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CORRECTING QHM OBSERVATIONS FOR ELASTIC CREEP

by

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Abstract

Elastic creep in the quartz suspension of a QHM (Quartz Horizontal Magnetometer) necessitates a small correction to the results if the time schedule for the observation is not the same as was used for calibrating the instrument. Especially where the standard scheme 0 + − − + 0 is not used, as is often the case in practice, the effect of elastic creep may be several nT, depending on the temperature and the time spent on the observation. The time dependence seems to follow the general behaviour of elastic creep, being a $T^{1/3}$ function of the time T for practical QHM measurements. A good approximation for temperature dependence is a second-degree polynomial. So the change of the angle ϕ corresponding to a constant torsion, usually 2π, follows the formula $\Delta\phi = \tan\phi(b + ct + dt^2)\, T^{1/3}$, where t is the temperature, and T is the time since the start of the torque.

1. *Introduction*

The Quartz Horizontal Magnetometer (QHM) (La COUR, [3]), now almost 50 years old, is the instrument most widely used for H observations. Recently, KRING LAURIDSEN, [2] has written an exhaustive study on the theory behind the OHM, including elastic creep. He presents elastic creep and its effect on observations both theoretically and experimentally. He compares different schedules of QHM observations and gives coefficients of correction. He also shows that the effect of elastic creep can practically be reduced to zero by using a standard scheme, *i.e.* 0 + − − + 0, and fixing the time interval between the observations and that the dependence of elastic creep on temperature is taken into account by the temperature coefficient of the QHM.

In this study we attack the problem of elastic creep from a practical point of view – looking for an easy way of correcting observations made with QHMs. For this purpose long series of observations were made with five QHMs and simple functions were fitted to the observations. We find our results to be consistent with those of Kring Lauridsen: a correction must be made to H measured with a QHM unless the standard scheme with standard time intervals is used.

In many cases the standard scheme is not feasible. Often it is desired to make several observations in same torque position so as to avoid turning and damping. In such cases the angle readings of the plus observations creep towards lower values and the corresponding readings of the minus observations towards higher values because the torque of the suspension fibre is decreasing as a function of time and temperature. According to the literature (*e.g.* CRUSSARD, [1]) the creep almost follows a $(\text{time})^{1/3}$ law.

The main difficulty in observing the effect of elastic creep on standard QHM observations arises from the fact that some time – one to three minutes – is needed to turn the instrument into the torque position and to damp the oscillations of the magnet. This means that the first observation includes the effect of the steepest part of the creep.

2. *Elastic creep*

The theory of elastic creep, or transient creep, is complex (*e.g.* CRUSSARD, [1]). Normally the creep

$$\Delta\phi = \beta T^m$$

where β and m are constants possibly depending on temperature, and T is the duration. The examples found in the literature give creep a roughly $T^{1/3}$ dependence. In the QHM the torque lasts 1 to, say, 30 minutes.

The observations made by KRING LAURIDSEN, [2] on several QHMs confirm this dependence, as do our observations. Observations made with 5 QHMs produced the curves shown in Figure 1. The dependence on temperature is clearly seen in Figure 1. Our observations showed that the exponent m in the formula for creep can be kept at a constant value of 1/3 if a proper function of the temperature t is derived for the factor β.

Ten series of observations were made in an H field of 15,000 nT with both QHM-85 (angle of deflection $\phi \approx 45°$) and QHM-286 ($\phi \approx 54°$) keeping the QHMs at a torsion of $+2\pi$ for half an hour, and at a torsion of -2π for the next half-hour after two minutes for turning. The temperatures ranged from 8 to 30°C. Each half series was first fitted to

