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SOME FEATURES OF AEROLOGICAL SOUNDINGS DEPENDENT ON INSTRUMENT WEIGHT

by

VILHO VÄISÄLÄ

Institute of Meteorology, University of Helsinki

Abstract

In this paper a study is made concerning the influence of increasing instrument weight upon the hydrogen consumption and the size of balloons at aerological soundings.

It is not without interest to study the influence of instrument (radiosonde) weight upon the economics of aerological soundings. It is, of course, self-evident that increasing the weight of the instrument involves increasing the costs. The purpose of this paper is to make some practical calculations concerning this point.

We shall start from the fact that the Finnish (Väisälä) radiosonde, the gross weight of which is 300 grams, ascends, at a mean rate of about 350 m/min. between 1000 and 100 mb, up to a mean height of about 22 km or 40 mb, where the balloon bursts, when the balloon weight is 350 g and the free lift 1000 g. Such an ascent we shall call the reference ascent or sounding. In the following, we shall assume that the performance of all balloons is the same, which means that the thickness of the balloon rubber when unstretched is always the same and that the balloon bursts when the rubber has reached the thickness of the said 350 g balloon at the bursting level of 40 mb. Let us examine the following questions:

I. Suppose that in every case a balloon of 350 g weight is filled up to 1000 g free lift. What is the influence of instrument weight upon the height reached, the rate of ascent, the time of ascent and the hydrogen consumption?

II. Suppose that in every case a balloon of 350 g weight is filled so that the rate of ascent becomes the same as in the reference sounding. What is the influence of instrument weight upon the height reached and the hydrogen consumption?

III. Suppose that the balloon is of such a size (weight) that when it is filled so as to get the same rate of ascent as the reference sounding balloon, it bursts at 40 mb level. What is the influence of instrument weight upon the balloon size (weight) and the hydrogen consumption?

The rate of ascent of an aerological balloon is not quite constant throughout the whole ascent. There are two kinds of changes affecting this rate: First a jump which in ascents comparable with the reference sounding, occurs once during the whole sounding at somewhere about 400—200 mb. This abrupt change is due to the change of the turbulent flow at the balloon. Secondly, a continuous increase in the rate of ascent which is inversely proportional to the sixth root of the air pressure. This increase involves that the pressure-time curve is a straight line on a diagram in which the ordinate changes linearly with the sixth root of the pressure [1]. In the following calculations we shall not take into account the jump in the rate of ascent but use a mean rate between 1000 and 100 mb. Above 100 mb the change with pressure will be taken into account.

We denote:

K = instrument weight
B = balloon weight
A = free lift
$L = K+B+A$ = total load at filling of the balloon (1)
V = volume of the balloon
ϱ = air density
σ = hydrogen density
p, T = air pressure and temperature
v = rate of ascent, m/min
$V_0, \varrho_0, \sigma_0, p_0, T_0$ values of the quantities at filling.

Index 1 refers to the reference sounding, index ν to sounding with the instrument K_ν. V_{01} means the volume of the reference balloon B_1 at filling, $V_{0\nu}$ the same of the balloon B_ν. V_1 and V_ν mean the volumes of the balloons B_1 and B_ν in the state (ϱ, p, T) during the ascent or at bursting.

It is assumed that the temperature and pressure of the hydrogen are the same as that of the surrounding air. This is not the case, however, especially as regards the temperature in the stratosphere in daytime. The pressure differences are negligible for our purposes [2]. Furthermore, we shall disregard any possible diffusion of the hydrogen through the balloon wall during the ascent. The air is supposed to be dry.

We have at filling of the balloon and throughout the ascent

$$V_0(\varrho_0-\sigma_0) = V(\varrho-\sigma) = L \tag{2}$$

for any ascent. By our assumptions we further have

$$\frac{\sigma_0}{\varrho_0} = \frac{\sigma}{\varrho} = s \tag{3}$$

where s is the specific weight of hydrogen with respect to air. From (2) we get

$$V_0 = \frac{L}{\varrho_0(1-s)} = \frac{L}{1.11} \tag{4}$$

Here we have put $s=0.07$ and $\varrho_0=1.19$ at $p_0=1000$ mb and $T_0=+20°C$ $=293°K$.

As is well known, the rate of ascent can be expressed as

$$v = k \sqrt[6]{\frac{\varrho_0}{\varrho}} \frac{\sqrt{A}}{\sqrt[3]{L}} \tag{5}$$

where k is nearly constant during the ascent except for the jump due to the turbulence conditions. We assume that k has the same value for balloons of different sizes.

Replacing the density by means of pressure and temperature, we have

$$v = k \sqrt[6]{\frac{T}{T_0} \cdot \frac{p_0}{p}} \frac{\sqrt{A}}{\sqrt[3]{L}} \tag{6}$$

In the following we shall consider the layer from 1000 mb up to 100 mb with the mean temperature $T=-34°C=239°K$, which is supposed to belong to the pressure $p=300$ mb. Because $p_0=1000$ mb and $T_0=293°K$ we get for 300 mb $\frac{\varrho_0}{\varrho} = \frac{1000}{300} \cdot \frac{239}{293} = 2{,}72$. Considering the balloon as ascending through the layer concerned at the mean rate of 350 m/min, which should be the actual rate at 300 mb level, we get for the reference ascent, from (5),

$$350 = k\sqrt[6]{2{,}72}\,\frac{\sqrt{1000}}{\sqrt[3]{1650}} = k\cdot 1{,}18\cdot 2{,}68$$

which gives the value $k=111$.

