Original Paper

# Comparative Analysis of Modeling Methods for Predicting Woven Fabric Properties

**B K Behera * and S B Muttagi**

*Department of Textile Technology, Indian Institute of Technology, Delhi-110016*

*Received September 10, 2003; Accepted for Publication February 18, 2005*

**Abstract**

Three modeling methodologies based on mathematical, empirical and artificial neural network based on radial basis function have been compared for their ability to predict fabric properties. It has been observed that artificial neural network model produced the least error as well as minimum range of error as compared to the other modeling methodologies.

*Keywords*: artificial neural network, empirical modeling, mathematical modeling, radial basis function, prediction error

## 1. Introduction

The main objective of many scientific studies in textiles is to reveal the complex functional relationships that exist between structural parameters of fibre, yarn and fabric properties. Modeling methodologies for predicting fabric properties are essential to design fabrics according to the specifications desired by the customer. If the relationships between different parameters that determine the specific fabric property are known, they can be used to optimize that particular property for different end-use applications so as to minimize the cost. Predictive modeling methodologies can also be used to identify the different levels of combinations of process parameters and material variables that yield the desired fabric property .

The relationship between fabric structure and property is complex and inherently nonlinear; to create a predictive model one must resolve the complexities. There are essentially three modeling tools for predicting fabric properties: *mathematical models* that are derived from first principles, *empirical models* that use statistical techniques, and *artificial neural network models* that are the part of the evolving field of artificial intelligence [1].

The aim of this analysis is to explore a more realistic system of the three modeling methodologies that can help in predicting fabric properties accurately for efficiently handling non-linear and complex fabric parameters and to examine the merit of these models.

A similar study for predicting the strength of air-jet spun yarns was made by Rajamanickam, *et al.* [1]. But no such studies are reported for analysis of modeling methodologies for fabric properties. In this analysis, artificial neural network methodology based on *radial basis function* (RBF) learning algorithm has been implemented.

## 2. Approach

To explore the predictability of the modeling methodologies, the published data and mathematical models of Leaf and Kandil [2] and Leaf *et. al.,* [3] for fabric initial tensile modulus and bending rigidity properties, respectively, were used to study the predictability of mathematical model.

In statistical method of modeling, multilinear regression equation between fabric property and constructional parameters was applied to the same data, to find the predictability of statistical method. Finally, artificial neural network, based on radial basis function algorithm was used to model the fabric structure-property relationship, using the same experimental data given in reference [2] and [3]. The predictive power of each methodology was estimated by comparing the predicted fabric property values with experimentally obtained results in terms of absolute error% of prediction.

## 3. Mathematical models

Mathematical models are very appealing because they have their basis in applied physics. They can be used to explain the reasons that determine structure-property relationships. To model the fabric properties one of the three idealized structures of unit cells of woven fabric is employed, namely, Peirce's *flexible thread model* [4], *rigid thread model* [4], or *saw-tooth* model [5] as a starting point.

Two fabric properties, namely, initial tensile moduli and bending rigidities have been considered, because the many experimental published data were available to model the structure-property relationships using regression analysis and artificial neural network modeling methodologies. The equations for initial tensile moduli were [2]:

Tensile moduli (mN/mm):

$$E_1 = \frac{12\beta_1}{p_1 p_2^2 (1 + c_1)^3 \sin^2\theta_1} \times \frac{1 + \beta_2 p_2^3 (1 + c_1)^3 \cos^2\theta_1}{\beta_1 p_1^3 (1 + c_2)^3 \cos^2\theta_2} \quad (1)$$

* Corresponding author Fax + 91-11-26562403, E-mail: behera@textile.iitd.ernet.in

$$E_2 = \frac{12\beta_2}{p_2 p_1^2(1+c_2)^3 \sin^2\theta_2} \times \left(\frac{1+\beta_1 p_1^3(1+c_2)^3\cos^2\theta_2}{\beta_2 p_2^3(1+c_1)^3\cos^2\theta_1}\right) \quad (2)$$

Bending moduli (mN mm$^2$/mm) [3]:

$$B_1 = \beta_1 p_2 \div p_1\left(p_2(1+c_1) - 0.8758D\theta_1\right) \quad (3)$$

$$B_2 = \beta_2 p_1 \div p_2\left(p_1(1+c_2) - 1.0778D\theta_2\right) \quad (4)$$

Table 1 Experimental data of tensile moduli of fabrics [2]

