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HARMONIC CALIBRATION OF THE WWSSN LONG-PERIOD SEISMOGRAPHS AT NURMIJÄRVI

by

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Abstract

Calibration of the Press-Ewing long-period seismograph set by the steady-state harmonic sine-wave method yielded very low standard deviations of magnification, about 2 per cent, in the pass band whereas deviations of observations were within 5 per cent. The adjusted magnification curves differed by less than 6 per cent from the standard course. Magnification at 15 seconds is in good agreement with the magnification derived from the recorded pulse. The differences were 1–3 per cent if the calibration constants calculated with the adjusted equivalent constants were used. An attempt was also made to determine directly the basic seismograph constants giving accurate adjusted magnification curves.

1. *Introduction*

One of the seismograph calibration methods is based on the steady-state harmonic driving of the seismometer lever by a current flowing through the calibration coil (WILLMORE, 1979). The direct excitation of the lever simulates the action of the inertial force originated by the seismograph frame motion during seismic waves. The galvanometer trace amplitudes recorded may be easily applied to determine seismograph magnification at discrete periods. The main advantage of this method is that the whole seismograph system is tested at the same time, and only few measurements and seismometer parameters are necessary.

With the pendulum seismometer (rotational system), the magnification (displacement sensitivity) M reads

$$M = \frac{4\pi^2 M_s}{G_c^\star} \frac{X_s}{T^2 i_s} \tag{1}$$

where M_s is the mass of the moving lever in kilograms, X_s is the peak-to-peak trace amplitude on the record in metres, i_s is the peak-to-peak amplitude of the harmonic calibration current in amperes and T its period in seconds. $G_c^\star$ is the adjusted motor constant of the calibration coil in newtons per ampere. It corresponds to the motor constant of the coil acting at the centre of gravity of the lever at the distance d_{cg} from the axis of rotation and giving the same moment of forces as the real calibration coil with the motor constant G_c, which is at the distance d_c from the axis of rotation: $G_c^\star = G_c\, d_c/d_{cg}$.

The magnification of the translatory type of seismograph is

$$M = \frac{4\pi^2 M_s}{G_c} \frac{X_s}{T^2 i_s} \tag{2}$$

in which the real motor constant of the calibration coil is used.

Two of these parameters, *i.e.* M_s, $G_c^\star$ and/or G_c, are fixed. The first parameter, which cannot be measured without dismantling the seismometer mass, is usually given by the manufacturer. The motor constant $G_c^\star$ can be checked by the weight-lift test. The value of d_{cg}, which is necessary for derivation of the adjusted motor constant of the calibration coil, is generally supplied with the seismometer data or it can be calculated using other parameters: the reduced length of pendulum ℓ and its moment of inertia K_s. Then $d_{cg} = K_s/(\ell M_s)$.

The calibration procedure requires only three quantities to be measured: X_s, T and i_p. It is easy to measure the amplitudes X_s on the recorded trace with sufficient accuracy. The maximum peak-to-peak trace deflection is convenient because no knowledge of the zero line is needed. Several sine-waves recorded during each period enable the influence of random noise on the amplitudes to be removed. With a highly sensitive seismograph the continuous seismic noise may considerably disturb the harmonic response. The signal amplitudes can be increased to improve the signal to noise ratio only in the range of linearity of the seismograph.

Calibration of the seismograph at the decreased sensitivity is correct only when changes in the coupling coefficient have negligible influence on the amplitude response course, and the decrease of sensitivity is constant in the whole pass band of the seismograph.

A good function generator enables the signal period to be checked according to the instrument scale without measuring it on the record. Accurate adjustment of periods is necessary owing to the square dependence of magnification on the period. On the other hand a calibrated scale of the output voltage provides constant current amplitudes at different periods to be adjusted, which accelerates the calibration procedure.

The harmonic driving of the seismometer is applicable without any knowledge of the other seismograph constants, i.e. free periods and damping constants of seismometer and galvanometer, the coupling coefficient between seismometer and galvanometer and the scheme of the signal and the damping circuits. There are however some limitations to the applicability of Eqs. (1) and (2), which are correct for the linear systems with two degrees of freedom. The basic conditions for calibration are as follows:

(i) There is no transformation of the electromotive force in the signal circuit, owing to the mutual inductance between the calibration and the signal coil. This effect can be checked through the negligible signal coil output if the seismometer mass is clamped and the calibration coil forced by the harmonic current in the higher frequency and amplitude ranges used for calibration. This effect is largely eliminated by using separate magnetic fields for both coils.

