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ESTIMATION OF THE ELECTRIC FIELD ON THE EARTH'S SURFACE DURING A GEOMAGNETIC VARIATION

by

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Abstract

Theoretical calculation of the induced electric field during a time variation in the geomagnetic field is discussed. Formulas expressing a horizontal electric field component in terms of the time derivative of the perpendicular magnetic component are given. The treatment is based principally on a formula valid for the field on the surface of a homogeneous earth. This formula is obtained assuming that the primary field is spatially almost constant on the surface.

Four different types of horizontal magnetic variations are discussed: linear change, change of level, pulse-like variation and periodic variation. The numerical calculations indicate the possible existence of horizontal electric fields in the order of 1 V/km on the earth's surface during geomagnetic storms, but usually the field is much smaller. A comparison indicates that a change of level and a periodic variation may be accompanied by smaller electric fields than the other two types of magnetic variations.

1. *Introduction*

According to Faraday's law of induction, a change of the geomagnetic field in time is always accompanied by an electric field. The observed geomagnetic variation and the electric field on the earth's surface depend on primary sources situated in the magnetosphere and ionosphere, and on secondary currents and charges induced in the earth (and in principle also slightly in the air) (*e.g.* PIRJOLA, 1982). The phenomenon is called electromagnetic or geomagnetic induction in the earth; this stresses the role of secondary sources in the earth.

The existence of a horizontal electric field implies voltages between different points at the earth's surface. These voltages give rise to electric currents in conductors like power transmission grids, and oil and gas pipelines. Currents in the former cause saturation in transformers, which leads to interference in the operation of the power system or even to permanent damage (WILLIAMS, 1979). In pipelines problems associated with corrosion and corrosion control arise (PEABODY, 1979; CAMPBELL, 1979).

Theoretical calculation of geomagneticlly-induced currents in a network of conductors is divided into two parts:

1. The electric field in the absence of the conductors is estimated. This can be regarded as a purely geophysical problem.
2. Currents caused in the network by this electric field are calculated, which can be viewed as an engineering problem.

For Part 2 we refer to LEHTINEN and PIRJOLA (1984), and PIRJOLA and LEHTINEN (1984). This paper deals with Part 1.

I assume that the geomagnetic variation is known and I will calculate the corresponding electric field induced on the earth's surface. The purpose is to discuss basic principles of the phenomenon and to consider simple specific cases usable in the practical evaluation of the geomagnetically-induced electric field. I will therefore describe the earth, when its effect is taken explicitly into accout, simply as an electromagnetically isotropic, linear and homogeneous half-space with a flat surface, and assume that the electromagnetic field within the earth associated with a geomagnetic variation depends only on the vertical space coordinate and time.

As explained on page 113 in PIRJOLA (1982), the latter assumption can be considered satisfied provided the primary field does not change much over horizontal distances equal to the relevant skin depths in the earth. This requires that the primary source is either an enormous horizontal current sheet or situated far enough from the point of observation. In practice, the primary field normally has an appreciable horizontal variation only in auroral zones or near the equatorial electrojet current. In this respect the present paper is well applicable, say, to southern Finland. ALBERTSON and VAN BAELEN (1970) theoretically discuss the magnetic variation and associated electric field produced by an ionospheric horizontal straight line current. They state that the electric field increases if the source current moves upwards and the magnetic variation is kept constant. Consequently the electric field to be found in the case of a horizontally constant primary field, which is created by a source current situated at an infinite height, is an upper limit for geomagnetically-induced electric fields. For theory on geomagnetic induction caused by straight line currents in the ionosphere, whose fields do not meet the above-mentioned requirement, PIRJOLA (1982, Chapters 3 and 4) can be referred to, and in LEHTO (1983) and LEHTO (1984) an even more general model is discussed. In these three papers the earth also has more complex structures than in the present article.

The magnitude of the electric field at the earth's surface during a geomagnetic variation depends both on the rate of change (*i.e.* the time derivative) of the magnetic field, and on the duration of the change. It can probably be assumed that the highest rates of change in practice are about 40 nT/s and the maximum durations of such rapid changes are of the order of a minute (*cf.* LANZEROTTI, 1983). The biggest time variations in the geomagnetic field are a few thousand nT.

As will be clear from the above, we are interested only in geomagnetic variations, i.e. deviations from the value of the main field, which is treated as being independent of time here, and can thus be disregarded. Hence »magnetic field» may refer only to the magnetic variation in this paper.

