Geophysica (1997),33(2),45-50

Extending the Similarity Theory for Atmospheric Boundary Layers: Contribution from Background Stratification

Sylvain M. Joffre

Finnish Meteorological Institute
P.O. Box 503
FIN-00101 Helsinki, Finland

Abstract

The structure of the atmospheric boundary layer has traditionally been described using only surface fluxes and the Coriolis parameter. We review here the proposal of Kitaigorodskii (1988) and Kitaigorodskii and Joffre (1988) that the background stratification of the atmosphere can influence the overall properties of the boundary layer through the process of entrainment. Theoretical predictions have been successfully compared to various data sets.

Key words: Atmospheric boundary layer, similarity theory, turbulence, stratification effects

1 *Introduction*

The atmospheric boundary layer (ABL) is the layer where interactions take place between the surface, which captures most of the solar energy and redistributes it under different forms, and the large-scald atmospheric flow which absorbs them in the end. This turbulent ABL is of great importance for transfer of momentum, heat and atmospheric constituents, dispersion of pollutants, for enhancing evaporation over the sea, in creating wind waves on water surface and producing drift currents in the upper layer of the water bodies. More recently, it has been recognized that transfers of many minor constituents (SO_2, NO_x, CO_2, O_3, C_xH_y) which often enter the atmosphere or are deposited to terrestrial and aquatic ecosystems via the ABL is one of the main link in the global biogeochemical cycles.

It is thus of primary importance to be able to understand, parametrize and simulate the structure and behaviour of the ABL. The classical way of describing its structure is through similarity theories (eg. *Kazanski* and *Monin,* 1960) where the only influencing agents are rotation and buoyancy. There are, however, other factors such as entrainment, the intensity of which is connected with background stratification. The height h of the ABL is the key parameter embedding all internal and external influences and having much importance for a host of applications is. Predictions of the ABL height has many theoretical and practical applications such as predictions of pollutant concentrations or of surface temperature. We will review in this paper new theoretical and empirical arguments proposed by *Kitaigorodskii* (1988) and *Kitaigorodskii* and *Joffre* (1988) for extending the similarity theory describing the height of the ABL.

Published by the Finnish Geophysical Society, Helsinki

2. *Theoretical background*

We assume an ABL structure where the external forcing parameters are the Coriolis parameter f, the surface roughness length z_0, and the background temperature lapse rate $\gamma = \partial\theta/\partial z$ above the ABL, and in which the internal turbulent parameters are the surface momentum flux (proportional to the friction velocity u_*) and the surface heat flux Q_0. The ABL profiles of mean quantities are not specified because we will only consider overall properties. The ABL profiles adjust to the overlying free stream values within an interfacial sublayer of thickness Δh. Classically the ABL height is assumed to be a function of the length scale $L_E = u_*/f$ and $L_* = -u_*^3/g\beta\kappa Q_0$, *where* $g\beta = g/T_0$ *is* the buoyancy parameter and $\kappa = 0.4$ the Karman constant.

A starting point for investigating the dynamics of the ABL is the equation of turbulent kinetic energy E_K since it contains the basic physics driving the turbulence field. Since we are not interested here in the detail of the structure of the ABL but rather in the overall energetics we integrate the E_K-equation across the ABL and the interfacial layer yielding:

$$\langle E_K \rangle \partial h / \partial t = u_*^2 V_{SL} + \frac{1}{2}(\Delta u^2 + \Delta v^2)\frac{\partial h}{\partial t} + \frac{1}{2}h(g\beta Q_0 - g\beta\Delta\Theta\,\partial h/\partial t) - h < \varepsilon > - \varepsilon_h \Delta h$$

where $h <s> = \int s(z)dz$ through the ABL from z=0 to $z=h$, ε is the viscous dissipation and ε_h its value integrated through the interfacial layer between h and $h+\Delta h$. The terms Δu, Δv and $\Delta\Theta$ express the jumps of the wind components and temperature through the interfacial layer, respectively, while V_{SL} is the horizontal wind velocity at the top of the surface layer. Assuming that the production of E_K by shear in the interfacial layer (2nd rhs term) is balanced by the local viscous dissipation (last rhs term) and that the nonstationarity term (Ihs term) is small, we are left with:

$$-\frac{1}{2}h(g\beta Q_0 + g\beta Q_h) = C_f u_*^3 - h < \varepsilon > \qquad (1)$$

where $Q_h = - \Delta\Theta\partial h/\partial t$ is the entrainment heat flux at the top of the ABL and C_f is a drag coefficient.

This equation can be solved using the concept of marginal Richardson number (*Pollard et al.* 1973; *Joffre* 1981) which leads to the result that the classical critical number $Ri_{\text{crit}}= 0.25$ fits the data in the case of small rotation effect ($hf/u_* < 0.\ 1$) but increases as hf/u_* increases (*Joffre* 1981).