(ii) The calibration and signal circuits contain only galvanic resistors; the inductances and capacitances of the coils should be negligible in the calibration frequency range. In the reverse case, complicated corrections of recorded amplitudes are required to remove this influence, or then the measurements at some frequency bands should be omitted.

(iii) The driving force acts at the centre of gravity of the seismometer lever in the direction of its free motion as does the inertia when the frame is in motion. If the calibration force is applied at another point on the pendulum and the axis of rotation is flexible, additional torque acts on it owing to the inertia of the pendulum mass. Then at high frequencies additional free resonances of pendulum may arise. These parts of the amplitude response should be omitted.

(iv) The calibration circuit should have sufficiently high resistance to ensure that the damping constant of the calibration circuit is negligible in comparison with the total damping constant of the seismometer.

The aforementioned conditions should be checked for a particular seismograph model before applying the calibration. In the present paper, the processing of the experimental data by the least-squares method and the difficulty of determining the seismograph constants are analysed in more detail for the long-period seismographs of the WWSSN at Nurmijärvi station. These instruments were precisely calibrated by the transient pulse and step-by-step constant measurements in 1970 (TOBYÁŠ, TEIKARI and VESANEN, 1976, 1977).

2. *Theoretical background*

The theoretical relation for seismograph magnification reads

$$M = \frac{2A}{\ell}\sqrt{\frac{K_s}{K_g}}\sqrt{\frac{4D_sD_g\sigma^2}{T_sT_g}}\,U = V_1U \tag{3}$$

where V_1 = scaling factor, U = dynamic magnification (amplitude response), A is the recording distance, K_g is the moment of inertia of the galvanometer, T_s, D_s is the free period and the damping constant of the seismometer, T_g, D_g are the corresponding constants of galvanometer and σ^2 is the coupling coefficient. With the translational seismometer, the same relation is valid if we put $\ell = 1$, and the seismometer moment of inertia K_s is replaced by its mass M_s. The amplitude response U may be written as follows:

$$\begin{aligned}
U &= (T^{-2} + a + bT^2 + cT^4 + dT^6)^{-1/2}, \quad a = m^2 - 2p, \quad b = p^2 - 2mq + 2s,\\
c &= q^2 - 2ps, \quad d = s^2, \quad m = 2(D_sT_s^{-1} + D_gT_g^{-1}),\\
p &= T_s^{-2} + T_g^{-2} + 4D_sD_gT_s^{-1}T_g^{-1}(1-\sigma^2), \quad q = 2(D_sT_s^{-1}T_g^{-2} + D_gT_g^{-1}T_s^{-2}),\\
s &= T_s^{-2}T_g^{-2}.
\end{aligned}$$

At the time of calibration the scaling factor is constant: Its factor $V_0 = 2A/\ell\sqrt{(K_s/K_g)}$ depends only on the constructional parameters of the seismometer and galvanometer, which are fixed during operation; the second factor equal to $[4D_sD_g\sigma^2/(T_sT_g)]^{1/2}$ is determined by the basic constants of the seismograph which may change spontaneously in some ranges during long-term operation. At the time of calibration they must not be altered by the calibration procedure.

The scaling factor V_1, which is independent of the period of motion, determines the magnification level. The group of five basic constants, T_s, D_s, T_g, D_g, σ^2, defines the amplitude response at different periods. These six constants should be derived by the least-squares fitting of the magnification obtained by measurement of amplitudes X_s at discrete periods T with equation (3). But such a procedure is not unique: the coupling coefficient may range between zero and one, and for each σ^2 we receive a minimum of one group of constants, T_s, D_s, T_g, D_g, yielding an identical magnification curve. Therefore we define constants T_1, D_1, T_2, D_2, which yield the equivalent course of amplitude response for $\sigma^2 = 0$. Then the amplitude response U of the electromagnetic seismograph may be written as the product of the »seismometer» response U_1 defined by the free period T_1 and the damping constant D_1, and the »galvanometer» response U_2 defined by the free period T_2 and the damping constant D_2 (TOBYÁŠ and HÜBNEROVÁ, 1969).

$$M = V_1 T U_1 U_2 \tag{4}$$

where $U_i = [(1 - T^2/T_i^2)^2 + 4D_i^2 T^2/T_i^2]^{-1/2}$.