2. *Theory*

It was pointed out above that the existence of an electric field during a geomagnetic variation is theoretically based on Faraday's law of induction. This is in integral form:

$$\oint_L \bar{E} \cdot \overline{ds} = -\int_S \bar{g} \cdot \overline{da} \tag{1}$$

where L is a closed curve bounding a surface S; $\bar{E}(\bar{r}, \bar{t})$ is the electric field depending on the space vector $\bar{r}$ and on the time t; $\bar{g}(\bar{r}, \bar{t})$ is the time derivative of the magnetic field: $\bar{g} = \partial\bar{B}/\partial t$. The simplest way to obtain an estimate for $\bar{E}$ is to assume that the component of $\bar{E}$ parallel to the line element $\overline{ds}$ and denoted by $E(t)$ is constant along L, and the component of $\bar{g}$ parallel to the surface element vector $\overline{da}$ and denoted by $g(t)$ is constant on S. Then

$$E(t) = -\frac{g(t)A_S}{P_L} \tag{2}$$

where A_S and P_L are the area of S and the length of L, respectively.

If we further assume that L is a circle of radius R on the earth's surface, and S the corresponding part of the earth's surface, the following formula is obtained

$$E(t) = -\frac{g(t)R}{2} \tag{3}$$

in which E is horizontal and g is the rate of variation of the vertical magnetic component. Let g = 20 nT/s and R = 200 km. Then E = –2 V/km (*cf.* PERSSON, 1979). The minus sign in the value of E merely expresses Lenz's law: the induced electric field is negative from the point of view of the magnetic variation.

Even if the component of $\bar{E}$ parallel to L or the component of $\bar{g}$ perpendicular to S is not constant, formulas (2) and (3) are still valid for the means of the components in question in the loop L and on the surface S. Equations (2) and (3) indicate that the electric field is proportional by a time-independent coefficient to the time derivative of the magnetic field. But it must be remembered that we have had to assume that the components of $\bar{E}$ and $\bar{g}$ are constants or that E and g are averages in equations (2) and (3). If a physical model is used for the whole induction phenomenon and the electric and magnetic fields observed at a point on the earth's surface are calculated, there does not need to be proportionality between $\bar{E}$ and $\bar{g}$, though equation (1) is, of course, satisfied. It can further be noted that, under »normal» conditions, the vertical component of a magnetic variation is zero (*e.g.* SCHMUCKER, 1970, p. 13). It also seems probable that the vertical component does not change rapidly in the same direction over a wide area on the earth's surface simultaneously. Hence $g(t)$ (*i.e.* the average value) is usually small in equation (3) if R is big; thus $E(t)$ is prevented from becoming very large.

I will now discuss a more detailed model, in which the effect of the earth is taken explicitly into account by describing it as a homogeneous half-space with a flat surface. The earth is also treated as electromagnetically isotropic and linear (see STRATTON, 1941, p. 10). The primary electromagnetic field originating from the magnetosphere and ionosphere is assumed to be a transverse plane wave, not necessarily harmonic, which propagates vertically downwards. This situation, in which all electric and magnetic fields are horizontally directed and independent of the horizontal position, is discussed on pages 20 to 25 of PIRJOLA (1982). The following equation expresses the relationship between a horizontal electric field component $E(t)$ and the time derivative $g(t)$ of the perpendicular horizontal magnetic field component $B(t)$ on the earth's surface:

$$E(t) = -\frac{1}{\sqrt{\mu_0 \epsilon}} \int_0^\infty g(t-u)\, e^{-\sigma u/2\epsilon} J_0(i\sigma u/2\epsilon)\, du \tag{4}$$

where σ and ϵ are the conductivity and permittivity of the earth, respectively, and J_0 denotes the Bessel function of the zeroth order. The integration variable u has the dimension of time. The permeability of the earth was set equal to the free space permeability μ_0 (RIKITAKE, 1966, p. 221; SCHMUCKER, 1970, p. 3). Without this assumption the right-hand side of formula (4) would have had to be multiplied by $\sqrt{\mu/\mu_0}$, where μ is the permeability of the earth. To be precise, the directions of the electric and magnetic-field components in equation (4) must be selected so that the magnetic component, the electric component and the downward vertical direction constitute a right-handed system (*e.g.* the magnetic north component and the electric east component).

The value of ϵ/σ could typically be of the order 0.01...1 μs (see SARAOJA, 1946, pp. 122...123) or less, *i.e.* much smaller than times relevant to geomagnetic phenomena (for the conductivity of the earth see also JONES, 1980). This indicates that $J_0(i\sigma u/2\epsilon)$ can be replaced by its asymptotic expression $e^{\sigma u/2\epsilon}\sqrt{\epsilon/\pi\sigma u}$ in equation (4), and so approximately

$$E(t) = -\frac{1}{\sqrt{\pi\mu_0\sigma}} \int_0^\infty \frac{g(t-u)}{\sqrt{u}}\, du \,. \tag{5}$$

This equation is also given by CAGNIARD (1953, p. 611) in his basic publication on magnetotellurics. The use of equation (5) instead of the exact formula (4) is equivalent to neglecting the displacement current in the earth. It is important to note that in both equations (4) and (5) the relation between E and g is not a simple proportionality, but E is also affected by past values of g.

