相空间路径积分中的Feynman 规则和广义 Ward 恒等式*

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1995-09-04 收稿

摘　　要

基于 Green 函数的相空间生成泛函，导出了广义正则 Ward 恒等式．指出无须作出相空间生成泛函中对正则动量的路径积分，即可求得树图近似下的 Feynman 规则．对 φ^4 场的拉氏量添加一个四维散度项后，场的传播子发生了改变．

关键词　路径积分，Ward 恒等式，Feynman 规则，散度项．

在动力系的路径积分(泛函积分)量子理论中，从 Green 函数的生成泛函导出理论的 Feynman 规则和 Ward 恒等式，通常是用位形空间中的拉氏量(或有效拉氏量)来表述的，它仅适用于相空间中生成泛函对正则动量的路径积分为 Gauss 型的情形．当对动量的路径积分不能积出或积分十分困难(特别是约束 Hamilton 系统)的情形，其相空间 Ward 恒等式已建立[1,2]．这里给出它的推广；讨论相应的 Feynman 规则；研究 φ^4 场论的拉氏量添加一个四维散度项后，其系统的量子 Green 函数的变化．

考虑一个用奇异拉氏量描述的系统，该系统在相空间中存在固有约束，为约束 Hamilton 系统[3]．设 $\Lambda_k(k=1, 2,\cdots, K)$ 为系统的第一类约束，$\theta_i(i=1,2, \cdots I)$ 为第二类约束，与第一类约束相应的规范条件为 $\Omega_k(k=1, 2, \cdots, K)$．对正则变量均引入外源，相空间 Green 函数的生成泛函为[1]

$$Z[J, K, L, M, N]=\int \mathscr{D}\varphi^{\alpha}\mathscr{D}\pi_{\alpha}\mathscr{D}\lambda_{m}\mathscr{D}C^{\alpha}\mathscr{D}p_{\alpha}\mathscr{D}\overline{C}^{\alpha}\mathscr{D}\overline{p}_{\alpha}$$
$$\times\exp\left\{\mathrm{i}\left[I_{\mathrm{eff}}^{\mathrm{P}}+\int \mathrm{d}^4x(J_{\alpha}\varphi^{\alpha}+K^{\alpha}\pi_{\alpha}+L^{m}\lambda_{m}+\overline{M}_{\alpha}C^{\alpha}+\overline{C}^{\alpha}M_{\alpha}+\overline{N}^{\alpha}p_{\alpha}+\overline{p}_{\alpha}N^{\alpha})\right]\right\}, \quad (1a)$$

其中

* 国家自然科学基金和北京市自然科学基金资助.

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$$I_{\text{eff}}^{\text{p}}=\int \mathrm{d}^4x\, \mathscr{L}_{\text{eff}}^{\text{p}}=\int \mathrm{d}^4x\Big\{\pi_\alpha\dot{\varphi}^\alpha-\mathscr{H}_{\text{c}}+\lambda_k\Lambda_k+\lambda_l\Omega_l+\lambda_i\theta_i+\int \mathrm{d}^4y\Big[\overline{C}_k(x)\{\Lambda_k(x),\ \Omega_l(y)\}C_l(y)$$
$$+\frac{1}{2}\overline{C}_i(x)\{\theta_i(x),\ \theta_j(y)\}C_j(y)\Big]\Big\}, \tag{1b}$$

而 $\lambda_m=(\lambda_i,\ \lambda_k,\ \lambda_l)$ 为乘子场，p_α 和 $\overline{p}_\alpha$ 为鬼场 C^α 和 $\overline{C}^\alpha$ 的正则动量．为简化记号，令 $\phi=(\varphi^\alpha,\ \lambda_m,\ C^\alpha,\ \overline{C}^\alpha)$，$\pi=(\pi_\alpha,\ p_\alpha,\ \overline{p}_\alpha)$，$J=(J_\alpha,\ L^m,\ M_\alpha,\ \overline{M}_\alpha)$，$K=(K^\alpha,\ N^\alpha,\ \overline{N}^\alpha)$，$I_{\text{eff}}^{\text{p}}=I^{\text{p}}$，于是(1a)式可写为

$$Z[J,\ K]=\mathrm{e}^{\mathrm{i}W[J,\ K]}=\int \mathscr{D}\phi\mathscr{D}\pi\exp\Big\{\mathrm{i}\,[I^{\text{p}}+\int \mathrm{d}^4x(J\phi+K\pi)]\Big\}. \tag{2}$$

对于非奇异(正规)拉氏量系统，$\mathscr{L}_{\text{eff}}^{\text{p}}=\pi_\alpha\dot{\varphi}^\alpha-\mathscr{H}_{\text{c}}$．

设 $F(\phi,\ \pi)$ 是正则变量的给定泛函，下列泛函积分

$$Z_F\,[J,\ K]=\int \mathscr{D}\phi\mathscr{D}\pi F\,(\phi,\ \pi)\ \exp\Big\{\mathrm{i}\Big[I^{\text{p}}+\int \mathrm{d}^4x(J\phi+K\pi)\Big]\Big\}. \tag{3}$$

