $r = 0.05$, $\delta = 0.25$, $\Delta T = 0.25$, $T = 10$, $a_0 = 0.1$, $a_1 = 0.05$, $a_2 = 0.05$, $b_0 = 0.1$, $b_1 = 0.05$, $b_2 = 0.05$, $c_0 = 0.1$, $c_1 = 0.05$, $c_2 = 0.05$.

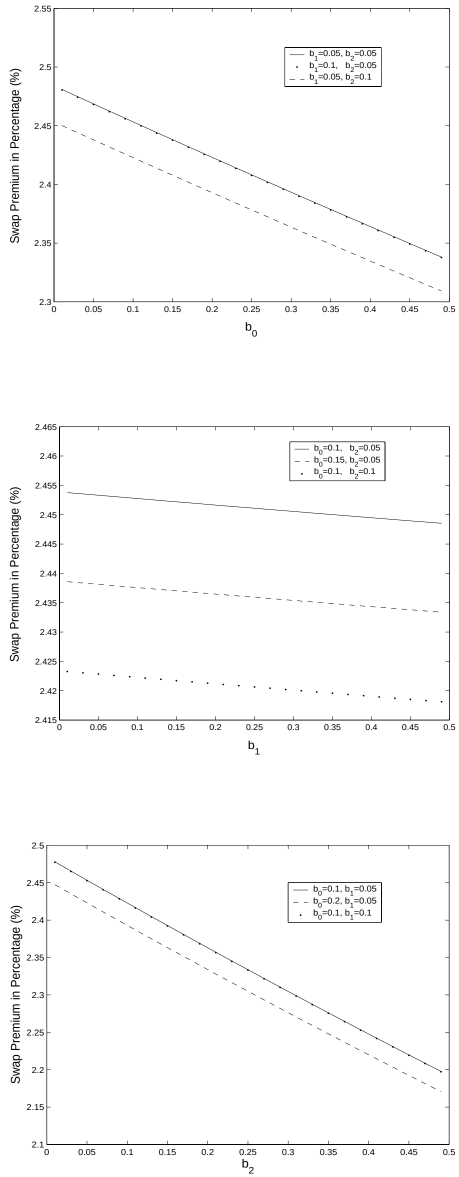


Figure 4 Impact of risk of default of the protection seller on swap premium. The base parameter values are: $r = 0.05$, $\delta = 0.25$, $\Delta T = 0.25$, $T = 10$, $a_0 = 0.1$, $a_1 = 0.05$, $a_2 = 0.05$, $b_0 = 0.1$, $b_1 = 0.05$, $b_2 = 0.05$, $c_0 = 0.1$, $c_1 = 0.05$, $c_2 = 0.05$.

C, this could severely overprice the swap rate due to neglect of default correlation. Our findings can be summarized as follows:

1. For CDS valuation, the swap premium can be significantly affected by counterparty risk. If the protection seller (firm B) has a higher correlation with the reference asset (firm C), then the swap premium becomes slightly lower. It is due to the fact that the swap buyer (firm A) is expected to pay a lower swap premium for less protection on its reference asset. On the other hand, when C has a higher correlation with A and B, the swap premium has no change at all. Though the occurrence of default either of A or B (prior to the default of C) increases the default probablility of C, the contract is then immediately terminated, so it has no impact on the swap premium. The same reasoning can be used to explain the insignificant change on the swap premium due to the impact of A's default on B. Due to the very nature of the CDS structure, the impact of default either of B or C on A does not give any change on the swap premium. Suppose B defaults prior to C, A can simply walk away and enter a new contract for the remaining period. When A survives longer than C during the life of the contract, A will receive compensation from B, independent of whether A defaults or not before the settlement date. In summary, the default risk of C is the primary determinant of the swap premium, and a higher value of c_0 leads to a significant increase in swap premium. Both results agree with financial intuition. Furthermore, the swap premium increases with a_0 and declines with b_0, but this effect is comparatively less pronounced.