A step-like change of the geomagnetic field is discussed as an example on pages 24 to 25 of PIRJOLA (1982), in which expressions obtained from formula (4) and formula (5) for the corresponding electric field are given. It can be shown that, with reasonable values of the parameters, the two expressions (formulas (2.80) and (2.81) in PIRJOLA, 1982) differ significantly from each other for less than 0.1...1 ms starting from the point in time of the change in the magnetic field. This is a very short time and does not seem to have any importance, so the use of formula (5), which will be applied in the next chapter, is »justifiable». As can be concluded from the discussion in PIRJOLA (1982, pp. 20...24), high frequencies are weighted excessively in equation (5). In the example of a step-like change mentioned above, the error can be seen most clearly in the fact that the approximate electric field goes to infinity at the moment of the change of the magnetic field, while its rigorous value remains finite. But as stated above, the two electric field values become practically equal after an extremely short period of time.

It was assumed above that the primary field is a transverse plane wave propagating vertically. Actually, the only assumption used in deriving formula (4) is that the electromagnetic field depends inside the earth solely on the vertical space coordinate and time. As indicated in Chapter 1, this is approximately true provided the spatial variation of the primary field is small in the horizontal direction (see also WAIT, 1954; WAIT, 1962).

3. *Numerical examples*

In this chapter it will be assumed that the variation $B(t)$ of a horizontal magnetic component on the earth's surface is known as a function of time. I will then calculate the induced perpendicular horizontal electric field component $E(t)$ on the earth's surface from equation (5) in which $g = dB/dt$. The numerical value for the conductivity of the earth is $10^{-2}\Omega^{-1}\mathrm{m}^{-1}$, which can be regarded as a kind of average conductivity in Scandinavia (see JONES, 1980). The permeability μ_0 equals $4\pi\cdot 10^{-7}$ Vs/Am. To ensure that the mathematical treatment involved in the derivation of formula (5) is permissible, the magnetic variation $B(t)$ must vanish when $t = \pm\infty$. Where this is not the case explicitly, I will assume implicitly that the expression of $B(t)$ is multiplied by a factor $e^{-\eta|t|}$ in which η is positive but arbitrarily small (*cf.* PIRJOLA, 1982, pp. 24...25).

3.1. Linear change

Let $B(t)$ be defined by the formula

$$B(t) = \begin{cases} 0, & t < t_1 \\ B_0\alpha(t - t_1), & t_1 \leqslant t \leqslant t_2 \\ \text{arbitrary}, & t > t_2 \end{cases} \tag{6}$$

where B_0 and α are constants. In other words, the magnetic field starts to change linearly at time t_1 and the linear change may stop at time t_2. The electric field is not considered after t_2, so the magnetic field can then have any value. It is evident that only the product of B_0 and α is significant here, so it could be denoted by one symbol. To make the discussion of this section analogous to the discussions in other sections of this chapter, however, the present notation will be kept, and it is assumed that B_0 has the dimension of a magnetic field, and α the dimension of the inverse of time.

At a moment t satisfying the inequalities $t_1 \leqslant t \leqslant t_2$, the electric field is

$$E(t) = -2B_0\alpha\sqrt{\frac{t - t_1}{\pi\mu_0\sigma}} \quad . \tag{7}$$

Hence $E(t)$ is proportional to the rate of change $B_0\alpha$ of the magnetic field. Here, however, we must stress that this is not the kind of proportionality which was discussed in Chapter 2, because the coefficient depends on time. The absolute value of $E(t)$ increases with $t - t_1$ (the duration of the change before t). When