| Fabric No | $p_1$ (mm) | $p_2$ (mm) | $l_1$ (mm) | $l_2$ (mm) | $\beta_1$ (mNmm$^2$) | $\beta_2$ (mNmm$^2$) | $E_1$ N/cm | $E_2$ N/cm |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.485 | 0.588 | 0.701 | 0.514 | 5.62 | 6.06 | 14.3 | 36.6 |
| 2 | 0.488 | 0.624 | 0.758 | 0.515 | 5.62 | 6.06 | 9.4 | 29.8 |
| 3 | 0.485 | 0.713 | 0.835 | 0.508 | 5.62 | 6.06 | 14.2 | 34.1 |
| 4 | 0.49 | 0.67 | 0.798 | 0.514 | 5.62 | 7.05 | 15.9 | 42.9 |
| 5 | 0.492 | 0.679 | 0.871 | 0.515 | 5.62 | 7.05 | 15.5 | 33.8 |
| 6 | 0.495 | 0.849 | 0.983 | 0.513 | 5.62 | 7.05 | 14.6 | 28.6 |
| 7 | 0.494 | 0.779 | 0.939 | 0.508 | 5.62 | 8.16 | 13.7 | 53.6 |
| 8 | 0.494 | 0.839 | 1.022 | 0.507 | 5.62 | 8.16 | 10.6 | 45.5 |
| 9 | 0.491 | 0.691 | 0.847 | 0.509 | 5.62 | 8.16 | 14.9 | 42.4 |
| 10 | 0.476 | 0.589 | 0.704 | 0.504 | 4.44 | 4.44 | 9.2 | 25.5 |
| 11 | 0.587 | 0.749 | 0.827 | 0.616 | 4.44 | 4.44 | 9.1 | 14.8 |
| 12 | 0.549 | 0.532 | 0.606 | 0.615 | 4.44 | 4.44 | 12.7 | 13.4 |
| 13 | 0.556 | 0.548 | 0.598 | 0.622 | 4.44 | 4.25 | 24 | 13.8 |
| 14 | 0.591 | 0.637 | 0.722 | 0.622 | 4.44 | 4.25 | 13.3 | 19.7 |
| 15 | 0.594 | 0.756 | 0.832 | 0.624 | 4.44 | 4.25 | 11.7 | 14.5 |
| 16 | 0.568 | 0.465 | 0.509 | 0.621 | 4.44 | 2.96 | 23.2 | 22 |
| 17 | 0.577 | 0.538 | 0.597 | 0.639 | 4.44 | 2.96 | 18 | 12.8 |
| 18 | 0.571 | 0.662 | 0.73 | 0.608 | 4.44 | 2.96 | 12 | 13 |

$l_1$ & $l_2$ : length of thread axis between axes of consecutive cross threads.

Table 2 Predictability of mathematical model for fabric tensile modulus

| Fabric No. | Predicted $E_1$ N/cm | Predicted $E_2$ N/cm | Percent error in $E_1$ | Percent error in $E_2$ |
|---|---|---|---|---|
| 1 | 11.76 | 37.53 | -17.76 | 2.54 |
| 2 | 9.67 | 36.51 | 2.84 | 22.52 |
| 3 | 13.99 | 32.89 | -1.45 | -3.55 |
| 4 | 12.69 | 39.56 | -20.17 | -7.78 |
| 5 | 6.49 | 43.08 | -58.12 | 27.46 |
| 6 | 17.65 | 36.15 | 20.86 | 26.41 |
| 7 | 12.83 | 61.59 | -6.32 | 14.91 |
| 8 | 12.05 | 59.83 | 13.70 | 31.50 |
| 9 | 10.78 | 58.03 | -27.62 | 36.85 |
| 10 | 9.13 | 29.25 | -0.75 | 14.71 |
| 11 | 11.29 | 16.86 | 24.12 | 13.90 |
| 12 | 10.80 | 14.05 | -14.94 | 4.85 |
| 13 | 17.01 | 12.48 | -29.13 | -9.53 |
| 14 | 8.18 | 22.24 | -38.52 | 12.89 |
| 15 | 11.07 | 15.49 | -5.38 | 6.81 |
| 16 | 15.55 | 23.61 | -32.97 | 7.34 |
| 17 | 10.87 | 12.83 | -39.63 | 0.27 |
| 18 | 10.17 | 13.25 | -15.27 | 1.91 |
| | | Mean Abs. Error % | 20.53 | 13.65 |

$$\text{Percent Error of prediction} = \frac{(\text{Predicted value} - \text{Experimental value}) \times 100}{\text{Experimental value}}$$

Notations:

*d* yarn diameter in mm
$= 4.44 \times 10^{-2}$ (yarn tex/fibre density) $^{0.5}$

*D* sum of the warp and weft diameters, *i.e.,* $D = d_1 + d_2$

$\beta$ yarn flexural rigidity(mN mm)

$\theta$ weave angle in radians = 1.88 $\sqrt{c}$

*p* thread spacing in mm, between two adjacent yarns in the fabric

c yarn crimp in fabric ( Excess of modular length over thread spacing from Peirce's geometry)

In all these equations, subscripts 1 and 2 refer to parameters in the warp and weft directions respectively.
The model equations for tensile and bending moduli *i.e.,* Equations 1, 2 and 3, 4 were based on *saw-tooth* or *straight-line* model of the plain-woven fabric. They had used Castigliano's theorem as the principal method of attack. The analysis for tensile moduli was based on work reported by Leaf and Kandil [2]. Yarn compression and extension were taken into account. In the analysis, the yarn was assumed to have following properties:

(a) They extend according to the law $T = \lambda\varepsilon_y$, where *T* is the tension in the yarn, $\varepsilon_y$ is the extension produced and $\lambda$ is elastic constant of the yarn. The strain energy of extension per unit length of yarn is then $T^2/2\lambda$.

(b) The inter-yarn force was assumed to be point force *V*, and the original diameter of the yarn *D*. $V = \mu\varepsilon_d$, where $\varepsilon_d D$ was the change in the yarn diameter that takes place when *V* is applied and $\mu$ is coefficient of friction along the fibres . The strain energy of the compression was then $V^2D/2\mu$..

(c) The yarns bent with flexural rigidity *B*. The strain energy per unit length of yarn was then $M^2/2B$, where *M* was the applied bending moment.