By the least-squares of empirical magnification data with (4) we have to find only five unknown constants, *i.e.* V_1, T_1, D_1, T_2, D_2. The real basic constants can then be determined in two ways:

(a) One basic constant of the seismograph is available. The free period of the seismometer is measured to a high degree of accuracy with the long-period seismograph. The measurement of the galvanometer free period takes longer time, and its accuracy is poorer owing to substantial open circuit damping. It is also too complicated to derive damping constants under operational conditions with the standard procedure. Determination of the coupling coefficient for this purpose is not recommended because all the forementioned constants must be known in advance. Taking into account the relations between equivalent and basic constants, we get

$$T_g = T_1 T_2 / T_s \tag{5a}$$

$$D_g = (c_3 - c_1 T_g^2)/[2T_g(T_s^2 - T_g^2)] \tag{5b}$$

$$D_s = (\tfrac{1}{2} c_1 - T_g D_g)/T_s \tag{5c}$$

$$\sigma^2 = 1 - (c_2 - T_g^2 - T_s^2)/(4T_1 T_2 D_s D_g) \tag{5d}$$

and consequently,

$$V_0 = V_1 / \sqrt{[(4D_s D_g \sigma^2)/(T_1 T_2)]} \tag{5e}$$

Here $c_1 = 2(T_1 D_1 + T_2 D_2)$, $c_2 = T_1^2 + T_2^2 + 4T_1 T_2 D_1 D_2$, $c_3 = 2(T_1 D_1 T_2^2 + T_2 D_2 T_1^2)$.

The applicability of this procedure depends on the precision of the period measurement and the changes in the equivalent constants according to the coupling, which are given by the amplitude response and the operational value of the coupling coefficient of the particular seismograph.

(b) If V_0 is given, the calculation of constants is more convenient. Eqs. (5a–e) yield five relations for five basic constants. Direct determination of V_0 from its definition requires two parameters of the seismometer, ℓ, K_s (and/or M_s for the translatory system) and two parameters of the galvanometer, A, K_g. The seismometer parameters required are not supplied if they are not necessary for the recommended calibration procedure, and in this case they must be measured with the dismantled system. The recording distance of the galvanometer is easy to measure; the free period, critical resistance and current sensitivity must be known for derivation of the moment of inertia K_g. The attainable accuracy of V_0 is 2–5 per cent. This way of determining the constants is cumbersome, but once V_0 has been found, its value can be used further on because V_0 is not subject to random variations during seismograph operation.

In the present paper V_0 was calculated using (3), where M was obtained by the harmonic calibration, and the basic constants by the step-by-step calibration procedure (Tobyáš, Teikari and Vesanen, 1976).

3. *Experimental results*

The procedure was tested with the standard long-period Press-Ewing seismographs. The conditions for the application of the harmonic calibration are satisfied as follows: the calibration coil is placed in a separate permanent magnetic field; the transformation effect and the inductance of circuits are negligible for relatively long periods; the driving force does not act in the centre of the seismometer pendulum gravity; the distance of centre of the calibration coil is about 0.348 and 0.356 m and the distance from the hinge to the center of gravity of the seismometer mass is 0.3081 m and 0.345 m (with the vertical and horizontal components, respectively). Some discrepancies at short periods may be caused by the flexible hinge when moment of forces acts with respect to the centre of gravity. As previous experience shows, the shortest period of excitation was therefore five seconds. This is useful with regard to the limited power output of the generator. The influence of electromagnetic damping of the calibration circuit on the seismometer damping was calculated in (Tobyáš, Teikari and Vesanen, 1977). The total resistance of the three calibration coils in series is 10 ohms, and the load resistance of the calibration circuit is greater than 1000 ohms. Therefore, the additional damping constant of a seismometer with a period of 15 s is less than 0.001, which is negligible as regards a seismometer operational damping constant of more than 0.8. To obtain the course of the amplitude response we must use periods over the whole pass band of the seismograph. The limiting

Table 1. Experimental data.

Period T(s)	Current i_s(mA)	Amplitude X_s (mm) at component		
		Z	E–W	N–S
5	2.4	24	25	22
7	2.4	62	63	59
10	2.4	156	160	148
15	0.24	37	39	36
20	|	63	64	61
30	|	108	111	104
40	|	145	148	140
50	|	172	176	166
70	|	211	211	197
100	|	226	225	209
200	0.24	178	180	168

periods for half of the maximum magnification are 4–5 s and 45–50 s. Shorter periods of excitation were excluded as mentioned above; longer periods up to 200 s were added, and calibration was performed at 11 selected periods: 5, 7, 10, 15, 20, 30, 40, 50, 70, 100 and 200 seconds. All three components were driven simultaneously, the calibration coils being connected in series.