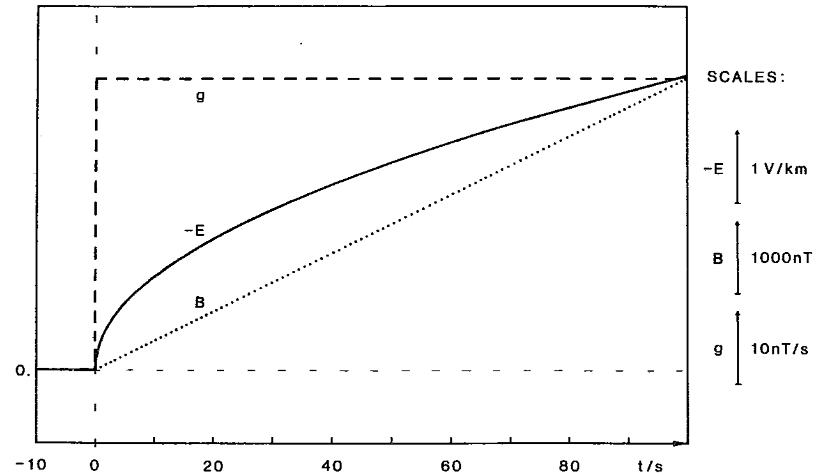

Fig. 1. A linear change $B(t)$ in a horizontal magnetic component, its time derivative $g(t)$, and the induced horizontal perpendicular electric field $-E(t)$. The conductivity of the earth is $10^{-2}\,\Omega^{-1}\mathrm{m}^{-1}$. If the derivative $g(t)$ differs from its value in the graph, the electric field must be changed linearly.

$B_0\alpha$ = 40 nT/s and $t - t_1$ = 1 min, the value of E is −3.1 V/km. The behaviour of $B(t)$, $g(t)$, and $-E(t)$ when $B_0\alpha$ = 40 nT/s (*e.g.* when B_0 = 1200 nT and $\alpha = 1/30$ s^{-1}) and $t_1 = 0$ is shown in Fig. 1.

3.2. Change of level

A change of level in the value of a magnetic field component can be described as a step function (*cf.* PIRJOLA, 1982, pp. 24...25). Here, however, we assume that the change is smoother, *i.e.* that

$$B(t) = \frac{B_0}{\pi}\left(\frac{\pi}{2} + \overline{\arctan}\,\alpha t\right) \tag{8}$$

where B_0 denoting the magnitude of the change and α (> 0) are constants. The integration involved in formula (5) can be performed by calculus of residues (*e.g.* ARFKEN, 1968, pp. 270...281), and the following expression is obtained for the electric field:

$$E(t) = -B_0\sqrt{\frac{\alpha}{2\pi\mu_0\sigma}}\sqrt{\frac{\alpha t + \sqrt{\alpha^2 t^2 + 1}}{\alpha^2 t^2 + 1}}\,. \tag{9}$$

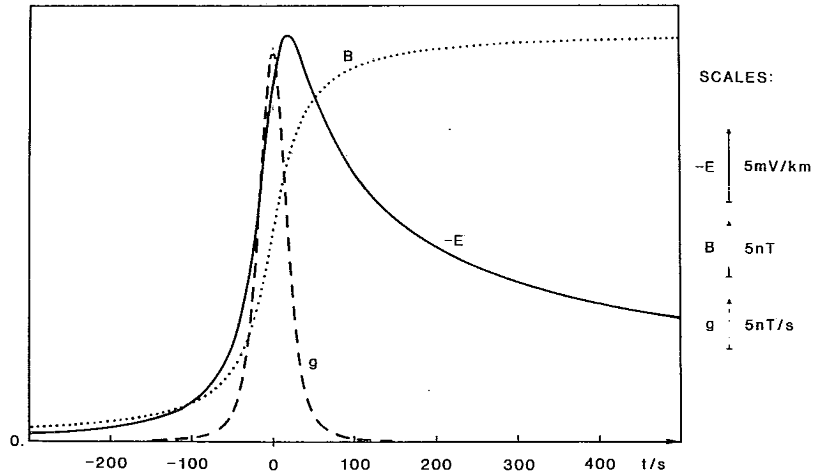

Fig. 2. $B(t)$ depicts a change in the level of a horizontal magnetic component, $g(t)$ is the time derivative of $B(t)$, and $-E(t)$ is the induced horizontal perpendicular electric field. The conductivity of the earth is $10^{-2}\Omega^{-1}m^{-1}$. If the magnitude of the change of the magnetic component differs, the curves $g(t)$ and $-E(t)$ must be changed linearly.

The change of the magnetic field becomes more abrupt if α increases, and is step-like when $\alpha = \infty$. So it is natural that the limit value of formula (9) as α approaches infinity equals equation (2.81) in PIRJOLA (1982). (When $t = 0$, the identity is not quite straightforward, but this has no practical importance.)

The curves in Fig. 2 depict $B(t)$, $g(t)$ and $-E(t)$ when $B_0 = 12\pi$ nT ≈ 38 nT and $\alpha = 1/30\ s^{-1}$. It can be seen that B and g are symmetric with respect to the moment 0, but E is asymmetric. The time of the largest electric field is $1/\alpha\sqrt{3}$, *i.e.* -0.0279 V/km at 17.3 s in Fig. 2.