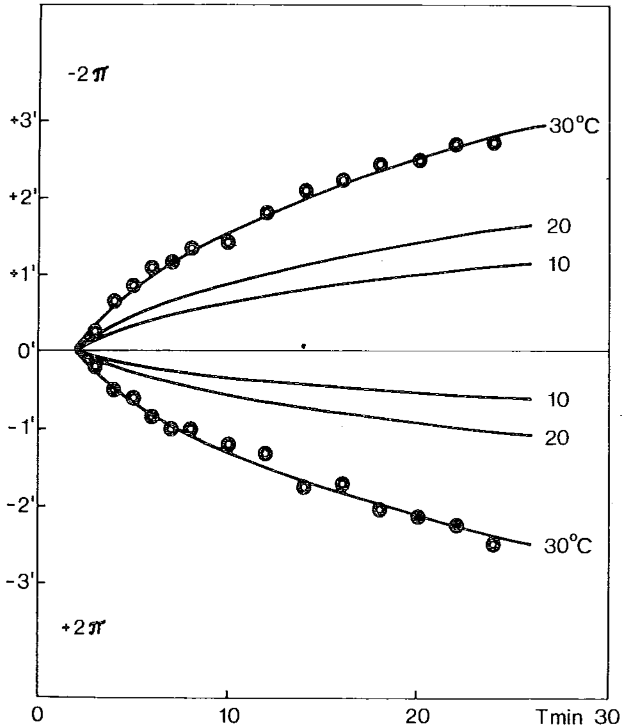

Fig. 1. Effect of elastic creep on the angle readings A of 5 QHMs at different temperatures. The curves correspond to a torsion of $\pm 2\pi$, *i.e.* one whole turn of the suspension fibre in the + or − direction. The curves represent the least square fits of all the observations. The circles denote one series of observations.

$$\Delta a = a_0 + \beta T^{1/3}$$

using the method of least squares. Δa is the amount by which the deflection angle a changes. The factor a_0 ($= -\beta 1.5^{1/3} = -\beta 1.14$) is the effect of the elastic creep included in the main constant K of the QHM. After that, an expression for β as a function of temperature t was sought. A simple second-degree polynomial was found to give a satisfactory fit. Again the method of least squares was applied. The results for both QHM-85 and 286 were:

$$\begin{aligned} \beta_{+2\pi} &= \tan\phi\,(0.60 - 0.051\,t + 0.0026\,t^2) \\ \beta_{-2\pi} &= \tan\phi\,(0.84 - 0.040\,t + 0.0023\,t^2) \end{aligned} \qquad (1)$$

The factor $\tan\phi$ follows from the general QHM formula, taking into account

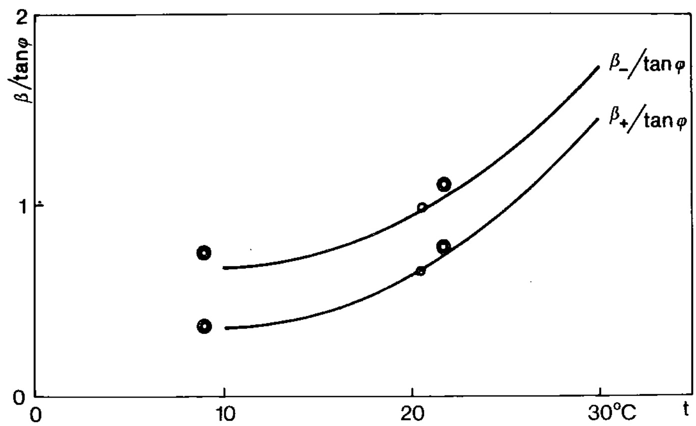

Fig. 2. The factor β of the correction to the angles of deflection as a function of temperature. The curves are calculated from all the 30 min + 30 min series, using least square fits. Large circles: 10 min + 10 min series. Small circles: 5 min + 5 min series.

the angle dependence of sensitivity (see next section). Formulas (1) give the angles in minutes of arc.

The few long series made with the other Finnish QHMs (QHM-84, 86 and 287) gave the same formulas within the errors of observation, which were roughly 0.2′ in a, corrected for changes of H and D. The factors β are shown in Figure 2.

