The Diagnostics of Density Distribution for Dense Hot DT Plasmas Using Fast Protons

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The density reconstruction for dense DT plasmas with high temperature has been studied. In simulation of a plasma density diagnostics, the energy loss of fast protons going through the plasmas is crucial. The fast protons used for the diagnostics may be generated in the laser-plasma interaction and the number of the proton sources is four. If the original and final energies of protons are given, we can do the density reconstruction using simultaneous reconstruction technique (SIRT) without knowing the noise level of the final energy. It turns that the accuracy is better than that of the Tikhonov regularization method based on the deviation principle of Morozov's principle. In addition, the SIRT is suitable when the data is incomplete.

Key Words: Density reconstruction, Dense DT plasmas, High temperature, Fast protons, SIRT

1. Introduction

For relatively low plasma densities, several methods of plasma diagnostics, such as laser interferometry,[1)] Thomson scattering,[2)] and spectroscopic measurements,[3)] have been successfully applied. With the increasing of the plasma density, the optical depth of the plasma volume becomes excessively high when the plasma areal density is beyond 10^{21} cm^{-2}.[4)] So the techniques above no longer work well. At the same time fast protons generated[5-7)] during the interaction of ultraintense ($I > 10^{19}$ W/cm^2) short laser pulses with thin solid targets become effective for the diagnostics of dense plasmas because of their large stopping range in plasmas, small source size, short duration, large number density, and quasi-monoenergy.[7)] There are also many proton imaging techniques which allow the distribution of electromagnetic fields in plasmas to be explored[8,9)] and the density gradient of the laser-driven implosion target to be obtained using the angle deflection of the density impact on the protons.[10)] The research above[8, 9)] has been done with the thickness of the probing targets much smaller than the collisional stopping distance for the protons employed. So the energy loss of the protons is mainly due to the electromagnetic fields they have passed through. In the following we will focus on the impact of background electrons' collision on the energy loss of protons, ignoring that of the electromagnetic fields, which is right when the proton number is low or protons move together with electrons of equal number.[11)] Therefore, we can use the protons for the density diagnostics for extremely dense and thick plasmas, such as in the case of laser fusion. Furthermore the protons can be supposed to go through the plasmas probed straightly,[12)] which makes it easier to use the Coulomb energy loss as a method of plasma density diagnostics.

The density of the order of 10^{19} cm^{-3} in the homogeneous cold plasma has been obtained through the above method with one single proton beam by A. Golubev et al.[4)] For the inhomogeneous plasma of high temperature, we will do the density reconstruction in this paper by SIRT solving the large linear equation set of the densities of all the grids.

2. Theory and formulas

Protons propagate in a plasma almost without any angle deflection, the stopping power for the fast protons in the plasma of high temperature is

$$\frac{dE_p}{dx} = -\frac{4\pi(e^2/4\pi\varepsilon_0)^2}{m_e {v_p}^2} n_{fe} L_{fe} (\frac{1}{3})\sqrt{\frac{2}{\pi}}(\frac{v_p}{v_{the}})^3 . \qquad (1)$$

Here v_p and E_p are, respectively, the velocity and the kinetic energy of fast protons in the probing beam, v_{the} is the velocity of the electron of the background plasma, n_{fe} is the number density of free electrons in the plasma, L_{fe} is the coulomb logarithm, ε_0 is the permittivity of free space, and m_e and e are, respectively, the electron mass and charge. The condition[12)] for the feasibility of equation (1) is that the velocity of the proton is smaller than the thermal velocity of the background plasma electron, which is satisfied in this article where the largest energy of the proton is 15MeV and the temperature of the plasma is 10 keV.

A two dimensional (2D) studied area is divided into many grids, for example, N = 14x14 as shown in Fig. 1, N is the picture element of the area. When the grid is small enough, the density of each grid can be assumed uniform.