$Ri_{\text{crit}}= 0.25$ fits the data in the case of small rotation effect ($hf/u_* < 0.1$) but increases as $hf/u,$ increases *(Joffre* 1981).

Another closure approach is to use the heat diffusion approach by which the term Q_h is pgametrized through an effective eddy diffusivity coefficient (*Kitaigorodskii* 1988; *Kitaigorodskii* and *Joffre* 1988), ie.:

$$Q_h = -\Delta\Theta\,\partial h/\partial t = -K_h \partial\Theta/\partial z = -K_h\gamma = -cw_m h\gamma \qquad (2)$$

where $w_m = a_0 u_* + a_1 w_*$ is the velocity scale including the convective velocity scale $w_* = [g\beta h Q_0]^{1/3}$. Assuming that the dissipation term can be parameterized as $h<\varepsilon> = m_i u_*^3$ with m_i being a function of the Monin-Kazanski stability parameter $\mu_0 = L_E/L_*$.

Thus, the final E_K-equation takes the nondimensional form:

$$a_0\kappa(h/L_N)^2 + (h/L_*) = 2\kappa C_f - 2\kappa m_i \qquad (3)$$

where has appeared the new length scale $L_N = u_*/N$, with $N = (g\beta\gamma)^{1/2}$ the Brunt-Väisälä frequency.

3. *Comparisons with observations*

This theoretical framework was tested by *Kitaigorodskii* and *Joffre* (1988) using several sets of data. They used data gathered during an intensive field campaign carried out over the frozen Northern Baltic Sea with a wide range of stable conditions and covering as well unstable conditions with a dual regime of thermal and mechanical production of turbulence (*Joffre* 1981, 1982). Additionally they use data from the Wangara campaign carried out in Australia (*Clarke et al.* 1971) and the North Atlantic JASIN data (*Nicholls* 1985).

a) Neutral conditions

In the asymptotic case of neutral conditions (i.e. $|L_*| \rightarrow \infty$) Eq. (3) tends towards the solution:

$$h = [(2C_f - m_0)/a_0]^{1/2} L_N = b L_N \approx 4\text{-}13\, L_N \qquad (4)$$

where $m_i(0) = m_0$. Note that if we use the factor $(2\pi)^{-1}$ to express N, the factor b is of order 1. The data compiled by *Kitaigorodskii* and *Joffre* (1988) confirm this prediction.

b) Stable conditions

The velocity scale $w_m \rightarrow u_*$ and $m_i \rightarrow m_s$ such that Eq.(3) takes the form:

$$a_o\, \kappa\, (h/L_*)^2 (LN/L_*)^{-2} - (h/L^*) = 2\, \kappa\, (C_f - m_s) = m' \qquad (5)$$

Thus $h/L_* = F^+(L_N/L_*)$. According to Eq. (5), in the case of a strong capping lid (i.e., N $\rightarrow \infty$), h. tends towards 14 L_N. In the case of no background stratification (i.e., N $\rightarrow$ 0) but with a surface negative flux Q_0, the ABL height h is proportional to the Monin-Obukhov length scale, i.e. $h \rightarrow m' L_*$ (with $m' \approx 40$). This result was also derived by *Kitaigorodskii* (1960) for the surface layer of the ocean but with a much smaller coefficient.

The general solutions for stable conditions read:

$$h/L_* = \{- 1 + [1 + 4\, a'\, m'\, (L_N/L_*)^{-2}]^{1/2}\}\ \{2\, a'(L_N/L_*)^{-2}\}^{-1} \qquad (6)$$

where $a' = a_0\, \kappa\, K_h$ (0.1-0.3). Comparison with atmospheric data show a good agreement up to values of $L_N/L_* \approx 5$ (*Kitaigorodskii* and *Joffre,* 1988). The scatter of the data can be explained for the most part by differentiating the data according to the remaining dimensionless parameter L_E/L_N (which has to be taken into account according to Buckingham's π-theorem). The degradation of the empirical fit under stronger stability conditions can be also due to problems connected to measuring turbulent fluxes and determining the ABL height under such regimes of intermitent weak turbulence.

c) Unstable conditions

Under unstable conditions we assume $m_i \rightarrow m_u$ and the general E_K-equation (3) takes the form:

$$[a' + a''\, |h/L_*|^{1/3}]\, (h/L^*)\, (L_N/L_*)^{-2} - |h/L_*| = m'' \qquad (7)$$

where $a'' = a_1\, \kappa^{2/3} K_h$ and $m'' = 2\, \kappa\, (C_f - m_u)$. Thus, in general $h/L_* = F(L_N/L_*)$. In the special case of free convection (i.e., $u_* \rightarrow 0$) the function F behaves asymptotically according to $|L_N/L_*|^{3/2}$. Considering only conditions with $m'' > 0$, i.e. where both mechanical and thermal turbulence are active, we obtain the general solution:

$$h/L_* = \{1 + [1 + 4\, a'm'' (L_N/L_*)^{-2}]^{1/2}\} \{2\, a'(L_N/L_*)^{-2}\}^{-1} \qquad (8)$$

Once again the data compiled by *Kitaigorodskii* and *Joffre* (1988) fitted very well with these solutions and even the start of the convective regime with the $|L_N/L_*|^{3/2}$ -slope could be empirically observed. Here again the internal scatter of the data is described by the dimensionless parameter $L_E/L_N = N/f$.