2. The swap premium shows almost a flat term structure for all maturities. This behavior is probably attributed to our CDS payment structure. When B defaults prior to C's default, the protection buyer can simply walk away and enters a new contract for the remaining period. However, when the protection buyer defaults prior to C's default, B has the right to terminate the contract and has no obligation on the protection. This leads to the insensitivity of the swap premium with respect to maturity.

3. The change of settlement risk premium with respect to the settlement period is highly sensitive: a longer settlement period, a higher settlement risk premium. This suggests that the protection buyer should be aware of the credit rating of its counterparty B and the settlement period in order to determine a fair swap premium.

Our work also provides the motivation for investigating other credit risk issues. It is worth to study the effect of counterparty risk on other credit derivatives and structured products. Since the default intensity of one party jumps until another party defaults, the contagion model is unable to capture the intermediate change in credit rating of counterparties prior to credit event. Unlike structural models, it is not appropriate to use our framework to price structured credit products with strong dependence on the prior-to-maturity change in credit rating. On the other hand, the contagion model provides nice analytic tractability for multi-asset instruments while most structural models have great difficulty to provide an extension to the inter-dependent default structure for a basket of multiple assets.

Appendix

A. Joint density of default times (τ^B, τ^C)

Let $E^C[\cdot]$ denote the expectation taken under the measure P^C. For $t_1 < t_2$, the joint distribution of the pair of default times is found to be

$$\begin{aligned}
& P[\tau^B > t_1, \tau^C > t_2] \\
= \; & E\left[1_{\{\tau^B > t_1\}} 1_{\{\tau^C > t_2\}}\right] \\
= \; & E^C\left[1_{\{\tau^B > t_1\}} \exp\left(-\int_0^{t_2} (c_0 + c_2 1_{\{\tau^B \le s\}})\, ds\right)\right] \\
= \; & e^{-c_0 t_2} E^C\left[1_{\{\tau^B > t_1\}} \exp\left(-c_2(t_2 - \tau^B) 1_{\{\tau^B \le t_2\}}\right)\right] \\
= \; & e^{-c_0 t_2}\left[\int_{t_1}^{t_2} b_0 e^{-b_0 u - c_2(t_2 - u)}\, du + \int_{t_2}^{\infty} b_0 e^{-b_0 u}\, du\right] \\
= \; & b_0 e^{-(c_0 + c_2) t_2}\left[\frac{e^{-(b_0 - c_2) t_1} - e^{-(b_0 - c_2) t_2}}{b_0 - c_2}\right] + e^{-(b_0 + c_0) t_2}.
\end{aligned}$$

The fourth equality follows from the fact that $\lambda_t^B = b_0$ for $t \le t_2$ under P^C. By a similar argument, for $t_2 < t_1$, the joint distribution is given by

$$P[\tau^B > t_1, \tau^C > t_2] = c_0 e^{-(b_0 + b_2) t_1}\left[\frac{e^{-(c_0 - b_2) t_2} - e^{-(c_0 - b_2) t_1}}{c_0 - b_2}\right] + e^{-(b_0 + c_0) t_1}.$$

The differentiation of $P[\tau^B > t_1, \tau^C > t_2]$ with respect to t_1 and t_2 gives the joint density of the default times in Eq. (7).

B. Swap premium $S_2(T)$ of the two-firm model

Using the joint density $f(t_1, t_2)$, we obtain

$$E\left[e^{-\int_0^{\tau^C + \delta} r\, ds} 1_{\{\tau^C \le T\}} 1_{\{\tau^B > \tau^C + \delta\}}\right] = \frac{c_0 e^{-(b_0 + b_2 + r)\delta}[1 - e^{-(b_0 + c_0 + r)T}]}{b_0 + c_0 + r}.$$

To evaluate $E\left[e^{-rT_i} 1_{\{\tau^B \wedge \tau^C > T_i\}}\right]$, we can take the advantage of the change of measure to avoid tedious integration involving $f(t_1, t_2)$. Specifically, we have

$$
\begin{aligned}
& E\left[e^{-rT_i} 1_{\{\tau^B \wedge \tau^C > T_i\}}\right] \\
= \; & e^{-rT_i} E\left[1_{\{\tau^B > T_i\}} 1_{\{\tau^C > T_i\}}\right] \\
= \; & e^{-rT_i} E^C\left[1_{\{\tau^B > T_i\}} \exp\left(-\int_0^{T_i} (c_0 + c_2) 1_{\{\tau^B \le s\}} \, ds\right)\right] \\
= \; & e^{-(c_0+r)T_i} E^C\left[1_{\{\tau^B > T_i\}}\right] \\
= \; & e^{-(b_0+c_0+r)T_i}.
\end{aligned}
$$