The mean rate of ascent between 1000 and 100 mb in any other sounding with the same atmospheric conditions is calculated from (6):

$$v_{300} = 131\,\frac{\sqrt{A}}{\sqrt[3]{L}} \tag{7}$$

In the stratosphere we can compute the rate of ascent for each balloon at any level from (6) by putting $T=221$°K. Denoting the rate of ascent at 100 mb with v_{100}:

$$v_{100} = 155\,\frac{\sqrt{A}}{\sqrt[3]{L}}\,, \tag{8}$$

we obtain from (6)

$$v = v_{100}\sqrt[6]{\frac{100}{p}} \tag{9}$$

valid in the stratosphere.

1. *The influence of instrument weight (K_ν) upon the height reached, the hydrogen consumption and the rate and time of ascent with $B_\nu=350$ g and $A_\nu=1000$ g.*

The bursting pressure with the instrument $K_1=300$ g is $p_1=40$ mb, the bursting volume may be V_1. As the weight B_ν of any balloon is 350 g, it bursts when its volume has reached the value $V_\nu=V_1$. The bursting pressure of the balloon B_ν may be p_ν. As the hydrogen temperature in the balloon is supposed to be equal to the air temperature, $T=221$°K, we have for the state of the hydrogen at bursting

$$p_1 = R_H\frac{M_1}{V_1}T\,,\qquad p_\nu = R_H\frac{M_\nu}{V_\nu}T$$

where M_1 and M_ν are the hydrogen masses and R_H the gas constant in the balloons B_1 and B_ν. As $V_\nu=V_1$ we get

$$\frac{p_\nu}{p_1} = \frac{M_\nu}{M_1} \tag{10}$$

At filling it is

$$p_0 = R_H \frac{M_1}{V_{01}} T_0 = R_H \frac{M_\nu}{V_{0\nu}} T_0$$

or

$$\frac{M_\nu}{M_1} = \frac{V_{0\nu}}{V_{01}} \tag{11}$$

From (4) we obtain

$$\frac{V_{0\nu}}{V_{01}} = \frac{L_\nu}{L_1} \tag{12}$$

In this way we get from (10), (11) and (12)

$$\frac{p_\nu}{p_1} = \frac{L_\nu}{L_1} \text{ or } p_\nu = \frac{L_\nu}{L_1} p_1 \tag{13}$$

This formula allows the computation of the bursting pressure p_ν when taking into account that, according to (1),

$$\begin{aligned} L_1 &= K_1 + B_1 + A_1 = 1650 \\ L_\nu &= K_\nu + 1350 \end{aligned} \tag{14}$$

Geopotential differences are computed by the means of the formula

$$\Phi - \Phi_0 = \frac{R_a}{0.98} \overline{T} \ln \frac{p_0}{p} = 67.4\, \overline{T} \log \frac{p_0}{p} \tag{15}$$

where $\Phi - \Phi_0$ denotes the geopotential difference (gpm) between pressures p_0 and p, R_a the dry air gas constant and $\overline{T}$ the mean temperature of the layer (dry air). Putting $\Phi_0 = 0$ and $\overline{T} = 239$°K, the height of the 100 mb level becomes

$$\Phi_{100} = 16080 \text{ gpm}. \tag{16}$$

The height of the level p mb above 100 mb with $\overline{T} = 221°$ is, according to (15), given by

$$\Phi = \Phi_{100} + 14870 \log \frac{100}{p} \tag{17}$$

From this we get the bursting height of the reference sounding as

$$\Phi_1 = 16080 + 14870 \log \frac{100}{40} = 22000 \text{ gpm}.$$

The mean temperature 239° was, of course, calculated so as to get this result. This happens, for instance, in an atmosphere where the temperature diminishes from the value +20°C at 1000 mb linearly with respect to the pressure logarithm up to about 200 mb and then is constant (221°K) upwards.

To calculate the time of ascent we shall start from

$$d\Phi = v dt \tag{18}$$

where the speed v is taken from (9). Instead of $d\Phi$ in (18) we introduce the pressure from the static balance equation

$$d\Phi = -R'T\frac{dp}{p} \text{ with } R' = \frac{R_a}{0.98} = 29.3\,. \tag{19}$$

Then we obtain from (18)

$$\frac{v_{100}\sqrt[6]{100}}{R'T}dt = -p^{-\frac{5}{6}}dp$$

which gives, integrated from 100 mb to p,

$$t - t_{100} = \frac{6R'T}{v_{100}}\left(1 - \sqrt[6]{\frac{p}{100}}\right). \tag{20}$$

This formula shows the linear dependence of the time upon the sixth root of the pressure. According to (9) the formula (20) can also be written

$$t_\nu - t_{100} = 6R'T\left(\frac{1}{v_{100}} - \frac{1}{v_\nu}\right) = 38850\left(\frac{1}{v_{100}} - \frac{1}{v_\nu}\right). \tag{21}$$

We shall use this formula to calculate the bursting time t_ν with the instrument K_ν. Then v_ν means the rate of ascent at bursting.