An Airmec signal generator type 422 was used for the calibration. This generator gives a frequency reading with 6 numbers; the driving periods T are therefore very accurate. The output voltage level is also stable over the whole frequency range and during the calibration time used in the work. Expected errors are smaller than 0.1 per cent. The peak value of the calibration current was determined by measuring the voltage across a precision resistor of 100 ohms in the calibrating circuit, using a driving frequency of 0.05 Hz and a high quality Knick »zero center» voltmeter. The voltage readings were observed before and after the calibration procedure from the left and right side of the scale, and the mean value of approximately five readings was finally used. The expected accuracy was 0.2 per cent.

The experimental data received at a standard operational magnification of 1500 at 15 s on January 27, 1982 are given in Table 1. This calibration was performed with instruments as found. No checking and adjustment of constants were carried out immediately before or after this test. The standard calibration procedure was carried out 4 1/2 months earlier.

The zero-to-peak amplitudes of current are given as well as the peak-to-peak trace amplitudes on the record. The latter are average values obtained by fitting the values by eye over several oscillations at each period. At the time of the measurements the level of microseismic noise was moderate with maximum peak-to-peak amplitudes at Z, E–W components of up to 4–5 mm and N–S component of up to 3 mm at periods of six seconds. In the winter season microseismic storms may cause peak-to-peak amplitudes of up to 22 mm on the recorded trace. The amplitudes listed were further used without any corrections for calculation of magnification and for the least-squares fitting of equivalent constants and the scaling factor. In (1) effective masses of the vertical component equal to 11.2 kg and of the horizontal component equal to 10.7 kg were used, and the measured and adjusted motor constants of calibration coil 0.101 N/A for Z and 0.097 N/A for H components were applied.

The motor constant $G_c^\star$ of the calibration coil of the vertical seismometer is easy to determine with a high degree of accuracy because the position of the test weight is well defined. With the horizontal seismometer the correct adjustment of the test weight on a jig-supported thread is more complicated. The horizontal force acting on the seismometer mass is very sensitive on adjustment of the angle Θ. A deviation of Θ from the standard value of 45° by only 1° yields an error of 3.5 %, and a deviation of 2° gives an error of 7 %. The distance of the calibration mass d_m = 0.356 m given in the Manual (ANONYMOUS, 1962) corresponds to the force arm when the jig plane is perpendicular to the seismometer arm. When the jig plane is in the direction of the coil axes as shown in Fig. 11 of the Manual, the distance of the testing weight from the hinge is smaller (354.7 mm). The error is then 0.4 %. The inaccuracy of the motor constant affects the whole magnification level of the seismograph derived from (1).

Magnifications of all components based on the least-squares approximation of observations are given in Table 2. Real deviations of observations in Figure 1 are compared with the s.d. of the magnification. For periods up to 50 seconds the s.d. is smaller than 2 per cent. Only for long periods does it increase up to 5 per cent with the Z and N–S components. An extremely small s.d. was found with the E–W component where the deviations of the observations are within 1 per cent. The results indicate the good quality of the measurements and the consistency between the response of the seismograph and the theoretical assumptions. The estimated accuracy of magnification calculated from particular constants is 5–10 per cent. The overall behaviour of the seismograph was not checked.

If we use the standard magnification of the seismograph we introduce some systematic errors due to non-identity of responses. To determine the deviations of magnification of a particular component we take as standard constants T_s = 15 s, D_s = 0.9, T_g = 98.1 s, D_g = 1 and σ^2 = 0.025 for the horizontal components and σ^2 = 0.035 for the Z component, the maximum magnification being 1500 at 15 seconds. The deviations given in Figure 2 are approximately within 5 per cent. Such differences are reasonable in routine operation.

Table 2. Magnification of long-period seismographs.

Period T(s)	Component		
	Z	E–W	N–S
2	379	382	346
4	730	738	671
6	1027	1045	954
8	1255	1283	1180
10	1409	1445	1339
12	1494	1536	1431
14	1524	1567	1467
15	1523	1565	1469
16	1514	1554	1461
18	1476	1514	1426
20	1422	1455	1374
25	1259	1283	1213
30	1096	1113	1053
35	951	963	912
40	827	837	791
45	722	730	690
50	633	639	603
60	492	496	467
70	389	390	367
80	311	311	291
90	252	250	234
100	206	204	189