Table 3 Experimental data for bending rigidity of fabrics [3]

| Fabric No | warp tex | weft tex | $d_1$ mm | $d_2$ mm | $p_1$ mm | $p_2$ mm | $l_1$ mm | $l_2$ mm | $\beta_1$ mNmm$^2$ | $\beta_2$ mNmm$^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 36.9 | 59 | 0.219 | 0.277 | 0.478 | 0.646 | 0.711 | 0.515 | 2.82 | 4.54 |
| 2 | 36.9 | 59 | 0.219 | 0.277 | 0.473 | 0.613 | 0.715 | 0.505 | 2.82 | 4 |
| 3 | 36.9 | 59 | 0.219 | 0.277 | 0.486 | 0.622 | 0.713 | 0.512 | 2.82 | 5.17 |
| 4 | 36.9 | 63.9 | 0.219 | 0.288 | 0.467 | 0.628 | 0.703 | 0.506 | 2.82 | 2.85 |
| 5 | 36.9 | 59 | 0.219 | 0.277 | 0.466 | 0.561 | 0.63 | 0.511 | 2.82 | 4.08 |
| 6 | 36.9 | 59 | 0.219 | 0.277 | 0.467 | 0.553 | 0.658 | 0.497 | 2.82 | 4.68 |
| 7 | 19.7 | 36.9 | 0.160 | 0.219 | 0.368 | 0.558 | 0.624 | 0.384 | 1.97 | 3.07 |
| 8 | 19.7 | 29.5 | 0.160 | 0.196 | 0.368 | 0.568 | 0.614 | 0.386 | 1.97 | 2.12 |
| 9 | 19.7 | 34.7 | 0.160 | 0.212 | 0.364 | 0.583 | 0.642 | 0.388 | 1.97 | 2.3 |
| 10 | 19.7 | 19.7 | 0.160 | 0.160 | 0.365 | 0.578 | 0.609 | 0.378 | 1.97 | 1.97 |
| 11 | 19.7 | 49.2 | 0.160 | 0.253 | 0.369 | 0.668 | 0.727 | 0.39 | 1.97 | 2.23 |
| 12 | 19.7 | 45.4 | 0.160 | 0.243 | 0.367 | 0.641 | 0.699 | 0.384 | 1.97 | 3.35 |
| 13 | 29.5 | 45.4 | 0.196 | 0.243 | 0.421 | 0.661 | 0.738 | 0.439 | 2.19 | 3.67 |
| 14 | 29.5 | 49.2 | 0.196 | 0.253 | 0.417 | 0.649 | 0.716 | 0.439 | 2.19 | 2.4 |
| 15 | 29.5 | 34.7 | 0.196 | 0.212 | 0.421 | 0.642 | 0.715 | 0.441 | 2.19 | 2.75 |
| 16 | 29.5 | 59 | 0.196 | 0.277 | 0.429 | 0.649 | 0.732 | 0.442 | 2.19 | 5.76 |
| 17 | 29.5 | 59 | 0.196 | 0.277 | 0.429 | 0.654 | 0.745 | 0.442 | 2.19 | 6.81 |
| 18 | 29.5 | 49.2 | 0.196 | 0.253 | 0.408 | 0.574 | 0.631 | 0.439 | 2.19 | 2.41 |
| 19 | 29.5 | 34.7 | 0.196 | 0.212 | 0.408 | 0.578 | 0.637 | 0.439 | 2.19 | 2.97 |
| 20 | 29.5 | 29.5 | 0.196 | 0.196 | 0.413 | 0.571 | 0.622 | 0.44 | 2.19 | 2.25 |
| 21 | 29.5 | 45.4 | 0.196 | 0.243 | 0.416 | 0.565 | 0.631 | 0.439 | 2.19 | 4.52 |
| 22 | 29.5 | 36.9 | 0.196 | 0.219 | 0.41 | 0.556 | 0.625 | 0.437 | 2.19 | 2.85 |
| 23 | 60 | 60 | 0.279 | 0.279 | 0.485 | 0.588 | 0.701 | 0.514 | 5.62 | 6.06 |
| 24 | 60 | 60 | 0.279 | 0.279 | 0.488 | 0.624 | 0.758 | 0.515 | 5.62 | 6.06 |
| 25 | 60 | 60 | 0.279 | 0.279 | 0.485 | 0.713 | 0.835 | 0.508 | 5.62 | 6.06 |
| 26 | 60 | 60 | 0.279 | 0.279 | 0.476 | 0.589 | 0.704 | 0.504 | 4.44 | 4.44 |
| 27 | 60 | 60 | 0.279 | 0.279 | 0.549 | 0.532 | 0.606 | 0.615 | 4.44 | 4.44 |
| 28 | 60 | 60 | 0.279 | 0.279 | 0.556 | 0.548 | 0.598 | 0.622 | 4.44 | 4.25 |
| 29 | 60 | 60 | 0.279 | 0.279 | 0.591 | 0.637 | 0.722 | 0.622 | 4.44 | 4.25 |
| 30 | 60 | 60 | 0.279 | 0.279 | 0.594 | 0.756 | 0.832 | 0.624 | 4.44 | 4.25 |
| 31 | 60 | 46 | 0.279 | 0.244 | 0.568 | 0.465 | 0.509 | 0.621 | 4.44 | 2.96 |
| 32 | 60 | 46 | 0.279 | 0.244 | 0.577 | 0.538 | 0.597 | 0.639 | 4.44 | 2.96 |
| 33 | 60 | 46 | 0.279 | 0.244 | 0.571 | 0.662 | 0.73 | 0.608 | 4.44 | 2.96 |

The analysis yielded Eqs. 1 and 2, which were very similar to the results, derived by Grosberg and Kedia [8]. The experimental data of Leaf and Kandil [2], for determination for tensile moduli is reproduced in Table 1. $E_1$ and $E_2$ are the experimental values of initial moduli of fabrics in warp and weft way directions in N/cm. It may be pointed out here, that there is an error in units of initial tensile moduli in the original work published by Leaf and Kandil [2], which were reported as mN/cm, which should have been N/cm. Accordingly corrections have been made in the units of initial tensile moduli shown in Table 1.

Theoretical values of initial moduli of the fabrics were calculated using Eqs. 1 and 2. The predictive power of mathematical model is shown in Table 2. In calculating average error %, absolute value of error % was considered. From the Table 2 it can be observed that predictability of warp tensile modulus ranges from -58.12 % to 24.12% with average absolute error percentage of 20.53 %. Similarly, the error of predictability ranges from -9.53 % to 36.85 % for weft-way initial modulus of fabric with average error of 13.65 %.