3.3. Pulse-like variation

It is assumed here that the geomagnetic field has a pulse-like variation, *i.e.* that

$$B(t) = \frac{B_0}{\alpha^2 t^2 + 1} \tag{10}$$

where B_0 denoting the amplitude of the variation and α (>0) are constants. To calculate the electric field from formula (5) we can make use of Section 3.2 by noting that $B(t)$ in equation (10) is the time derivative of that in formula (8) with-

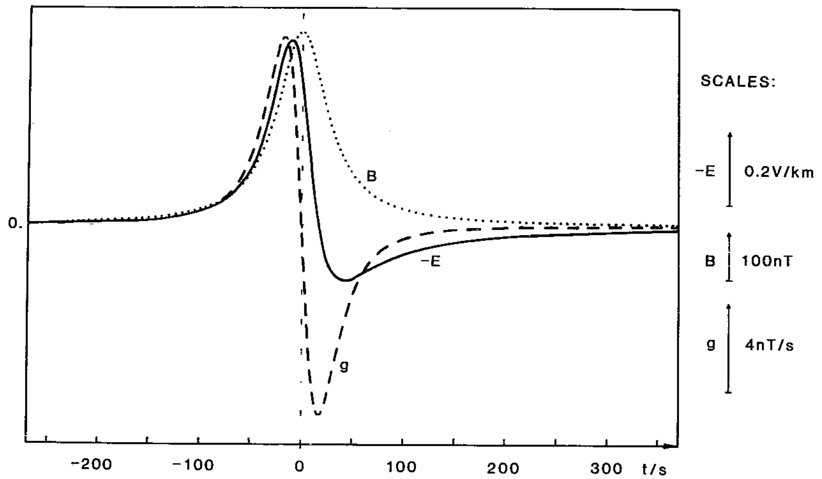

Fig. 3. A pulse-like variation $B(t)$ of a horizontal magnetic component, its time derivative $g(t)$, and the induced horizontal perpendicular electric field $-E(t)$. The conductivity of the earth is $10^{-2}\Omega^{-1}m^{-1}$. If the amplitude of the variation of the magnetic component differs, the curves $g(t)$ and $-E(t)$ must be changed linearly.

out a constant multiplier. The final result is

$$E(t) = -\frac{B_0}{2}\sqrt{\frac{\pi\alpha}{2\mu_0\sigma}}\,\frac{1-\alpha t(\alpha t+\sqrt{\alpha^2t^2+1})}{\sqrt{\alpha t+\sqrt{\alpha^2t^2+1}}\,(\alpha^2t^2+1)^{3/2}} \tag{11}$$

If we replace B_0 by $B_0\alpha/\pi$ and let α go to infinity, $B(t)$ of formula (10) approaches $B_0\delta(t)$, where $\delta(t)$ is the Dirac delta function or distribution (*e.g.* ARFKEN, 1968, p. 324). Multiplying the right-hand side of formula (11) by α/π and letting α approach infinity thus produce the electric field associated with a delta-type impulse in the geomagnetic field:

$$E(t) = \begin{cases} 0\,, & t<0 \\ \left[-\dfrac{B_0\alpha^{3/2}}{2\sqrt{2\pi\mu_0\sigma}}\right]_{\alpha=\infty} = -\infty\cdot\mathrm{sign}(B_0), & t=0 \\ \dfrac{B_0}{2\sqrt{\pi\mu_0\sigma}\,t^{3/2}}\,, & t>0 \end{cases} \tag{12}$$

Equation (12) can also, of course, be derived directly from formula (5) when

$g = B_0 d\delta/dt$. Difficulties will then arise in connection with the value $t = 0$, though obviously these are of no practical importance. Here again, it is necessary to emphasize that equation (5), from which formula (12) is derived, is only approximate. With the rigorous equation (4) the negative (if $B_0 > 0$) infinity when $t = 0$ would clearly be »smaller» and $E(t)$ would be bounded for all positive values of t.

Fig. 3 shows $B(t)$, $g(t)$ and $-E(t)$ (formulas (10) and (11)) when $B_0 = 400$ nT and $\alpha = 1/30\ \mathrm{s}^{-1}$. As in Section 3.2, E is asymmetric, but B and g are symmetric. The largest electric field value is attained when $t \approx -0.325/\alpha$, *i.e.* -0.509 V/km at -9.75 s in Fig. 3.