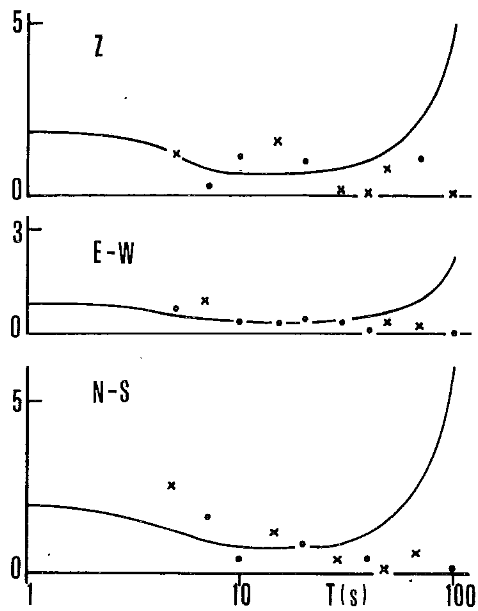

Fig. 1. Standard deviation of magnification in per cent and the real deviations of observation: x minus, • plus.

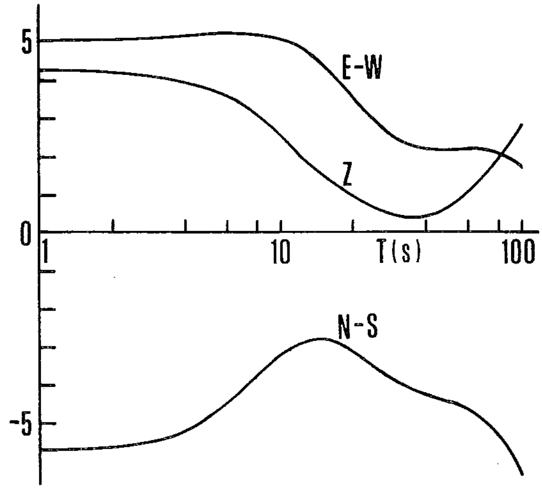

Fig. 2. Deviation of adjusted magnification from the standard in per cent.

Equivalent constants with their standard deviations are presented in Table 3. The greatest discrepancies are in the equivalent constants of the »galvanometer». These show substantially greater standard deviations than the »seismometer» equivalent constants and the scaling factor V_1. The ratio of the standard deviations of the periods is 5.7–7. and of the damping constants 5.0–5.8. The best results were for the E-W component: its standard deviations are 2.3–2.8 times smaller than those of the corresponding constants of Z and N-S components. Their values differ by up to 3 per cent from the standard equivalent constants which, for the horizontal component with $\sigma^2 = 0.025$, are $T_1 = 15.2$ s, $D_1 = 0.91$, $T_2 = 96.9$ s, $D_2 = 1.01$. Systematically greater standard deviations of the »galvanometer» can be explained by the insufficient number of measurements at long periods; only one measurement of 200 seconds was greater than the free period. Systematically greater standard deviations of equivalent constants of the Z and N-S components are caused by a greater scatter of observations: the s.d. of one measurement of

Table 3. Adjusted constants with standard deviations in per cent.

Component	T_1(s)	D_1	T_2(s)	D_2	V_1(s^{-1})
Z	14.9	0.918	101.9	1.04	191.9
	2.7	3.4	18.9	19.6	1.9
E–W	14.8	0.893	95.6	0.978	193.3
	1.1	1.4	6.8	7.3	0.8
N–S	15.1	0.879	92.8	0.961	174.6
	2.7	3.4	15.3	16.9	1.9

E-W component is 6.0; of Z it is 13.6 and of N-S 13.8. The number of observations was the same in all cases. These discrepancies cannot be explained by differences in the harmonic driving of the seismometers. The harmonic course and the current amplitudes are identical in all the calibration coils; the stability of current amplitudes at different periods is manifested by the small s.d. of the scaling factor V_1, which does not depend on the excitation periods.

Nor is the different level of disturbance of trace amplitudes by microseismic noise responsible for differences in the standard deviations. The microseismic noise on the Z component is approximately the same as on the E-W component. The perturbations on the N-S component are lower. The discrepancies in the standard deviations are probably caused by some non-linearity of particular instruments.

The phase response of the seismograph may be calculated with the aid of adjusted equivalent constants. The phase shift in seconds reads

$$\varphi = \frac{T}{2\pi} \tan^{-1} \frac{-1 + pT^2 - sT^4}{mT - qT^3} \tag{6}$$

In this case parameters m, p, q, s are defined by T_1, D_1, T_2, D_2 instead of the

Table 4. Phase response φ(s) and its deviation Δ(s) from the standard course.