The mathematical models for initial bending behaviour of plain-woven fabric (Eqs. 3 and 4) were considered from work of Leaf and *et. al.* [3]. The experimental data for

Table 4 Predictability of mathematical model for bending rigidity of fabrics

| Fabric No. | Experimental $B_1$ mN mm | Experimental $B_2$ mN mm | Predicted $B_1$ mN mm | Predicted $B_2$ mN mm | Error% $B_1$ mN mm | error% $B_2$ mN mm |
|---|---|---|---|---|---|---|
| 1 | 7.76 | 12.32 | 8.65 | 15.02 | 10.3 | 18.0 |
| 2 | 8.64 | 14.76 | 9.95 | 13.28 | 13.2 | -11.1 |
| 3 | 10.83 | 23.49 | 9.32 | 14.97 | -16.2 | 56.9 |
| 4 | 9.88 | 7.54 | 8.56 | 8.89 | -15.4 | 15.2 |
| 5 | 8.26 | 17.96 | 10.25 | 18.27 | 19.4 | 1.7 |
| 6 | 10.58 | 20.31 | 11.64 | 17.09 | 9.1 | -18.8 |
| 7 | 7.02 | 8.06 | 7.46 | 9.32 | 5.9 | 13.5 |
| 8 | 6.27 | 4.56 | 6.9 | 6.24 | 9.1 | 26.9 |
| 9 | 6.36 | 6.09 | 7.19 | 7.73 | 11.5 | 21.2 |
| 10 | 6.34 | 4.38 | 6.47 | 4.97 | 2.0 | 11.9 |
| 11 | 9.3 | 3.48 | 6.9 | 6.78 | -34.8 | 48.7 |
| 12 | 9.48 | 9.41 | 7.01 | 9.56 | -35.2 | 1.6 |
| 13 | 7.11 | 9.25 | 7.13 | 9.44 | 0.3 | 2.0 |
| 14 | 6.21 | 6.16 | 7.26 | 6.97 | 14.5 | 11.6 |
| 15 | 5.92 | 8.17 | 6.96 | 7.14 | 14.9 | -14.4 |
| 16 | 6.23 | 15.64 | 7.48 | 14.21 | 16.7 | -10.1 |
| 17 | 7.32 | 16.99 | 7.53 | 16.67 | 2.8 | -1.9 |
| 18 | 6.28 | 6.88 | 7.92 | 9.64 | 20.7 | 28.6 |
| 19 | 9.12 | 14.37 | 7.49 | 10.42 | -21.8 | -37.9 |
| 20 | 9.43 | 7.85 | 7.18 | 7.14 | -31.3 | -9.9 |
| 21 | 9.54 | 13.33 | 7.94 | 15.05 | -20.2 | 11.4 |
| 22 | 9.68 | 9.79 | 7.92 | 9.92 | -22.2 | 1.3 |
| 23 | 19.59 | 22.09 | 24.26 | 22.16 | 19.2 | 0.3 |
| 24 | 18.48 | 20.55 | 22.89 | 19.95 | 19.3 | -3.0 |
| 25 | 20.45 | 22.5 | 18.83 | 16.42 | 8.6 | -37.0 |
| 26 | 17.25 | 20.25 | 19.6 | 16.48 | 12.0 | -22.9 |
| 27 | 18.25 | 19.62 | 17.32 | 22.26 | -5.4 | 11.9 |
| 28 | 28.5 | 16 | 14.19 | 20.04 | -100.8 | 20.2 |
| 29 | 14 | 13.06 | 12.87 | 11.22 | -8.8 | -16.4 |
| 30 | 16.47 | 13.68 | 10.7 | 9.32 | -53.9 | -46.8 |
| 31 | 20.5 | 12 | 15.64 | 12.78 | -31.1 | 6.1 |
| 32 | 15.2 | 10 | 13.83 | 11.49 | -9.9 | 13.0 |
| 33 | 11.6 | 7.36 | 11.65 | 7.82 | 0.4 | 5.9 |
| | | | Average absolute error % | | 18.7 | 16.9 |

bending rigidities is given in Table 3. For finding the bending rigidity of fabric, they had adopted 'best fit' technique to find the yarn contact lengths between the threads in fabric.

As seen from the Table 4, the predictive error of this model varies from -100.8 % to 20.7 % with an average prediction error of 18.7 % for warp-way bending, and from -46.8 % to 56.9 % with an average of 16.9 % for weft-way fabric bending rigidity.

From these observations, it can be concluded that the predictive power of the mathematical models depend on the assumptions used to build the models. Therefore, they yield high prediction error. This makes it unsuitable as a prediction tool. These models are applicable to specific problems only. These models cannot be applied to predict fabric properties with asymmetric weaves. Effects of yarn compression, and extension increase the error of prediction.

But nonetheless, mathematical models can be used to understand the relationships between structural and material parameters and fabric properties. However, it is unlikely that stand-alone mathematical models will become predictive tools because they are very difficult to use in practice. They are the basis for computer simulation model such as MECHFAB [6]. Therefore, alternative methodologies are required to predict the fabric properties more accurately.