By letting $\beta = b_0 + c_0 + r$ and observing $\Delta T = T_{i+1} - T_i$ for $1 \le i \le n-1$, we obtain

$$
\sum_{i=1}^{n} e^{-(b_0+c_0+r)T_i} = \frac{e^{-\beta\Delta T}(1 - e^{-\beta n \Delta T})}{1 - e^{-\beta\Delta T}}.
$$

In Appendix D, we will derive $A_3(T)$. Since $A_3(T)$ is an extension of $A_2(T)$ with the inclusion of the default possibility of the protection buyer, $A_3(T)$ is reduced to $A_2(T)$ by taking $a_0 = 0$. Combining all these results, we obtain the swap premium $S_2(T)$ in Eq. (10). As a result, we obtain

$$
\begin{aligned}
A_2(T) \; = \; & \frac{c_0}{\Delta T}\left[\frac{1 - e^{-(b_0+c_0+r)T}}{(b_0 + c_0 + r)^2} - \frac{T e^{-(b_0+c_0+r)T}}{b_0 + c_0 + r}\right] \\
& - \frac{c_0}{b_0 + c_0 + r} \sum_{i=1}^{N} T_{i-1}\left[e^{-(b_0+c_0+r)T_{i-1}} - e^{-(b_0+c_0+r)T_i}\right].
\end{aligned}
$$

C. Joint density of default times (τ^A, τ^B, τ^C)

Suppose $t_1 < t_2 < t_3$, we have

$$
\begin{aligned}
& P[\tau^A > t_1, \tau^B > t_2, \tau^C > t_3] \\
= \; & E\left[1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}} 1_{\{\tau^C > t_3\}}\right] \\
= \; & E^C\left[1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}} \exp\left(-\int_0^{t_3} (c_0 + c_1 1_{\{\tau^A \le s\}} + c_2 1_{\{\tau^B \le s\}}) \, ds\right)\right] \\
= \; & e^{-c_0 t_3} E^C\left[1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}} \, e^{-c_1 (t_3 - \tau^A) \, \mathbf{1}_{\{\tau^A \le t_3\}} - c_2 (t_3 - \tau^B) \, \mathbf{1}_{\{\tau^B \le t_3\}}}\right].
\end{aligned}
$$

Note that

$$
\begin{aligned}
& 1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}} \\
= \; & 1_{\{t_1 < \tau^A \le t_2\}} 1_{\{t_2 < \tau^B \le t_3\}} + 1_{\{t_2 < \tau^A \le t_3\}} 1_{\{t_2 < \tau^B \le t_3\}} + 1_{\{\tau^A > t_3\}} 1_{\{t_2 < \tau^B \le t_3\}} \\
& + 1_{\{t_1 < \tau^A \le t_2\}} 1_{\{\tau^B > t_3\}} + 1_{\{t_2 < \tau^A \le t_3\}} 1_{\{\tau^B > t_3\}} + 1_{\{\tau^A > t_3\}} 1_{\{\tau^B > t_3\}}.
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
& P[\tau^A > t_1, \tau^B > t_2, \tau^C > t_3] \\
= \quad & e^{-(c_0+c_1+c_2)t_3} E^C \left[1_{\{t_1<\tau^A\le t_2\}} 1_{\{t_2<\tau^B<t_3\}} e^{c_1\tau^A + c_2\tau^B} \right] \\
& + e^{-(c_0+c_1+c_2)t_3} E^C \left[1_{\{t_1<\tau^A\le t_2\}} 1_{\{\tau^B>t_3\}} e^{c_1\tau^A} \right] \\
& + e^{-(c_0+c_1+c_2)t_3} E^C \left[1_{\{t_2<\tau^A\le t_3\}} 1_{\{t_2<\tau^B\le t_3\}} e^{c_1\tau^A + c_2\tau^B} \right] \\
& + e^{-(c_0+c_1+c_2)t_3} E^C \left[1_{\{t_2<\tau^A\le t_3\}} 1_{\{\tau^B>t_3\}} e^{c_1\tau^A} \right] \\
& + e^{-(c_0+c_1+c_2)t_3} E^C \left[1_{\{t_2<\tau^B\le t_3\}} 1_{\{\tau^A>t_3\}} e^{c_2\tau^B} \right] \\
& + e^{-(c_0+c_1+c_2)t_3} E^C \left[1_{\{\tau^A>t_3\}} 1_{\{\tau^B>t_3\}} \right].
\end{aligned}
$$