T(s)	Z		E–W		N–S	
	φ(s)	Δ(s)	φ(s)	Δ(s)	φ(s)	Δ(s)
2	−0.409	0.0018	−0.410	0.0006	−0.413	−0.0020
4	−0.638	0.0070	−0.642	0.0027	−0.652	−0.0071
6	−0.694	0.014	−0.701	0.0073	−0.721	−0.0125
8	−0.591	0.022	−0.597	0.0150	−0.628	−0.0152
10	−0.347	0.029	−0.348	0.0256	−0.388	−0.0132
12	0.015	0.033	0.025	0.038	−0.021	−0.0061
14	0.476	0.035	0.498	0.051	0.449	0.0049
16	1.01	0.034	1.05	0.063	1.00	0.018
18	1.62	0.031	1.67	0.074	1.62	0.032
20	2.27	0.026	2.34	0.083	2.30	0.046
25	4.09	0.006	4.19	0.099	4.17	0.078
30	6.11	−0.020	6.2	0.108	6.24	0.105
35	8.29	−0.053	8.46	0.115	8.48	0.132
40	10.6	−0.090	10.82	0.122	10.9	0.162
45	13.0	−0.133	13.31	0.131	13.4	0.196
50	15.6	−0.180	15.91	0.143	16.0	0.235
60	21.0	−0.287	21.4	0.178	21.6	0.333
70	26.7	−0.406	27.3	0.227	27.6	0.454
80	32.7	−0.532	33.5	0.287	33.8	0.592
90	38.9	−0.662	39.9	0.356	40.3	0.740
100	45.4	−0.791	46.5	0.430	47.0	0.891

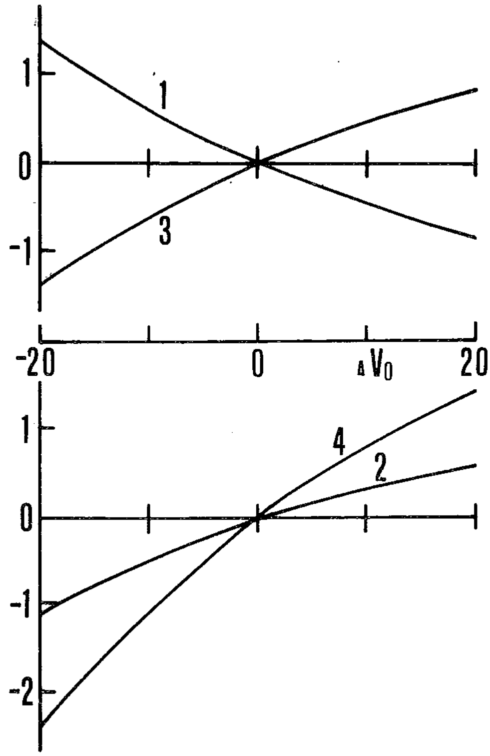

Fig. 3. Relative deviations of partial constants $-T_S$ (1), D_S (2), T_g (3), D_g (4) – in per cent due to variation of the static magnification within 20 per cent.

corresponding constants T_s, D_s, T_g, D_g and the coupling coefficient is equal to zero. If the period $T > \sqrt{m/q}$, π should be added to the inverse tangent (in radians) to get the real value. The phase shift for all three components is given in Table 4. Negative values of φ mean that the recording of the trace amplitude is delayed in comparison to the ground motion amplitude. The differences between the calculated phase shift and the theoretical course for the standard constants are also given in Table 4. The phase response of all three components fits well the standard response.

Measurement of the real period of the seismometer at the time of calibration, which is needed for calculation of the other seismograph constants, is not appropriate. The dependence of V_0 and σ^2 on basic constants is very poor and even a small error in period determination yields large errors of V_0 and of the coupling coefficient. This is clear from the example in Figure 3, which shows the relations for the Z component. An error of periods of one per cent gives an error of about 18–20 per cent of V_0 and 20–60 % of the coupling coefficient (Figure 4). The effect of the inaccuracy of the seismometer damping constant is even worse; the

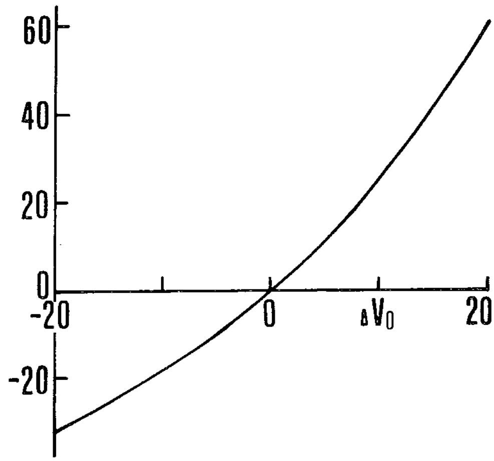

Fig. 4. Relative deviation of the coupling coefficient in per cent versus the static magnification as in Fig. 3.