3.4. Periodic variation

Let the geomagnetic field change periodically, *i.e.* let

$$B(t) = B_0 \sin \alpha t \tag{13}$$

where B_0 denoting the amplitude of the variation, and $\alpha\,(> 0)$ expressing the period of the oscillation as $T = 2\pi/\alpha$ are constants. The electric field can easily be obtained from equation (5), using the formulas

$$\int_0^\infty \frac{\sin \nu}{\sqrt{\nu}}\, d\nu = \int_0^\infty \frac{\cos \nu}{\sqrt{\nu}}\, d\nu = \sqrt{\frac{\pi}{2}} \tag{14}$$

(SPIEGEL, 1968, p. 97), and the result is that

$$E(t) = -B_0 \sqrt{\frac{\alpha}{2\mu_0 \sigma}}\,(\cos \alpha t + \sin \alpha t) = -B_0 \sqrt{\frac{\alpha}{\mu_0 \sigma}}\, \sin\left(\alpha t + \frac{\pi}{4}\right) . \tag{15}$$

The function $B(t)$ has only two non-zero Fourier components: those associated with the angular frequencies α and $-\alpha$. Equation (15) could therefore be derived, without using the general formula (5), directly from formula (2.66) in PIRJOLA (1982), which expresses the relationship between the Fourier components of $E(t)$ and $B(t)$.

As can be expected, the bigger the value of α is, *i.e.* the more rapidly the magnetic field varies, the greater is the amplitude of the electric field. However, $E(t)$ is not proportional to the derivative of $B(t)$ (*cf.* Chapter 2); the amplitude of $E(t)$ is proportional only to the square root of α. This phenomenon can be understood with the aid of the »skin-depth rectangle» concept (PIRJOLA, 1982, pp. 18...20): the electric field at a given frequency can be estimated by assuming that it is

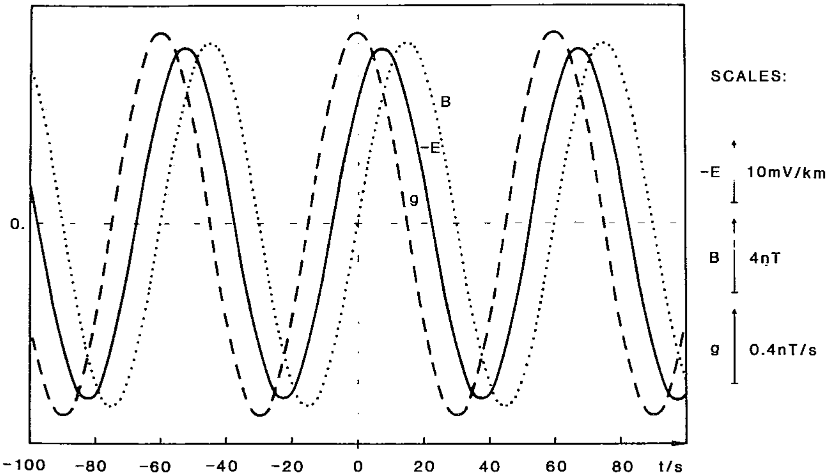

Fig. 4. A periodic variation $B(t)$ of a horizontal magnetic component, its time derivative $g(t)$ and the induced horizontal perpendicular electric field $-E(t)$. The conductivity of the earth is $10^{-2}\Omega^{-1}m^{-1}$. If the amplitude of $B(t)$ differs, the curves $g(t)$ and $-E(t)$ must be changed linearly.

induced by the magnetic-field change which is observed on the earth's surface in the skin-depth rectangle, whose height equals the skin depth in the earth at the particular frequency. The amplitude of the time derivative of the magnetic field, which signifies the intensity of the magnetic-field change, increases in ratio to the frequency, but the skin depth decreases in ratio to the inverse of the square root of the frequency. Thus the electric field has a square-root dependence on the frequency.

The functions $B(t)$, $g(t)$ and $-E(t)$ (equations (13) and (15)) when $B_0 = 10$ nT and $\alpha = \pi/30\ s^{-1}$, *i.e.* $T = 60$ s, are illustrated in Fig. 4.

4. *Discussion and concluding remarks*

This paper deals with theoretical estimates of induced electric fields during time variations of the geomagnetic field, assuming the latter to be known. The discussion is based mainly on equation (5), which expresses a horizontal electric-field component as a function of the time derivative of the perpendicular horizontal magnetic-field component. Both field values are measured on the earth's surface, and it should be noted that the electric and magnetic components involved in formula (5) do not depend on the point of observation on the earth's surface.

Derivation of equation (5) requires that the primary electromagnetic field caused by magnetospheric and ionospheric sources is spatially roughly constant on the earth's surface over horizontal distances equal to the relevant skin depths in the earth, and that the area considered is small enough for the earth to be described as a half-space with a flat surface (*e.g.* dimensions $\lesssim 2000$ km). Obviously, the use of this half-space model also restricts the frequency range to periods less than some hours, because the skin depths must be small compared to the dimensions of the area. In formula (5) it is further assumed that the conductivity and permeability of the earth are constant scalars, the latter equalling the permeability of free space. Equation (5) is such an approximation of the rigorous formula (4) that can be used provided the time variations discussed are not much more rapid than normal geomagnetic changes; obviously the error involved in this approximation is unimportant compared to the other simplifications in the present model. The content of formula (5) is also much more easily comprehensible than that of equation (4), and formula (5) is very easy to be applied to practical computations.