在 $J=K=0$ 时，恰为 $\hat{F}\,(\varphi,\ \pi)$ 的期望值[4]．(3) 式在下列积分变量的变换

$$\begin{cases}\phi(x)\to\ \phi'(x)=\phi(x)+S_\sigma\varepsilon^\sigma(x),\\ \pi(x)\to\ \pi'(x)=\pi(x)+T_\sigma\varepsilon^\sigma(x)\end{cases} \tag{4}$$

下是不变的，其中 S_σ 和 T_σ 为线性微分算符[5]．$\varepsilon^\sigma(x)(\sigma=1,2,\cdots,\ r)$ 为无穷小任意函数，它们及其微商在四维区域的边界上为零．在(4)式变换下，I^{p} 的变分为[3]

$$\delta I^{\text{p}}=\int \mathrm{d}^4x\left[\frac{\delta I^{\text{p}}}{\delta\phi}\ \delta\phi+\frac{\delta I^{\text{p}}}{8\pi}\ \delta\pi+\frac{\mathrm{d}}{\mathrm{d}t}\ (\pi\delta\phi)\right], \tag{5a}$$

其中

$$\delta I^{\text{p}}/\delta\phi=-\dot{\pi}-\delta H/\delta\phi\ ,\quad \delta I^{\text{p}}/\delta\pi=\dot{\phi}-\delta H/\delta\pi\ . \tag{5b}$$

假设变换(4)的 Jacobi 行列式为 1，根据 $\varepsilon^\sigma(x)$ 的边界条件，泛函(3)式在(4)式变换下不变，就得相空间中广义 Ward 恒等式[1]

$$\Big[\widetilde{S}_\sigma\Big(\frac{\delta F}{\delta\phi}\Big)+\widetilde{T}_\sigma\Big(\frac{\delta F}{\delta\pi}\Big)+\mathrm{i}\widetilde{S}_\sigma\Big(F\frac{\delta I^{\text{p}}}{\delta\phi}\Big)+\mathrm{i}\widetilde{T}_\sigma\Big(F\frac{\delta I^{\text{p}}}{\delta\pi}\Big)$$
$$+\mathrm{i}\widetilde{S}_\sigma(FJ)+\mathrm{i}\widetilde{T}_\sigma(FK)\Big]_{\substack{\phi\to\frac{1}{\mathrm{i}}\frac{\delta}{\delta J}\\ \pi\to\frac{1}{\mathrm{i}}\frac{\delta}{\delta K}}}Z_F[J,\ K]=0. \tag{6}$$

其中 $\widetilde{S}_\sigma$，$\widetilde{T}_\sigma$ 分别为 S_σ，T_σ 的伴随算符[3]．当 $F=1$ 时，(6)式即为相空间中的 Ward 恒等式[6]．如果取 $F=\phi(x_1)\phi(x_2)\cdots\phi(x_n)$，考虑正则变量的平移变换，由(6)式可导出场的 Green 函数间的关系．设 $F(\phi,\ \pi)$ 在(4)式变换下不变，则相空间中广义 Ward 恒等式为

$$\left[\widetilde{S}_\sigma\left(F\ \frac{\delta I^{\mathrm{p}}}{\delta\phi}\right)+\widetilde{T}_\sigma\left(F\frac{\delta I^{\mathrm{p}}}{\delta\pi}\right)+\widetilde{S}_\sigma(FJ)+\widetilde{T}_\sigma(FK)\right]_{\substack{\phi\to\frac{1}{\mathrm{i}}\frac{\delta}{\delta J}\\ \pi\to\frac{1}{\mathrm{i}}\frac{\delta}{\delta K}}} Z_F[J,\ K]=0. \tag{7}$$

利用泛函 Legendre 变换，引入正规顶角生成泛函 $\Gamma[\varphi,\ \pi]$[1],

$$\Gamma[\phi,\ \pi]=W[J,\ K]-\int \mathrm{d}^4x\,[J(x)\phi(x)+K(x)\pi(x)]\ , \tag{8a}$$

$$\phi(x)=\delta W\ /\ \delta J(x),\qquad J(x)=-\delta\Gamma\ /\ \delta\phi(x), \tag{8b}$$

$$\pi(x)=\delta W\ /\ \delta K(x),\qquad K(x)=-\delta\Gamma\ /\ \delta\pi(x). \tag{8c}$$