damping constant of the galvanometer has slightly better conditions. The standard seismometer free-period test is the most accurate of all the constant determinations, but is not sufficiently accurate for this purpose (0.1–0.2 per cent). Errors of period can create irregular relations between equivalent and basic constants of the seismograph. For the long-period seismograph with $T_s < T_g$, and D_s and D_g near critical value it should be $T_s < T_1$, $T_g > T_2$ and with a small coupling coefficient $D_s < D_1$, $D_g < D_2$ (Tobyáš and Hübnerová, 1969). For determination of V_0, the values of the moment of inertia K_s and the reduced pendulum length ℓ of the seismometer are not available. A more suitable method for finding V_0 is to use earlier calibrations with harmonic current at 15 and 32.2 seconds and simultaneous measurement of the basic constants (Tobyáš, Teikari and Vesanen, 1977). The value of $M/V_0 = \sqrt{(4D_sD_g\sigma^2)/(T_sT_g)}\ U = V_1U/V_0$ is then calculated for both periods and V_0 derived from (3). Basic parameters from Tables 10 and 11 in (Tobyáš, Teikari and Vesanen, 1977) with the average values of V_0 are given in Table 5. The E-W component had nearly identical values of V_0 after magnification at both

Table 5. Data for derivation of V_0.

Component	T = 15 s		T = 32.2 s		V_0
	M	V_1U/V_0	M	V_1U/V_0	
Z	1600	0.0798	1070	0.0520	20300
E–W	1570	0.0642	1070	0.0438	24450
N–S	1480	0.0664	1000	0.0437	22600

Table 6. Calculated constants of long-period seismographs.

Component	T_s(s)	D_s	T_g(s)	D_g	σ^2	V_0
Z	14.5	0.877	104.8	1.02	0.038	20300
E–W	14.5	0.868	97.3	0.965	0.026	24450
N–S	14.8	0.851	94.6	0.947	0.026	22600

15 and 32.2 seconds; with the Z component the differences were 2.5 per cent and with the N-S component 2.7 per cent from the average.

The resulting constants of long-period seismographs in Table 6 show the theoretical values which approach the standard constants. If we suppose that the calculated free periods are correct and that the damping constants are influenced only by random deviations of the period from the standard we get the following relations: the damping constants of Z, E-W and N-S seismometers are equal to 0.91, 0.90 and 0.85 and the damping constants of the corresponding galvanometers are 0.97, 0.99 and 1.0. This requires almost correct adjustment of the damping coefficient of the E-W component, and slight differences in the Z galvanometer and N-S seismometer.

4. *Comparison of magnification with pulse calibration*

The magnification at 15 seconds received by the least-squares fitting of the harmonic excitation of the seismometer may be compared with the magnification derived from the pulse calibration, *i.e.* the current step flowing through the calibration coil. In this case the magnification reads (Manuals, 1962, 1970)

$$M = K X_p / (G_c^\star i_p) \tag{7}$$

where K is the calibration constant of the long-period seismograph in newtons per metre, i_p is the amplitude of the direct current in mA, and X_p is the first trace deflection on record in mm. For i_p = 0.2 mA their values measured on an eye-smoothed record disturbed by microseisms are given in Table 7.

With the correct standard seismograph constants the calibration constant of the Z component is 0.449 N/m and of the horizontal components 0.429 N/m for magnification at periods of 15 seconds (Tobyáš, Teikari and Vesanen, 1977). The magnification M_{st} calculated using these constants and given in Table 7 is close to the steady-state harmonic calibration result, the differences being 0.8–2.6 per cent.

Table 7. Magnification at periods of 15 seconds for the pulse calibration.