## 4. Empirical modeling

Data from Tables 1 and 3 were used to develop regression models to predict initial tensile moduli and bending moduli of fabrics. The following were the empirical equations using linear multiple regression technique.

$$E_1 = -21.429 + l_1(-64.582) + l_2(87.234) + p_1(-51.112) + p_2(50.034) + \beta_1(4.814) + \beta_2(1.217) \quad (5)$$
$$[R^2 = 0.527]$$

$$E_2 = 38.161 + l_1(-48.801) + l_2(-370.5280) + p_1(362.866) + p_2(-1.122) + \beta_1(3.09) + \beta_2(5.100) \quad (6)$$
$$[R^2 = 0.904]$$

$$B_1 = 69.363 + d_1(-385.299) + d_2(-181.981) + l_1(-13.631) + l_2(232.493) + p_1(-289.970) + p_2(24.084) + tex_1(1.529) + tex_2(.491) + \beta_1(-.703) + \beta_2(1.336) \quad (7)$$
$$[R^2 = 0.842]$$

$$B_2 = 18.916 + d_1(300.380) + d_2(-438.798) + l_1(132.455) + l_2(159.367) + p_1(-174.684) + p_2(-137.467) + tex_1(-.798) + tex_2(1.137) + \beta_1(-.334) + \beta_2(2.425)$$
$$[R^2 = 0.903] \quad (8)$$

where $tex_1$ and $tex_2$ are the linear densities of warp and weft respectively.

Table 5 Predictability of initial tensile moduli of fabric using empirical model

| Fabric sample No (From Table No 1) | Experimental values (N/cm) | | Predicted values (N/cm) | | Percentage error | |
|---|---|---|---|---|---|---|
| | $E_1$ | $E_2$ | $E_1$ | $E_2$ | $E_1$ | $E_2$ |
| 1 | 14.3 | 36.6 | 17.2 | 37.1 | 20.26 | 1.37 |
| 7 | 13.7 | 53.6 | 12.96 | 41.47 | -5.43 | -22.63 |
| 11 | 9.1 | 14.8 | 13.15 | 18.08 | 44.5 | 22.18 |
| 18 | 12 | 13 | 13.38 | 12.52 | 11.49 | -3.66 |
| Average absolute error percentage | | | | | 20.4 | 12.33 |

Table 6 Predictability of bending moduli of fabric using empirical model

| Fabric sample No (From Table No.3) | Experimental values (mN mm) | | *Predicted values* (mN mm) | | Percentage error | |
|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_1$ | $B_2$ | $B_1$ | $B_2$ |
| 4 | 9.88 | 7.54 | 9.96 | 13.35 | .81 | 77.1 |
| 7 | 7.02 | 8.06 | 6.32 | 6.76 | -9.99 | -16.13 |
| 13 | 7.11 | 9.25 | 6.23 | 10.72 | -12.37 | 15.85 |
| 15 | 5.92 | 8.17 | 5.71 | 9.81 | -3.55 | 20.02 |
| 24 | 18.48 | 20.55 | 19.36 | 24.91 | 4.77 | 21.19 |
| 27 | 18.25 | 19.62 | 23.44 | 19.17 | 24.46 | -2.31 |
| 31 | 20.5 | 12 | 16.56 | 9.02 | -19.24 | -24.87 |
| Average absolute error percentage | | | | | 10.74 | 25.35 |

The coefficient of multiple determination ($R^2$) defines the fraction of variability in the dependent variable explained by the regression model. Except for $E_1$, the $R^2$ values of other models are high, and suggest that empirical model fits the data reasonably well. In further analysis of models, these empirical equations, namely, 5 to 8, were used to predict initial tensile moduli and bending rigidities of the fabric. The data used to test predictability of fabric properties by empirical models were *not* the part of data set used to develop the regression models. Fabric samples No. 1, 7, 11, and 18 from Table 1 were used to test the model represented by Eqs 5 and 6 and rest all data for development of regression equations. Similarly, Fabric samples Nos. 4, 7, 13, 15, 24, 27, and 31 from Table 3 were used to predict bending moduli of the fabrics and rest all fabric data was used to develop the regression model of fabric bending rigidity. Table 5 and Table 6 show the results of prediction for fabric initial tensile moduli and bending rigidities, respectively.

It can be observed from Table 5, that results of empirical models range from -5.43 % to 44.5% prediction error in warp-way fabric tensile modulus, and –22.63% to 22.18% in weft modulus, with average error of 20.4% and 12.33% in warp and weft modulus, respectively.

Table 6 shows, prediction error in bending rigidities range from –19.24% to 24.46% and –24.87% to 77.1%, in warp and weft directions, respectively. The average error in bending rigidities is 10.74% and 25.35% in warp and weft direction, respectively.

The high error in prediction of fabric properties by empirical modeling may be due to small data size and inability of the multilinear regression techniques to model the nonlinearities in the fabric structure-property relationships.

## 5. Artificial neural network modeling

Radial basis function (RBF) network was used to model the fabric structure-property relationships.

### 5.1 Principle of RBF network

The strategy used in RBF networks consists of approximating an unknown function with a linear combination of non-linear functions, called *basis functions.*

The basis functions are *radial functions*, i. e., they have radial symmetry with respect to a center. The schematic of

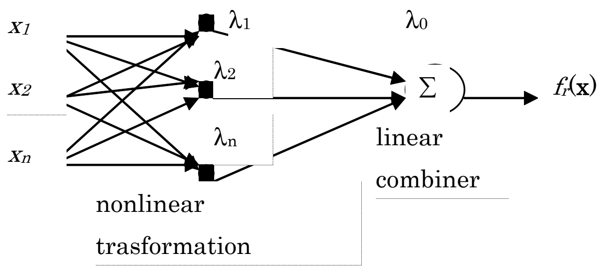


Fig. 1 Schematic of RBF network

the RBF network with $n$ inputs and a scalar output is shown in the Fig. 1.