The factors β were also determined from shorter series of observations, namely 0, 1.5 min for turning, ten times $+2\pi$ with one minute between observations, 2 minutes for turning, and ten similar -2π observations. These observations, which were made at two temperatures, are marked with large circles in Figure 2. Several tens of series were made in the form: 0, 5 times $+2\pi$ and 5 times -2π, again with 1.5 min for turning at the beginning and 2 min in the middle of the series. The β values determined from these series are shown by small circles in Figure 2. As can be seen the β values for different types of observations seem to fit fairly well in the curves determined from the 30 + 30 min series.

3. *Correction of QHM observations for elastic creep*

The basic formula for QHM observations (Kring Lauridsen, [2]) is:

$$H = \frac{K}{(1 - \mathrm{k}_1 t)(1 + \mathrm{k}_2 H \cos\phi) \sin\phi \cos\left(\frac{a_+ - a_-}{2} + \alpha\right)}, \qquad (2)$$

where K is the main constant of the instrument, k_1 is the temperature coefficient and k_2 is the induction coefficient. ϕ is the deflection of the magnet caused by a known torque of the quartz suspension, usually a torsion of 2π. a_+ and a_- are the angles of deflection for plus and minus torques, and α is the residual deviation of the magnet, *i.e.* the angle between the direction of H and the direction of the magnet in the zero position. α is obtained from the formula

$$\tan\alpha = \frac{\sin a_+ - \sin a_-}{2 - \cos a_+ - \cos a_-} \tag{3}$$

Elastic creep is caused by a slow change in the torsion constant of the quartz fibre. In other words, the main constant K changes slightly. As mentioned in the previous section, this change is a $T^{1/3}$ function of time and depends on temperature. On differentiating the QHM equation and omitting the small term $k_2 H \cos\phi$, we find that the change of the deflection angle ϕ caused by a change of K is as follows:

$$\Delta\phi = \frac{\Delta K}{K} \tan\phi \tag{4}$$

Hence the factor $\tan\phi$ in the formulas in Section 2 depicting the observed elastic creep.

From the experimental results expressed in formulas (1) we obtain

$$\frac{\Delta K}{K} = (b + ct + dt^2)\, T^{1/3} \tag{5}$$

where the factors b, c and d are the means of the corresponding factors b_+, b_-; c_+, c_- and d_+, d_- in formulas (1).

The constants of all QHMs were originally determined at the Rude Skov Standard Observatory, using the standard scheme $0 + - - + 0$ for the observation. This means that the elastic creep was effective for some 1.5 min in the first observation, the same time in the last observation and a somewhat longer time in the second minus observation. The constants of the QHMs make due allowance for this creep. If observations are made using a slower scheme or maintaining the same torsion for a longer time during several observations, then the effect of elastic creep has to be corrected for. The factors b, c and d in formula (5) remain the same, but we must replace $T^{1/3}$ by

$$\gamma = \frac{1}{n} \sum_{i=1}^{n} T_i^{1/3} - T_0^{1/3} \tag{6}$$

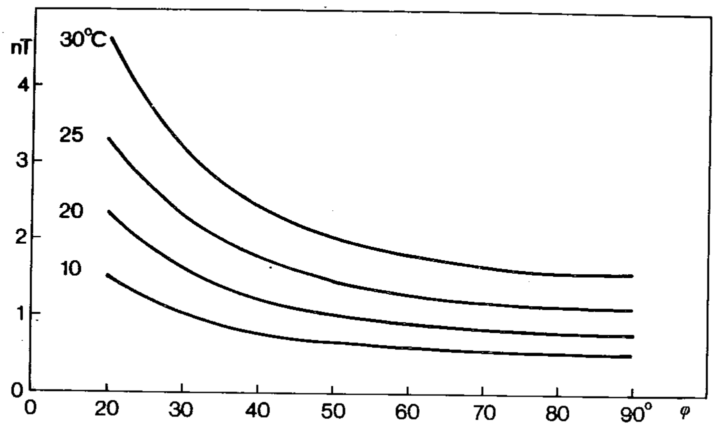

Fig. 3. Error caused by elastic creep in a 5 + 5 min series made with QHM-84, compared to results obtained with a 0 + − − + 0 series. The error is given as a function of the angle of deflection ϕ at different temperatures.