In order to reveal the density distribution, we will have M proton beams going through the zone studied. Then we will have M linear equations of the densities of all grids and obtain the following large linear equation set,

$$LX = b, \text{ ie. } b_i = \sum_{j=1}^{N} l_{ij} \times x_j , \qquad (2)$$

where x_j is the density of grid j, l_{ij} is the propagating length of proton beam i in grid j, b_i is proportional to the difference of the square root of the initial and final energies of proton beam i.

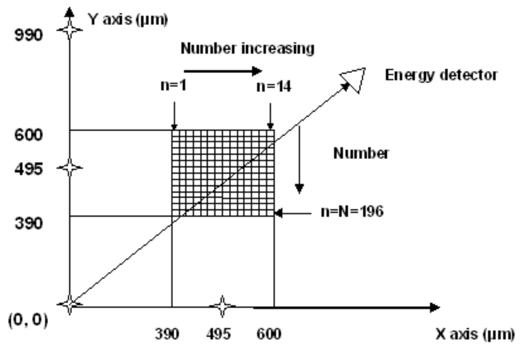

Fig. 1 The zone studied is the square of 210 μm × 210 μm extending from 390μm to 600μm on both the x axis and the y axis and divided into 196 grids. The coordinates of the proton sources are (0, 0), (0,990), (0,445), (445, 0). (This figure is only the sketch map, not represents the actual size.)

3. SIRT reconstruction

The matrix L can be calculated as long as the probing area, the initial position and the propagating direction of proton beams can be known. The matrix b can be obtained as the initial and final energies of proton beams can be known. We can't use general methods to solve this large linear equation set because the condition number of matrix L is very large and the measured final proton energies always include noises. To solve this equation set, we will use SIRT for the density reconstruction.[13)]

The iterative process of the SIRT is described in the followings:

$$\begin{aligned} &(\text{i})\ x^0 = (\text{b/L})' \\ &(\text{ii})\ x_j^{k+1} = x_j^k + \lambda(k) \times \sum_{i=1}^{M} (b_i - L_i \times x^k) \times L_{ij} \end{aligned} , \qquad (3)$$

where k is the number of the iterative times, x^k is the density distribution after iterated for k times and $\lambda(k)$ is the relaxing factor. When k is from 0.025 to 0.25, the reconstruction result is better than those of other relaxing factors for the algorithm ART. So we have one relaxing factor $\lambda(k)$ =0.065/M in this paper where M is the number of the equations in the large equation set.

4. Numerical simulations

4.1 Set plan for the numerical simulation

The 2D dense plasma slice considered has the density of order 10^{26} cm^{-3}. The zone studied extends from 390 m to 600 μm both on x direction and y direction in our coordinate system. The density distribution we will study has the expression:

$$n = \begin{cases} 10^{26}/\text{cm}^3 \times \exp(-r/36) & r < 60 \\ 0 & r \geq 60 \end{cases}, \qquad (4)$$

where r is the distance between the position and the center of the area(495,495).

The plasma densities of each grid can be thought as the density of the center of each grid. Then a simulated density matrix D of 196 elements is obtained.

Four identical proton sources are placed at (0, 0), (0, 990), (0,445) and (445, 0) respectively, with the mono-energy of 15 MeV. The energy detectors facing proton sources are around 1mm away from probing area with the minimum detecting angle interval of 0.2^{o}.The matrix L obtained this way has a condition number of 3.4×10^{17}, so the above equation set is quite ill-posed.