Under the measure P^C and for $t < t_3$, the default intensities λ_t^A and λ_t^B are given by

$$
\begin{aligned}
\lambda_t^A &= a_0 + a_1 1_{\{\tau^B \le t\}} \\
\lambda_t^B &= b_0 + b_1 1_{\{\tau^A \le t\}}.
\end{aligned}
$$

Using the joint density of (τ^A, τ^B)

$$
f(u_1, u_2) = a_0(b_0 + b_1) e^{-(b_0+b_1)u_2 - (a_0-b_1)u_1}, \quad u_1 < u_2,
$$

one can compute $E^C \left[1_{\{t_1<\tau^A\le t_2\}} 1_{\{\tau^B>t_3\}} e^{c_1\tau^A} \right]$ and other similar terms.

Once we have obtained $P[\tau^A > t_1, \tau^B > t_2, \tau^C > t_3]$, we differentiate the distribution function with respect to t_1, t_2 and t_3 to give the joint density function

$$
f(t_1, t_2, t_3) = a_0(b_0 + b_1)(c_0 + c_1 + c_2)\, e^{-(a_0-b_1-c_1)t_1 - (b_0+b_1-c_2)t_2 - (c_0+c_1+c_2)t_3}, \qquad \text{for } t_1 < t_2 < t_3.
$$

We can obtain $f(t_1, t_2, t_3)$ for other permutation in a similar manner and get the results in Eq. (16).

D. Swap premium $S_3(T)$ of the three-firm model

Using the joint density function $f(t_1, t_2, t_3)$ for $t_3 < t_2 < t_1$ and $t_3 < t_1 < t_2$, we obtain

$$
\begin{aligned}
& L_\chi(T) \\
\triangleq \quad & E\left[e^{-r(\tau^C+\delta)} 1_{\{\tau^C\le T\}} 1_{\{\tau^A>\tau^C\}} 1_{\{\tau^B>\tau^C+\delta\}} \right] \\
= \quad & \frac{c_0(a_0+a_2)e^{-(b_0+b_1+b_2+r)\delta}}{(a_0+a_2-b_1)(a_0+b_0+c_0+r)} \left[1 - e^{-(a_0+b_0+c_0+r)T} \right] \\
& - \frac{c_0(a_0+a_2)(b_0+b_1+b_2)e^{-(a_0+a_2+b_0+b_2+r)\delta}}{(a_0+a_2-b_1)(a_0+b_0+c_0+r)(a_0+a_2+b_0+b_2)} \left[1 - e^{-(a_0+b_0+c_0+r)T} \right] \\
& + \frac{c_0(b_0+b_2)(b_0+b_1+b_2)e^{-(a_0+a_2+b_0+b_2+r)\delta}}{(a_0+b_0+c_0+r)(a_0+a_2+b_0+b_2)} \left[1 - e^{-(a_0+b_0+c_0+r)T} \right]
\end{aligned}
$$

where the vector of parameters $\chi = (a_0, a_2, b_0, b_1, b_2, c_0)$ captures the correlated default "characteristic." The expectation $E\left[e^{-rT_i}1_{\{\tau^A\wedge\tau^B\wedge\tau^C>T_i\}}\right]$ can be handled in a similar fashion as that in the two-firm model. The expectation calculations are given by