Let $n$, denote the dimension of the input space, and then in an overall fashion, the network represents a map from $n$-dimensional input space to single-dimensional output space. Such a network implements a mapping $f_r$: $\boldsymbol{R}^n \rightarrow \boldsymbol{R}$ according to

$$f_r(\mathbf{x}) = \lambda_0 + \sum_{i=1}^{nr} \lambda_i \varphi(\ \|\mathbf{x} - \mathbf{c}_i\|\ ) \qquad (9)$$

where $\boldsymbol{x} \in \boldsymbol{R}^n$ is the input vector, $\varphi(\ .\ )$, is a given function from $\boldsymbol{R}^+$ to $\boldsymbol{R}$, $\|\ .\ \|$, denote the Euclidean norm, $\lambda_i$, $0 \le i \le n_r$, are the weights or the parameters, $\boldsymbol{c}_i \in \boldsymbol{R}^n$, $1 \le i \le n_r$, are known as the RBF centers, and $n_r$ is the number of centers. In RBF networks the functional form $\varphi(\ .\ )$ and the centers $\boldsymbol{c}_i$ are assumed to have been fixed. By providing a set of input $\boldsymbol{x}$ ($t$) and the corresponding desired output $d$ ($t$) for $t$ = 1 to $N$, values of the weights $\lambda_i$ can be determined using linear Least Square (LS) method.

Theoretical investigation and practical results suggest that the choice of the non-linearity $\varphi(\ .\ )$ is not crucial to performance of the RBF network. The functions may be thin-plate-spline function: $\varphi(v) = v^2 \log(v)$; or the Gaussian function: $\varphi(v) = \exp(-v^2 / \beta^2)$, or multiquadric function: $\varphi(v) = (v^2 + \beta^2)^{1/2}$; or inverse multiquadric function: $\varphi(v) = (v^2 + \beta^2)^{-1/2}$, where $\beta$ is a real constant. However the performance of RBF networks critically depends on the chosen centers.

In practice the centers are normally chosen from the data points $\{x(t)\}_{t=1}^{N}$. Such a mechanism results in poor performance and numerical ill conditioning frequently occurs owing to near linear dependency caused by, for example, some centers being too close.

Table 7 Effect of hidden layer neurons on prediction performance of RBF network

| No. of neurons | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Avg. error % of prediction | 19.5 | 21.5 | 20.9 | 11.5 | 10.4 | 9.41 | 9.41 | 9.41 | 9.41 | 9.41 | 9.41 |

Table 8 Effect of error goal on prediction performance of RBF network

| Error goal | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.1 | 0.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Neurons | 13 | 12 | 11 | 11 | 10 | 9 | 8 | 8 | 7 | 6 | 2 |
| Avg. error % of Prediction | 99.3 | 24.7 | 16.5 | 16.5 | 9.41 | 10.4 | 11.5 | 11.5 | 20.9 | 21.5 | 29.1 |

### 5.2 Selection of centers

Another way of center selection is *Orthogonal Least Square* (OLS) *learning procedure* [7]. This method is rooted in *Linear regression models*, according to which the desired response $d(n)$ is defined by

$$d(n) = \sum_{i=1}^{M} x_i(n)a_i + e(n) \qquad n = 1, 2, \ldots, N$$

where, $a_i$ are the model parameters, the $x_i(n)$ are the regressors, and $e(n)$ is the residue. Using matrix notation, the above equation can be written as-

$$\mathbf{d} = \mathbf{Xa} + \mathbf{e} \qquad (10)$$

where,

$$\mathbf{d} = [d(1), d(2), \ldots, d(N)]^T$$
$$\mathbf{a} = [a_1, a_2, \ldots, a_M]^T$$
$$\mathbf{X} = [\mathbf{x_1}, \mathbf{x_2} \ldots, \mathbf{x_M}]^T$$
$$\mathbf{x_I} = [x_I(1), x_I(2), \ldots, x_I(N)]$$
$$\mathbf{e} = [e(1), e(2), \ldots, e(N)]^T$$

The regressor vectors $\mathbf{x_I}$ form a set of basis vectors, and LS solution of equation 10 satisfies the condition that matrix product **Xa** be the *projection* of the desired response vector **d** on to the space spanned by the basis vector. The OLS method involves the transformation of the regressor vectors $\boldsymbol{x_1}, \boldsymbol{x_2} \ldots, \boldsymbol{x_M}$ into a set of orthogonal basis vectors $\boldsymbol{u_1}, \boldsymbol{u_2}, \ldots \boldsymbol{u_M}$. For this transformation, using *Gram- Schmidt orthogonalization procedure*

$$\mathbf{u_I} = \mathbf{x_I}$$
$$\alpha_{ik} = \mathbf{u_I}^T \mathbf{x_i} / \mathbf{u_I}^T \mathbf{u_I} \qquad 1 \le i \le k$$
$$\mathbf{u_I} = \mathbf{x_I} + \sum_{i-1}^{k-1} \alpha_{ik} \mathbf{x_I} \qquad \text{where } k = 2 \text{ to } M.$$

OLS learning procedure chooses the RBF centers $\mathbf{t}_1$, $\mathbf{t}_2$, ……$\mathbf{t}_M$ as a subset of training data vectors $\mathbf{x_1}, \mathbf{x_2} \ldots, \mathbf{x_M}$, where M < N. The centers are determined one-by-one (following Gram- Schmidt orthogonalization procedure), until the network of adequate performance is constructed. At each stage of the procedure, the increment to the explained variance of the desired response is maximized. In doing so, The OLS learning procedure will generally produce an RBF network whose hidden layer is smaller than that of an RBF network with randomly selected centers, for a specified level of unexplained variance of the desired response. Furthermore, the problem of numerical ill-conditioning encountered in random selection of centers is avoided. Thus, the OLS learning procedure provides another useful approach for construction of parsimonious RBF network with good numerical properties.