4.2 Density reconstruction calculation

4.2.1 Calculation without noises to the final energies

The final energies of each proton beam in the specific direction are calculated analytically using equation (1) and equation (4) for the simulated plasma density, so that an energy matrix of 196 elements is given. With this vector and equation (3), using the SIRT (k=4000), the revealed density matrix D' of 196 elements is obtained and the plasma density profile simulated can be repeated. The contour lines of the simulated density D are shown in Fig. 2, also the contour lines of the revealed density D' are shown in Fig. 3. Comparing the two figures, we can find that Fig. 3 can reflect the correct density distribution. In the zone where the simulated plasmas appear, the density revealed is a little smaller than the simulated one and on the same order of 10^{26} cm^{-3}. Defining Er=norm(D-D')/norm(D) as the revealing error, we can obtain

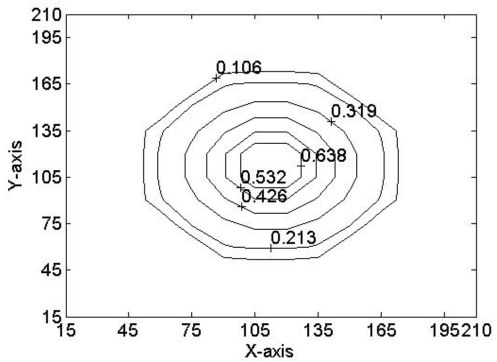

Fig. 2 The contour lines of the simulated density distribution. (The density values are the multiple of the value of each contour line and 10^{26} cm^{-3}, and this will apply to all the density distribution figures).

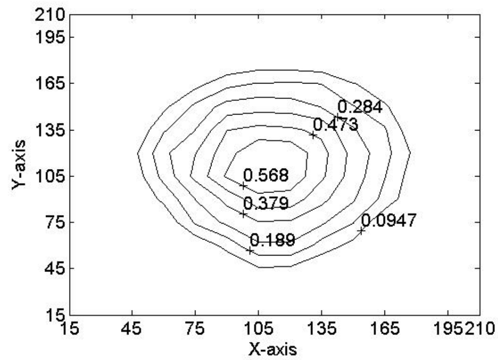

Fig. 3 The contour lines of the density revealed without noises to the final proton energies by SIRT method. The numbers of the grids and the protons sources are 196 and 4 respectively.

Er=16%. The error of the revealed density D' calculated by the Tikhonov regularization method[14] is described as Er=18%. Comparing these three errors, we can find that the result from SIRT can reflect the simulated density distribution better.

4.2.2 Calculation with noises to the final energies

Radio-Chromic Film (RCF) is one kind of energy detectors and has been used for high-flux proton detection in several laser–plasma experiments.[8,9] The uncertainty of the measurement is no more than 5%,[15] even can decrease to 2% in some measurement.[16] So we add two kinds of random noises (2% and 5%) to the final proton energies. The errors of the revealed density by SIRT are described as Er=18% and Er=22%. The accuracies are better than the results by the Tikhonov regularization method[14] with the errors of Er=20% and Er=26% for the same noises to the final energies.

4.2.3 Calculation with final energy matrix of 98 elements

The SIRT is mostly efficient for the situation when we don't have enough data. So we have done the density reconstruction when only 98 final energies are obtained with the same noises (2% and 5%) to the final energies. We obtain the errors of revealing as Er=22% and Er=26%, so the accuracy only decreases some percents, which proves the feasibility of the SIRT when we don't have enough data.

5. Conclusion

From the Coulomb energy loss of protons propagating in the unhomogenous, dense and hot plasmas, we obtain the large linear and ill-posed equation set for the densities of all grids, solved by the SIRT and the Tikhonov regularization method[14] based on the deviation principle of Morozov's principle. It turns out that the accuracy of SIRT is better than that of the Tikhonov regularization method.[14] Errors of without and with two kinds of noises (2% and 5%) to the final proton energies are described as Er=16%, 18% and 22% respectively, while those for the Tikhonov regularization method are described as Er=18%, 20% and 26%. SIRT also can do the reconstruction when not enough data of final proton energies are obtained. Because we don't assume the symmetric density distribution for the density reconstruction, so the SIRT should also be feasible for general density distribution. When more proton sources are used, the accuracy of revealing can be improved but the experiment will be more difficult. The errors are very high for two or three proton sources. Also with one proton beam, only the areal density $\int n_e dl$ can be obtained.[17]

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