The most important consequence of formula (5) is that the electric component is not proportional to the simultaneous time derivative of the magnetic component with a time-independent coefficient, but also depends on past values of the derivative. The effect of the past values decreases with time, but this decrease is not very rapid, being proportional only to the inverse of the square root of the time difference. Equation (5) is also congruent with the fact that the horizontal electric field at the surface of a conductor diminishes with increasing conductivity, but the dependence of the electric field on the conductivity is not very strong, being only an inverse-square-root dependence.

Four different types of magnetic variations are discussed as examples in this paper:

1. Linear change in a horizontal magnetic component
2. Change of level in the value of a horizontal magnetic component
3. Pulse-like variation of a horizontal magnetic component
4. Periodic variation of a horizontal magnetic component.

Expressions for the corresponding horizontal perpendicular electric-field component are given. In Example 1 the electric field grows with time and with the rate of change of the magnetic field. In Examples 2 to 4 the electric field as a function of time has a maximum value which is proportional to the quantity $B_0\sqrt{\alpha}$, where B_0 denotes the magnitude of the magnetic variation and α is a time constant of the variation. The timing of the peak depends on α. But the role played by α is not the same in each example because the functions whose variable is the product of α and time differ.

For numerical calculations the conductivity of the earth was assumed to be $10^{-2}\Omega^{-1}m^{-1}$, which is a reasonable value. But as noted above, the electric field is not very sensitive to the value of the conductivity, so this choice is not particularly critical. Moreover, results based on other values of the conductivity are easy to obtain from the present ones. ALBERTSON and VAN BAELEN (1970, p. 582) state that the conductivity of the upper layers of the earth to a depth of about 20 km has a much greater effect than that of lower layers, and JONES (1980) demonstrates a distribution of conductivity for Scandinavia. These findings indicate that the value $10^{-2}\Omega^{-1}m^{-1}$ used here is too high rather than too low for Scandinavia. This would favour the possible existence of even ten times higher electric fields in Scandinavia than those presented in this paper.

Characteristic changing times in the variations were assumed to be of the order of a minute in duration. The value of the amplitudes of the variations were not chosen with any special care because the electric field is linear with respect to the magnitude of the magnetic variation, which makes it simple to convert the results to correspond to any amplitude. But I did try to use realistic values. It should be noted that the vertical scales vary widely from one graph to another in this paper.

The results of this paper support the conclusion that horizontal electric fields in the order of 1 V/km may occur during geomagnetic disturbances. But in many cases the field is much smaller; values like 10 to 20 V/km (PERSSON, 1979), or even 45 to 55 V/km (RAMLETH, 1982), seem exceptionally high (*cf.*, however, the above comment on the value of the conductivity of the earth).

As is seen from above, the calculations of this paper involve approximations. The two most significant of them from the practical point of view are probably the assumptions of the spatial constancy of the primary field (*cf.* Chapter 1) and of the homogeneity of the earth. However, the estimates of electric field values obtained using the results of this paper certainly represent correct orders of magnitude.

I will end by making a simple and rough comparison between the horizontal electric fields in the four examples quoted above. In each case the time parameter α retains the value it has in the corresponding graph, but the parameter B_0 is scaled so that the magnitude of the total magnetic field variation is 400 nT, *i.e.* the scaling is rendered uniform by making the variations equal to that in Fig. 3. Concerning Example 1 we only consider the time from 0 to 60 s ($t_1 = 0$). In Example 4 the magnitude of the variation is twice the amplitude of the sinusoidal oscillation of the magnetic field. This gives us 0.52, 0.30, 0.51 and 0.58 V/km for the highest electric fields in Examples 1, 2, 3 and 4, respectively. If it is desired that α be the same in each case, the value obtained in Example 4 must be further divided by $\sqrt{\pi}$, yielding 0.33 V/km. This rough comparison indicates that a change in level of the value of a magnetic component is accompanied by a smaller electric field than the other three types of magnetic variations. A value of 0.33 V/km in Example 4 seems to imply that lower electric fields are also connected with periodic variations.

In this comparison it is necessary to bear in mind the assumptions made. It does not seem easy to make any simple and reliable comparison between the induced electric fields associated with the four types of magnetic variations; the main difficulty is a rational choice of the values of the parameter α (*cf.* the comment of the role of α above).