where n is the number of observations at the same torsion, T_i is the duration of time since the start of the torsion, and T_0 is the duration of time since the start of the torsion in a standard scheme when the constants of the QHM were determined. We found 1.5 min to be a suitable value for T_0. For a 5 + 5 series, with 1.5 min for turning at the beginning, 1 min between observations, and 2 min between the plus and minus positions, we obtain the following value for γ

$$\gamma_{5+5} = 0.34$$

For a series with 30 observations in the plus position and 30 observations in the minus position, again with one minute between observations, we obtain

$$\gamma_{30+30} = 1.26$$

The correction can be made 1) to the final result of a QHM measurement, 2) to the deflection angles, which tend to get smaller owing to the creep, or 3) to the main constant K according to formula (5). Correction of the deflection angles a_+ and a_- has the advantage that it also corrects α (eq. (3)), which is a good control on the stability of the measurements and the instrument. The correction of α has no practical effect on the final H because the factor $\cos\left(\frac{a_+ - a_-}{2} + \alpha\right)$ varies very little.

From equations (4), (5) and (6) we have

$$\Delta\phi = (b + ct + dt^2)\,\gamma \tan\phi \tag{7}$$

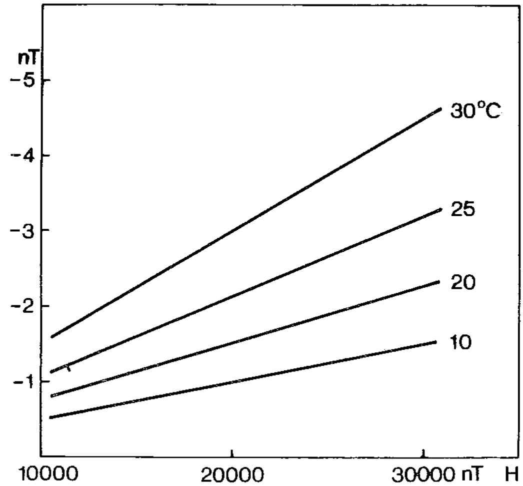

Fig. 4. Corrections to be made to H obtained with QHM-84 in a 5 + 5 min series, compared to the results of a 0 + − − + 0 series. The correction is given as a function of the H field at different temperatures.

The factors b, c and d have slightly different values for plus and minus observations. The observed mean values for QHMs 84, 85, 86, 286 and 287 are

Torque	b	c	d
$+2\pi$	0.010°	−0.00085°/°C	0.000043°/°C^2
-2π	0.014	−0.00067	0.000038

The change of angles a_+ and a_- with temperature can be seen in Figure 1. In different fields (different ϕ values) the change of a_+ and a_- produces different effects on the final H. These errors are shows in Figure 3. The corresponding corrections, as a function of H, are given in Figure 4.

Figure 4 shows that, if a QHM is calibrated in a standard observatory with say, an H field of 20.000 nT, at a temperature of 20°C, using a 5 + 5 min series, and a similar measurement is then made at 30°C in a field of 30.000 nT, the final result will have an error of 3 nT. If the calibration in the standard observatory is made using the standard 0 + − − + 0 scheme the error will be 4.5 nT, unless the effect of elastic creep is taken into account.

4. *Form for QHM observations*

A form for recording QHM observations in a 5 + 5 min series is reproduced in Fig. 5. Also included is observation of the declination D. The formulas used are given at the bottom. As can be seen, the angles of deflection a_+ and a_- have been corrected for the change of D and for elastic creep. a_- has also been corrected for

Place .... Date ....