Neural Network Toolbox of MATLAB software of The MathWorks Inc. version 5.1.0.421 was used for RBF network implementation.

### 5.3 Training and predicting with RBF network

#### 5.3.1 Procedure

To study the predictability of initial tensile moduli of fabric, the fabric data given in Table 1 was randomly divided into 14 sets of input-output pairs for training of Radial basis function network and 4 input-output data set was used for testing the generalization ability of the trained network. The inputs to the net were fabric constructional parameters $p_1$, $p_2$, $l_1$, $l_2$, and yarn bending rigidities $\beta_1$, and $\beta_2$. The output set consisted of fabric initial tensile moduli, $E_1$ and $E_2$. Before feeding to the network, the input-output data set was scaled down to be with in (0,1), by dividing each value of the data by the maximum value of the overall data. The outputs of the network are de-scaled by multiplying the network outputs by the maximum value of the overall data. For each

Table 9 Effect of hidden layer neurons on prediction performance of RBF network

| Bias constant | 0.1 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.5 | 2 | 5 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Avg. error % of prediction | 21.2 | 15.1 | 13.7 | 9.41 | 12.4 | 11.8 | 11.4 | 11.8 | 11.7 | 11.8 |

Table 10 Design parameters optimized RBF network

| Fabric property | Radbas neurons | Sum squared error goal | Bias constant |
|---|---|---|---|
| Initial modulus | 10 | 0.05 | 0.8 |
| Bending rigidity | 6 | 0.07 | 0.8 |

Table 11 Predictability of initial tensile moduli of fabric using Radial Basis Function Network (RBFN) model

| Fabric sample No (From Table No. 1) | Experimental values (N/cm) | | RBFN values (N/cm) | | Percentage error | |
|---|---|---|---|---|---|---|
| | $E_1$ | $E_2$ | $E_1$ | $E_2$ | $E_1$ | $E_2$ |
| 1 | 14.3 | 36.6 | 13.78 | 32.77 | -3.63 | -10.47 |
| 7 | 13.7 | 53.6 | 13.18 | 46 | -3.77 | -14.17 |
| 11 | 9.1 | 14.8 | 10.54 | 14.88 | 15.86 | -.0.58 |
| 18 | 12 | 13 | 14.1 | 14.2 | 17.53 | 9.27 |
| | | [illegible] | | | [illegible] | [illegible] |

training run, the input-output data pairs are fed randomly to the network. Once, the maximum neurons of the hidden layer is fixed, the only design parameters that are to be optimized are the error goal and the bias of the hidden neuron. Therefore, training of network is quite fast. Several combinations of net parameters are experimented with, to ensure relatively small prediction error. The training is stopped when the prediction error with test-data-set was minimum.

Similar process was followed to train RBF network to predict fabric-bending rigidities. The fabric data shown in Table 3 was randomly divided into 26 sets of input-output pairs for training and 7 input-output data pairs for testing the network. The inputs to the net were fabric constructional parameters, namely, warp tex, weft tex, $d_1$, $d_1$, $p_1$, $p_2$, $l_1$, $l_2$, and yarn bending rigidities $\beta_1$, and $\beta_2$. The output set consisted of fabric bending rigidities, $B_1$ and $B_2$.

### 5.3.2 Effect of network design parameters on error of prediction

The analysis that follows is done on prediction of initial tensile moduli of the fabric. The average percentage error of prediction of initial moduli by RBF network is the average of both warp-way and weft-way error percentages.

(1) Number of neurons of the hidden layer

Number of neurons required to reach the error goal depends on the size of the input vector, desired training error goal, and the spread of the neuron in the hidden layer. Therefore, the values of - input size of data at 14, sum squared error goal at 0.05 and spread constant at 0.8, were kept constant and neurons in the hidden layer were varied from 5 to 15. The percentage of prediction of error in initial moduli for test data set was recorded.

It can be seen from Table 7, that the prediction error is high for neurons up to 9 in the hidden layer, and minimum at 10 neurons in the hidden layer. Thereafter, the network learns the input-output relationship and reaches the set error goal to produce a good generalization of the network with minimum error in prediction, and further addition of the neurons in the hidden layer is not necessary.

(2) Sum square error (error goal) on performance of RBF network

To study the effect of error goal on performance of the network, bias constant of the neuron was fixed at 0.8 and number of neurons in hidden layer was varied to meet the error goal. It is obvious, that as error goal for training the network becomes smaller, higher neurons are required in the hidden layer to meet the error goal. But higher number of neurons in hidden layer in combination of small error goal do not necessarily yield low prediction error. In fact, as depicted in the Table 8, this particular network shows an error of 99.3 % in the prediction of initial modulus of fabric. This may be due to over-fitting of data, and the function the network forms does not generalize well. The prediction error reaches minimum at error goal of 0.05. Thereafter, the prediction error of the network increases with increase in error goal.

Table 12 Predictability of bending moduli of fabric using Radial Basis Function Network (RBFN) model

| Fabric sample No (From Table No. 3) | Experimental values (mN mm) | | RBFN model (mN mm) | | Percentage error | |
|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_1$ | $B_2$ | $B_1$ | $B_2$ |
| 4 | 9.88 | 7.54 | 9.41 | 7.38 | -4.7 | -2.1 |
| 7 | 7.02 | 8.06 | 7.37 | 7.74 | 4.36 | -3.86 |
| 13 | 7.11 | 9.25 | 6.17 | 12.18 | -13.17 | 31.74 |
| 15 | 5.92 | 8.17 | 6.45 | 9.38 | 8.98 | 14.92 |
| 24 | 18.48 | 20.55 | 20.5 | 22.1 | 10.96 | 7.56 |
| 27 | 18.25 | 19.62 | 20.33 | 18.65 | 11.43 | -4.91 |
| 31 | 20.5 | 12 | 19.01 | 11.48 | -7.24 | -4.3 |
| Average absolute error percentage | | | | | 8.7 | 9.92 |