REFERENCES

1. ALBERTSON, V.D. and J.A. VAN BAELEN, 1970: Electric and magnetic fields at the earth's surface due to auroral currents. *IEEE Trans. on Power Apparatus and Systems, PAS-89*, 578–584.
2. ARFKEN, G., 1968: *Mathematical methods for physicists.* Academic Press International Edition, Fourth Printing, Enlarged with Additional Exercises.
3. CAGNIARD, L., 1953: Basic theory of the magneto-telluric method of geophysical prospecting, *Geophysics*, 18, 605–635.
4. CAMPBELL, W.H., 1979: Considerations of telluric current effects on pipelines. The Alaskan pipeline. *Impacts of ionospheric/magnetospheric processes on terrestrial science and technology*, edited by L.J. Lanzerotti, Chapter III.2.1. *Solar System Plasma Physics, Vol. III*, edited by L.J. Lanzerotti, C.F. Kennel, E.N. Parker, North-Holland Publishing Company, 352–356.
5. JONES, A.G., 1980: Geomagnetic induction studies in Scandinavia: I. Determination of the inductive response function from the magnetometer array data, *J. Geophys.*, 48, 181–194.
6. LANZEROTTI, L.J., 1983: Geomagnetic induction effects in ground-based systems, *Space Sci. Rev.*, 34, 347–356.
7. LEHTINEN, M. and R. PIRJOLA, 1984: Currents produced in earthed conductor networks by geomagnetically-induced electric fields, Submitted for publication in *Annales Geophysicae.*
8. LEHTO, K., 1983: Electromagnetic field caused by a space- and time-dependent electrojet current system above a conducting earth, *Thesis for a Ph.Lic. degree, Department of Theoretical Physics, University of Helsinki*, Finland.
9. LEHTO, K., 1984: Electromagnetic field caused by a three-dimensional time- and space-dependent electrojet current system, Abridgement of LEHTO (1983), submitted for publication in *Geophysica.*
10. PEABODY, A.W., 1979: Considerations of telluric current effects on pipelines. Pipelines at middle to low latitudes. *Impacts of ionospheric/magnetospheric processes on terrestrial science and technology*, edited by L.J. Lanzerotti, Chapter III.2.1. *Solar System Plasma Physics, Vol. III*, edited by L.J. Lanzerotti, C.F. Kennel, E.N. Parker, North-Holland Publishing Company, 349–352.
11. PERSSON, P.-O., 1979: Disturbances in directly earthed transmission power systems caused by geomagnetic storms, *Proceedings of the 3rd Symposium and Technical Exhibition on Electromagnetic Compatibility, Rotterdam, Holland, May 1–3, 1979*, edited by T. Dvořák, 13–18.

12. PIRJOLA, R., 1982: Electromagnetic induction in the earth by a plane wave or by fields of line currents harmonic in time and space, *Geophysica,* 18, 1–161.
13. PIRJOLA, R. and M. LEHTINEN, 1984: Currents produced in the Finnish 400 kV power transmission grid and in the Finnish natural gas pipeline by geomagnetically-induced electric fields, Submitted for publication in *Annales Geophysicae.*
14. RAMLETH, K.J., 1982: Geomagnetiske forstyrrelser. Innvirkning på Televerkets linjenett (in Norwegian), *Telektronikk*, Nr. 1, 10–12.
15. RIKITAKE, T., 1966: *Electromagnetism and the earth's interior, Developments in Solid Earth Geophysics, 2.* Elsevier Publishing Company.
16. SARAOJA, E.K., 1946: Maa- ja kallioperän ominaisvastus Suomessa ja voimajohtojen vaikutus viestiavojohtoihin (Soil resistivity in Finland and induction between power and telephone circuits) (in Finnish with an English summary), *Thesis for a Tech.D. degree, Technical University of Finland, Helsinki,* Sanoma Oy.
17. SCHMUCKER, U., 1970: Anomalies of geomagnetic variations in the southwestern United States, *Bull. Scripps Inst. Oceanogr.,* 13, University of California, La Jolla, California, USA.
18. SPIEGEL, M.R., 1968: *Mathematical handbook of formulas and tables including 2400 formulas and 60 tables.* Schaum's Outline Series, McGraw-Hill Book Company.
19. STRATTON, J.A., 1941: *Electromagnetic theory.* International series in physics, McGraw-Hill Book Company, Inc.
20. WAIT, J.R., 1954: On the relation between telluric currents and the earth's magnetic field, *Geophysics,* 19, 281–289.
21. WAIT, J.R., 1962: Theory of Magneto-Telluric Fields, *J. Res. Nat. Bur. Stand.,* 66D, 509–541.
22. WILLIAMS, D.J., 1979: Magnetosphere impacts on ground-based power systems, *Impacts of ionospheric/magnetospheric processes on terrestrial science and technology,* edited by L.J. Lanzerotti, Chapter III.2.1. *Solar System Plasma Physics, Vol. III,* edited by L.J. Lanzerotti, C.F. Kennel, E.N. Parker, North-Holland Publishing Company, 327–330.