QHM ...... Theodolite .... Observer ....

K = ... nT; k_1 = ... $1/^\circ C$; k_2 = ... $\cdot 10^{-10}$/nT; ψ = ...$^\circ$

$\gamma = \frac{1}{n}\sum_{i=1}^{n} T_i^{1/3} - T_0^{1/3}$ = ...; b_+ = ...$^\circ$; c_+ = ...$^\circ/^\circ C$; d_+ = ...$^\circ/^\circ C^2$

Az = ...$^\circ$; b_- = ...$^\circ$; c_- = ...$^\circ/^\circ C$; d_- = ...$^\circ/^\circ C^2$

ε_H = ... nT/mm; ε_D = ...$^\circ$/mm

	UT	$t^\circ C$	A°	A_1	A_2	$(A_1+A_2)'$	nH_{mm}	nD_{mm}
Mire								
0								
+ 2π								
+ 2π								
+ 2π								
+ 2π								
+ 2π								
− 2π								
− 2π								
− 2π								
− 2π								
− 2π								
Mire								

$$a_+ = A_+ - A_0 - D_+ + D_0 + \gamma\,(b_+ + c_+t + d_+t^2)\tan\phi;$$

$$a_- = A_0 - A_- + D_- - D_0 + \gamma\,(b_- + c_-t + d_-t^2)\tan\phi + \frac{57.3\,k_1}{(1 - k_1t)}\tan\phi\,(t_+ - t_-) - \frac{57.3}{H}\tan\phi\,(H_+ - H_-);$$

$$\phi = \frac{a_+ + a_-}{2}; \quad \tan\alpha = \frac{\sin a_+ - \sin a_-}{2 - \cos a_+ - \cos a_-}; \quad D = \psi + A_0 + A_z - A_{Mire};$$

$$H = \frac{K}{(1 - k_1t_+)(1 + k_2H\cos\phi)\sin\phi\cos(0.5\,(a_+ - a_-) + \alpha)}\ \text{nT} \qquad \alpha = \boxed{\quad ^\circ}$$

$D = \boxed{\quad ^\circ}$ $D_{BL} = \boxed{\quad ^\circ}$ $H = \boxed{\quad \text{nT}}$ $H_{BL} = H - \varepsilon_H \cdot nH_+ = \boxed{\quad \text{nT}}$

Fig. 5. Form for QHM observations in a 5 + 5 series. The angles A and D have to be used in same angle units. The symbols 0, + and − refer to zero, plus or minus torsions.

the change of H and temperature t. The symbols used on the form are the same as in this paper. ψ = collimation angle between the magnet and normal to the mirror minus α.

Alternatively, the correction can made to the final H with a simple function:

$$\Delta H = -(b + ct + dt^2)\,\gamma H \qquad (8)$$

or to the main constant K as stated in formula (5).

5. *Conclusions*

QHMs are widely used in differing fields and temperatures. Elastic creep causes an error that depends on the field, the temperature, and the schedule of the observations. In a standard 0 + − − + 0 scheme with fixed time intervals, elastic creep is almost completely taken into account in the constants of the QHM. However, as shown above, the error in a non-standard QHM observation may be several nT if the results are not corrected for elastic creep.

As demonstrated above, elastic creep is fairly easy to take into account as a correction of the deflection angles a_+ and a_-, or as a correction of the final H. Correction to a_+ and a_- has the advantage that all other calculations involving a_+ and a_- will also be corrected.

The five QHMs tested in this study have similar elastic creeps within the accuracy of the observations, though their main constants differ. But according to KRING LAURIDSEN's experience with numerous QHMs (private communication), elastic creep sometimes varies from one QHM to another, possibly as a function of their age. Coefficients of elastic creep must therefore be determined separately for each QHM. During the latest 15 years of observations, our over 30-year-old QHMs have shown no difference between one another, and no time dependence, which leads to the conclusion that their quartz suspensions have become stabilized with age.

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