Component	X_p (mm)	K_{st} (N/m)	M_{st}	K_t (N/m)	M_t	X_{pt} (mm)
Z	68	0.449	1511	0.438	1474	70.2
E–W	69	0.429	1525	0.445	1583	68.3
N–S	65	0.429	1437	0.451	1511	63.1

Because the adjusted equivalent constants differ from the standard ones we tried to derive correct calibration constants for each component. It holds

$$K = M_s T U_1 U_2/(2\pi F_1) \tag{8}$$

where T is the period of which the magnification should be derived, U_1, U_2 are the amplitude responses calculated for the same period T (with adjusted equivalent constants) and F_1 is the first maximum of the pulse response function F (TOBYÁŠ, 1979). If $T_1 \neq T_2$, $D_1 \neq D_2 \neq 1$ we arrive at

$$\begin{aligned} F = \operatorname{sign} D'[&\exp(-D_1 n_1 t)\sin(\beta_1 n_1 t - \kappa_1)/(\beta_1 n_1) + \\ &+ \exp(-D_2 n_2 t)\sin(\beta_2 n_2 t - \kappa_2)/(\beta_2 n_2)]/D \end{aligned} \tag{9}$$

for $D_1 < 1$, $D_2 < 1$ and $\beta_i = \sqrt{1-D_i^2}$.

Here $n_i = 2\pi/T_i$, $D' = (n_2^2 - n_1^2)^2 + 4n_1n_2(D_2n_2 - D_1n_1)(D_2n_1 - D_1n_2)$, $D = \sqrt{|D'|}$, $\sin\kappa_k = 2(D_l n_l - D_k n_k)\beta_k n_k/D$, $\cos\kappa_k = [n_l^2 - n_k^2 - 2D_k n_k(D_l n_l - D_k n_k)]/D$ for $k, l = 1, 2$. If $D_i > 1$, β_i should be replaced by $\beta_i' = \sqrt{(D_i^2 - 1)}$ and the trigonometric functions by the corresponding hyperbolic functions. The relations for $\sinh\kappa_k$ and $\cosh\kappa_k$ are then the same as for $\sin\kappa_k$ and $\cos\kappa_k$, respectively.

The theoretical calibration constants, further denoted as K_t, are also given with the magnification M_t in Table 7. Agreement was best for the E-W component (1 per cent). The other components differ by about 3 per cent from the adjusted magnification at 15 seconds.

We have to point out that these differences cannot be caused by errors of M_s and $G_c^\star$, for example. These parameters play the same role in (3) and (7) when calculating magnification. The differences can only be due to deviations in the observed response from the theoretical response for adjusted constants. The theoretical first deflection reads

$$X_{pt} = 2\pi V_1 G_c^\star i_p F_1/M_s \tag{10}$$

Values of X_{pt} are presented in Table 7. Relative deviations of X_{pt} from the observed X_p are the same (except for the opposite sign) as the relative deviations of M_t from the adjusted magnification.

5. *Conclusion*

If the basic conditions for application of the harmonic calibration are satisfied, the adjusted magnification is very accurate in the working passband of the seismograph. With the long-period seismographs at Nurmijärvi the s.d. values were 5 per cent at maximum and the deviations of observations within 2 per cent. Higher s.d. values of magnification at long periods are caused by lower accuracy of the galvanometer equivalent constants, their standard deviations being 5–7 times greater than the s.d. of seismometer equivalent constants. All three components of the seismograph were adjusted so that they were consistent with the standard course for 1500 maximum magnification and so that their deflections were smaller than 6 per cent.

The harmonic calibration data were calculated using a computer programme. It includes the standard pulse determination of magnification, the derivation of the calibration constant and the theoretical pulse response with the aid of adjusted equivalent constants. Comparison between two different calibration procedures safely demonstrates the correctness of seismograph magnification and the linear dynamic properties of seismographs.

The accuracy of actual seismograph constants is limited because static magnification V_0 was determined indirectly and the standard measurement of some other constant is not sufficiently accurate. A similar problem arises with the practical application of equivalent constants derived from the pulse response (JAROSCH and CURTIS, 1973, GERSHANIK & GERSHANIK, 1981). Then the constants of the galvanometer can be found with great accuracy by the steady-state calibration of the galvanometer alone. This method gave extremely small s.d. values: 0.1 % with the free period and 0.3 % with the damping constant (TOBYÁŠ and TEIKARI, 1982).

Owing to the long decay of transient motion, the setting of the steady-state seismograph motion at discrete frequencies takes longer than the simple pulse excitation of the seismometer. On the other hand, with the pulse method more time is needed for the record evaluation where great accuracy is necessary when sampling the time ordinates. The harmonic method is more convenient for short-period seismographs where observations in the seismic vault are not so time-consuming and the free period of the galvanometer with a negligible open circuit damping constant is more accessible. The high accuracy of the harmonic calibration enables us to analyse the long-term stability of seismographs under operational conditions and to test the adjustment of constants and spontaneous changes in them.

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