Table 13 Summary of range and prediction error%

| Fabric property | *Mathematical model* | | *Empirical model* | | Artificial neural network model (RBF) | |
|---|---|---|---|---|---|---|
| | Range | Error% | Range | Error% | Range | Error% |
| $E_1$ ( N/cm) | -58.12 to 24.1 | 20.53 | -5.43 to 44.5 | 20.4 | -13.17 to 1.43 | 10.2 |
| $E_2$ (N/cm) | -9.53 to 36.85 | 13.65 | -22.6 to 22.18 | 12.33 | -14.17 to 9.27 | 8.63 |
| $B_1$ (mNmm) | -100.8 to 20.7 | 18.7 | -19.24 to 24.5 | 10.74 | -13.2 to 11.47 | 8.7 |
| $B_2$ (mNmm) | -46.8 to 56.9 | 16.9 | -24.9 to 77.1 | 25.35 | -4.91 to 31.74 | 9.22 |

(3) Bias constant on prediction performance RBF network performance

The bias constant affects the bias value of the hidden layer neuron

$$bias = 0.8326 /(\text{bias constant})$$

Therefore bias constant affects the response space of the hidden neuron. To analyze the effect of bias constant on network generalization, neurons in the hidden layer and the error goal were kept constant at 10 and 0.05 respectively.

Too high or too low values of *bias* constant inhibit good function generalization, which can be seen from Table 9. The *bias* constant chosen should be larger than the distance between adjacent input vectors, but smaller than the distance across the whole input space

### 5.4 Performance of the RBF network

Considering above factors the design of RBF network was optimized and trained to produce minimum error of prediction. Table 10 shows the network parameters of the trained and optimized RBF network model.

Table 11 shows the experimental values, predicted outputs, and the percentage error of prediction for initial tensile moduli of the fabric. Here, it can be observed that the predictive errors of trained networks for $E_1$ and $E_2$ are very low, *i. e.,* 10.2% and 8.63%, respectively. The range of errors as shown in Table 11 for $E_1$ is from -3.77% to 17.53% and for $E_2$ it is from -14.17% to 9.27%.

The network predictions for fabric bending rigidities are shown in Table12. The average prediction error percentage in warp way fabric bending rigidity is 8.7% and 9.92% in the weft way direction, respectively. The prediction error ranges from -13.17% to 11.43%, and from -4.91% to 31.74% for warp and weft bending rigidity, respectively.

## 6. Comparative analysis of prediction error in fabric properties

Table 13 shows the summary of percentage prediction error with range for all the three methodologies. From the Table 13 it can be seen that RBF network produces least error of 10.2%, as compared to 20.4% and 20.53% for warp way fabric tensile modulus. For same fabric property, RBF network prediction shows lowest range amongst the three models. The Table 13 indicates similar results in prediction of weft way fabric modulus.

The percentage of prediction error is 8.63%, 12.33% and 13.65% for RBF network, regression model and mathematical model, respectively. The range of prediction error in weft way fabric initial modulus is lowest for RBF network, as compared to other two networks

The prediction error in fabric bending rigidities in warp and weft as shown in Table 13, are 8.77% and 9.22%, 10.74% and 25.35%, 18.7% and 16.9%, respectively for RBF network, empirical model and mathematical models. The same table indicates lower spread of prediction error for RBF network. From this analysis it can be concluded that RBFN can model the input-output relationships more accurately than other modeling methodologies.

## 7. Conclusions

Artificial neural network based on RBF learning algorithm produced the least error, as well as, lower spread in the error, as compared to mathematical and regression methods of modeling. It is also concluded that ANN have excellent property of approximating any functional relationship between large numbers of input-output (independent variables-dependent variables) parameters. No prior assumptions are required to be made on the statistical nature of the variables of the data, since ANN are nonparametric in nature. ANN require a much smaller data set than the one required for conventional regression analysis for capturing the nonlinear relationships between the input and output parameters. The size of the data sets for training of RBFN to predict fabric initial moduli and bending rigidity were only 14 and 25. With such a small training data set, the network was able to generalize the functional relationships very well. In the industry, where the large data is continuously available, ANN can be expected to perform significantly better. The major advantage of ANN is that there is no restriction on the levels of interaction between the variables; therefore, it can capture the dynamics of the real world situation very well. Neural networks once trained, are very easy to use and require very little human expertise. Since the network can accurately capture the nonlinear relationships between input-output parameters, they have extremely good predictive power. From this study it can be concluded that ANN can be a better and accurate predicting tool to design and engineer woven fabric products.

## References

[1] Rajmanickam, R., Steven, H., Jayaraman, S.; *Text Res. J.,* **67,** 39, (1997)
[2] Leaf, G. A. V., Kandil, K. H.; *J Text. Inst.* **71**, 1 (1980)
[3] Leaf, G. A. V., Chen, Y., Chen, X.; *J. Text. Inst.*, **84**, 419 (1993)
[4] Peirce, F. T.; *J. Text. Inst.* **28**, T45 (1937)
[5] Kawabata, S., Niwa, M. Kawai. H.; *J. Text. Inst.*, **64**, 21 (1973)
[6] Chen, X., Leaf, G. A.V.; *Text. Res. J.*, **70**, 437 (2000)
[7] Chen. S., Cowan, C. F. N., Grant, P. M.; *IEEE Transaction on Neural Networks,* **2**, No. 2, 302 (1991)
[8] Grosberg, P., Kedia, S.; *J. Text. Inst.,* **57**, 71 (1966)