4. Discussion

Classical ground-based measurements of turbulent fluxes do not always provide a realistic description of the important contributions influencing the dynamics of the whole ABL. Rather, background stratification through the parameter N plays a very important role. This leads to the inclusion of the length scale $L_N = u_*/N$ into the similarity description of the ABL height, and the dimensionless ABL height h/L_* is a function of the dimensionless parameter L_N/L_* and L_E/L_N. This approach was tested against very different sets of data by *Kitaigorodskii* and *Joffre* (1988) which leads to the conclusion that, in both stable and unstable conditions, this theory did not contradict the assumption that the balance between entrainment and surface heat flux can be parameterized in the simple form (1 -3).

More recently, *Overland* and *Davidson* (1992) also have found a clear dependence between the height of the inversion base height and the scale L_N defined at a fixed height of 500 m from measurements performed above the Arctic Ocean. Their proportionality coefficient was of the same of order of magnitude (within the large scatter) as the one proposed by *Kitaigorodskii* and *Joffre* (1988).

Van Pul et al. (1994) also have found a good correlation between the height h of the stable ABL determined by lidar occultation and the scale L_N with a proportionality coefficient of 6. *Vogelezang* and *Holtslag* (1995) have derived an expression for stable and neutral ABL based on observations and large eddy simulation results showing that the backgound gradients are essential for describing *h.* They show that their formulation reduced to our expression (4) with compatible numerical coefficients.

Kitaigorodskii (1992) has also shown that the present theory applies to the benthic boundary layer which was also confirmed by the modelling effort of *Rahm* and *Svensson* (1989) and by direct observations of the benthic boundary layer in the Norwegian Sea by *Nabatov* and *Ozmidov* (1987).

References

Clarke, R.H., A.J. Dyer, R.R. Brook, D.G. Reid and AI Troup, 1971. The Wangara experiment: boundary layer data. *Techn. Pap.* No. 19, CSIRO, Division of Meteorological Physics, Aspendale, Australia, 362 p.

Joffre, S.M., 1981. The physics of the mechanically-driven atmospheric boundary layer as an example of air-sea ice interactions. *Report* No. 20, Department of Meteorology, University of Helsinki, 75 p.

Joffre, S.M., 1982. Momentum and heat transfers in the surface layer over a frozen sea. *Boundary-Layer Meteorol.,* **24**, 211-229.

Kazanski A.B. and A.S. Monin, 1960. The turbulent regime above the surface atmospheric layer. *Izv. Acad. Sci.,* Ser. Geof. 1, 165-168.

Kitaigorodskii, S.A., 1960. Calculating the thickness of the wind-induced mixing layer in the ocean. *Izv. Acad. Sci.,* Ser. Geof. 3, 425-431 (english edition 284-287).

Kitaigorodskii, SA, 1988. A note on similarity theory for atmospheric boundary layers in the presence of background stable stratification. *Tellus,* **40A**, 434-438.

Kitaigorodskii, SA, 1992. The location of thermal shelf fronts and the variability of the heights of tidal benthic boundary layers. *Tellus,* **44A**, 425-433.

Kitaigorodskii, S.A. and S.M. Joffre, 1988. In search of a simple scaling for the height of the stratified atmospheric boundary layer. *Tellus,* **40A**, 419-433.

Nabatov, VN. and R.V Ozinidov, 1987. Investigation of the bottom boundary layer in the ocean. *Oceanologia,* XXVII, 1, 5-11.

Nicholls, S., 1985. The structure of the marine atmospheric boundary layer observed during JASIN. *Quart. J. R. Met. Soc.,* **111**, 391-426.

Overland, LE. and K.L. Davidson, 1992. Geostrophic drag coefficients over sea ice. *Tellus,* **44A**, 54-66.

Pollard, RX, P.B. Rhines and R.O.R.Y. Thompson, 1973. The deepening of the wind-mixed layer. *Geophys. Fluid Dyn.,* **3**, 381-404.
Ralun, L. and U. Svensson, 1989. Dispersion in a stratified benthic boundary layer. *Tellus,* **41A**, 148-161.
Van Pul, W.XL, A.A.M. Holtslag and RRA Swart, 1994. A comparison of ABL heights inferred routinely from lidar and radiosondes at noontime. *Boundary-Layer Meteorol.,* **68**, 173-191.
Vogelezang, D.N.P. and AAM. Holtslag, 1995. Evaluation and model impacts of alternative boundary-layer height formulations. *Boundary layer Meteorology,* **81**, 245-269.