$$\begin{aligned}
& E\left[e^{-rT_i}1_{\{\tau^A\wedge\tau^B\wedge\tau^C>T_i\}}\right] \\
= \; & e^{-rT_i}E^A\left[1_{\{\tau^B>T_i\}}1_{\{\tau^C>T_i\}}\exp\left(-\int_0^{T_i}(a_0+a_1)1_{\{\tau^B\le s\}}+a_2 1_{\{\tau^C\le s\}}\,ds\right)\right] \\
= \; & e^{-(a_0+r)T_i}E^A\left[1_{\{\tau^B>T_i\}}1_{\{\tau^C>T_i\}}\right].
\end{aligned}$$

For $t \le T_i$, the dynamics of the default intensities of firm B and firm C under P^A are

$$\lambda_t^B = b_0 + b_2 1_{\{\tau^C\le t\}}$$
$$\lambda_t^C = c_0 + c_2 1_{\{\tau^B\le t\}}.$$

Using the result in the two-firm model (see Appendix B), we have

$$E^A\left[1_{\{\tau^B>T_i\}}1_{\{\tau^C>T_i\}}\right] = e^{-(b_0+c_0)T_i},$$

and so

$$E\left[e^{-rT_i}1_{\{\tau^A\wedge\tau^B\wedge\tau^C>T_i\}}\right] = e^{-(a_0+b_0+c_0+r)T_i}.$$

This leads to

$$\sum_{i=1}^n E\left[e^{-rT_i}1_{\{\tau^A\wedge\tau^B\wedge\tau^C>T_i\}}\right] = \frac{e^{-\alpha\Delta T}(1-e^{-\alpha n\Delta T})}{1-e^{-\alpha\Delta T}},$$

where $\alpha = a_0 + b_0 + c_0 + r$. It remains to evaluate $A_3(T)$, which is defined by

$$A_3(T) = \sum_{i=1}^n E\left[e^{-r\tau^C}\left(\frac{\tau^C - T_{i-1}}{\Delta T}\right)1_{\{T_{i-1}<\tau^C<T_i\}}1_{\{\tau^A\wedge\tau^B>\tau^C\}}\right].$$

Using $f(t_1, t_2, t_3)$ for $t_3 < t_2 < t_1$ and $t_3 < t_1 < t_2$, and performing straightforward integration, we obtain

$$\begin{aligned}
& E\left[e^{-r\tau^C}1_{\{T_{i-1}<\tau^C<T_i\}}1_{\{\tau^A\wedge\tau^B>\tau^C\}}\right] \\
= \; & \frac{c_0}{a_0+b_0+c_0+r}\left[e^{-(a_0+b_0+c_0+r)T_{i-1}} - e^{-(a_0+b_0+c_0+r)T_i}\right],
\end{aligned}$$

and

$$\begin{aligned}
& E\left[\tau^C e^{-r\tau^C}1_{\{T_{i-1}<\tau^C<T_i\}}1_{\{\tau^A\wedge\tau^B>\tau^C\}}\right] \\
= \; & c_0\left[\frac{e^{-(a_0+b_0+c_0+r)T_{i-1}} - e^{-(a_0+b_0+c_0+r)T_i}}{(a_0+b_0+c_0+r)^2}\right. \\
& \left. + \frac{T_{i-1}e^{-(a_0+b_0+c_0+r)T_{i-1}} - T_i e^{-(a_0+b_0+c_0+r)T_i}}{a_0+b_0+c_0+r}\right]
\end{aligned}$$

As a result, we obtain

$$A_3(T) = \frac{c_0}{\Delta T}\left[\frac{1-e^{-(a_0+b_0+c_0+r)T}}{(a_0+b_0+c_0+r)^2} - \frac{Te^{-(a_0+b_0+c_0+r)T}}{a_0+b_0+c_0+r}\right] - \frac{c_0}{(a_0+b_0+c_0+r)\Delta T}$$
$$\sum_{i=1}^{n} T_{i-1}\left[e^{-(a_0+b_0+c_0+r)T_{i-1}} - e^{-(a_0+b_0+c_0+r)T_i}\right].$$

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