将(2)式右端的指数因子在$[\phi_0(x),\ \pi_0(x)]$邻域展开，有

$$\begin{aligned}
&\int \mathrm{d}^4x\,[\pi(x)\dot{\phi}(x)-\mathscr{H}(x)+J(x)\phi(x)+K(x)\pi(x)]\\
&=\int \mathrm{d}^4x[\pi_0(x)\dot{\phi}_0(x)-\mathscr{H}_0(x)+J(x)\phi_0(x)+K(x)\pi_0(x)]\\
&+\int \mathrm{d}^4x\left\{\left[\frac{\delta I^{\mathrm{p}}}{\delta\phi(x)}+J(x)\right][\phi(x)-\phi_0(x)]\right.\\
&+\left.\left[\frac{\delta I^{\mathrm{p}}}{\delta\pi(x)}+K(x)\right][\pi(x)-\pi_0(x)]\right\}\\
&+\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^4y\ \frac{\delta^2 I^{\mathrm{p}}}{\delta\phi(x)\delta\phi(y)}\ [\phi(x)-\phi_0(x)][\phi(y)-\phi_0(y)]+\cdots\cdots\ .
\end{aligned} \tag{9}$$

采用最陡下降法，取展开点$[\phi_0(x),\ \pi_0(x)]$适合

$$\delta I^{\mathrm{p}}\ /\ \delta\phi(x)+J(x)=0,\qquad \delta I^{\mathrm{p}}\ /\ \delta\pi(x)+K(x)=0. \tag{10}$$

由(8a)和(9)式可得：在树图近似下正规顶角的生成泛函等于正则作用量，即 $\Gamma[\phi_0,\ \pi_0]=I^{\mathrm{p}}[\phi_0,\ \pi_0]$. 从而勿需作出相空间生成泛函对正则动量的泛函积分，就可导出树图近似下的各次正规顶角，导出相应的 Feynman 规则.

如所周知，对系统的拉氏量添加一个四维散度项，不会改变系统的经典运动方程. 在量子水平上对系统有否影响，近来从高阶微理论角度作了探讨[7, 8]. 这里对 ϕ^4 场论的拉氏量添加一个四维散度后，使其仍为一阶微商的情形来讨论:

$$\mathscr{L}(x)=-\frac{1}{2}\ (\partial_\mu\varphi\partial_\mu\varphi+\mu^2\varphi^2)-\frac{\lambda}{4!}\ \varphi^4-a_\mu\varphi\partial_\mu\varphi\ , \tag{11}$$

其中 a_μ 为常矢，$a_\mu=(1,\ 1,\ 1,\ i)$. $\varphi(x)$ 的正则动量为

$$\pi(x)=\partial\mathscr{L}/\partial\dot{\varphi}(x)=\dot{\varphi}(x)-\varphi(x). \tag{12}$$

正则 Hamilton 量密度为

$$\mathscr{H}(x)=\pi(x)\dot{\varphi}(x)-\mathscr{L}(x)=\frac{1}{2}\pi^2(x)+\frac{1}{2}\nabla\varphi(x)\cdot\nabla\varphi(x)+\frac{1}{2}(\mu^2+1)\varphi^2(x)$$
$$+\varphi(x)\pi(x)+\frac{\lambda}{4!}\varphi^4+a_k\varphi(x)\partial_k\varphi(x). \tag{13}$$

正则作用量为

$$I^{\mathrm{p}}=\int \mathrm{d}^4x\,[\pi(x)\dot{\varphi}(x)-\mathscr{H}]=\int \mathrm{d}\,x\left\{\frac{1}{2}[\pi^2(x)-\nabla\varphi\cdot\nabla\varphi-(\mu^2+1)\varphi^2(x)]\right.$$
$$\left.-\frac{\lambda}{4!}\varphi^4(x)-a_k\varphi(x)\partial_k\varphi(x)\right\}. \tag{14}$$

树图近似下两点正规顶角为

$$\left.\frac{\delta^2\Gamma}{\delta\varphi(x)\delta\varphi(y)}\right|_{\varphi=\pi=0}=\left.\frac{\delta^2 I^{\mathrm{p}}}{\delta\varphi(x)\delta\varphi(y)}\right|_{\varphi=\pi=0}$$
$$=\int \mathrm{d}^4z\,\mathrm{d}^4w\,\frac{\delta}{\delta\pi(w)}\left(\frac{\delta I^{\mathrm{p}}}{\delta\pi(z)}\,\frac{\delta\pi(z)}{\delta\varphi(x)}\right)\left.\frac{\delta\pi(w)}{\delta\varphi(y)}\right|_{\varphi=\pi=0}+\left.\frac{\delta^2 I^{\mathrm{p}}}{\delta\varphi(x)\delta\varphi(y)}\right|_{\varphi=\pi=0}$$
$$=-[\partial^2+a_\mu\partial_\mu+(\mu^2+1)]\delta(x-y). \tag{15}$$

它与通常 φ^4 场论的结果不同．从而场的传播子发生变化．不难验证，添加上述四维散度项后，最低次(四次)顶角不变．

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Feynman Rules and Generalized Ward Identities in Phase Space Functional Integral

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Received 4 Septmber, 1995

Abstract

Based on the phase-space generating functional of Green function, the generalized canonical Ward identities are derived. It is point out that one can deduce Feynman rules in tree approximation without carring out explicit integration over canonical momenta in phase-space generating functional. If one adds a four-dimensional divergence term to a Lagrangian of the field, then, the propagator of the field can be changed.

Key words path integral, Ward identity, Feynman rule, divergence term.