By means of the derived formulas we have calculated in Table 1 certain data for six radiosonde weights 240, 300, 400, 600, 1000, 2000 grams. The contents of the table are as follows: Instrument weight K_ν, total load at filling L_ν, bursting pressure p_ν (13), bursting height Φ_ν (17),

Table 1. B_ν = 350 gr, A_ν = 1 000 gr

K_ν g	L_ν g	p_ν mb	Φ_ν gpkm	$V_{0\nu}$ m³	$H_\nu - H_1$ m³	v_{300} m/min	v_{100} m/min	v_ν m/min	t_ν min
240	1 590	38.5	22.25	1.45	−36	355	420	491	58.6
300	1 650	40	22	1.50	0	350	414	481	58.9
400	1 750	42.5	21.6	1.59	66	344	406	467	59.3
600	1 950	47	20.9	1.77	197	332	392	444	59.9
1 000	2 350	57	19.7	2.14	466	312	368	404	60.8
2 000	3 350	81	17.5	3.04	1 120	276	327	338	62.0

filling volume $V_{0\nu}$ (4), excess of hydrogen used yearly (730 soundings) compared with the reference soundings $H_\nu - H_1$, mean rate of ascent between 1000 and 100 mb v_{300} (7), rate of ascent at 100 mb v_{100} (8), ditto at bursting v_ν (9) and time of ascent up to bursting t_ν min.

2. *The influence of the instrument weight K_ν upon the height reached and the hydrogen consumption when the same balloon (B_ν=350 g) is filled up each time to the rate of ascent of the reference sounding.*

When the balloon is filled at 1000 mb and 20°C then $\varrho = \varrho_0$ in (5) and it follows that for any instrument K_ν the rate of ascent at the ground

$$v_{0\nu} = k \frac{\sqrt{A_\nu}}{\sqrt[3]{L_\nu}}. \tag{22}$$

As $k = 111$, we obtain for the reference sounding

$$v_{01} = 111 \frac{\sqrt{1000}}{\sqrt[3]{1650}} = 111 \cdot 2{,}68 = 297 \ \text{m/min}.$$

Since it was assumed that for any instrument $v_{0\nu} = v_{01}$, it follows that

$$\frac{\sqrt{A_\nu}}{\sqrt[3]{L_\nu}} = 2.68 \text{ with } L_\nu = K_\nu + 350 + A_\nu. \tag{23}$$

This equation is practically solved in respect to A_ν by constructing the curve (A_ν, K_ν). Table 2 contains the values of the quantities A_ν and L_ν for the different instruments K_ν. The filling volume $V_{0\nu}$ of each balloon is given by (4). To compute the bursting pressure p_ν of the balloon B_ν we note that the balloon B_ν bursts when its volume V_ν has reached the bursting volume V_1 of the reference balloon B_1 which bursts at 40 mb. Therefore the bursting pressure in this case is also

Table 2. B_ν = 350 g, $v_{0\nu} = v_{01}$ = 297 m/min.

K_ν g	A_ν g	L_ν g	$V_{0\nu}$ m³	p_ν mb	Φ_ν gpkm	$H_\nu - H_1$ m³
240	963	1 553	1.41	37.6	22.4	−66
300	1 000	1 650	1.50	40	22	0
400	1 070	1 820	1.65	44	21.4	109
600	1 194	2 144	1.95	52	20.3	328
1 000	1 413	2 763	2.51	67	18.7	737
2 000	1 875	4 225	3.84	102	16.0	1 710

given by (13). The height reached is computed from formula (17). The excess of hydrogen consumed yearly, $H_\nu - H_1$ m³, is also given in Table 2.

It is convenient that all soundings should be carried out with the balloon ascending at about the same rate. Therefore Table 2 is of greater practical value than Table 1. The heavier the instrument, of course, the shorter is the time of ascent, and hence the lower the bursting height.

3. *The influence of the instrument weight upon the balloon weight and the hydrogen consumption when the balloon weight and its filling are adapted so that the rate of ascent and bursting height are the same as in the reference sounding.*

The ideal conditions for soundings would be that all balloons would rise to a fixed bursting height in a given time, i.e., with the same rate of ascent. This would involve bigger balloons for heavier instruments. Should greater heights than that fixed upon be needed, then still bigger balloons would have to be used. To solve this problem we start from the total load equation (1):

$$L_\nu = K_\nu + B_\nu + A_\nu \,. \tag{24}$$

Here the instrument weight K_ν is given. To compute the three unknown quantities L_ν, B_ν and A_ν we need two further equations. As the rate of ascent is the same in all soundings, this means, on account of (5), that

$$\frac{\sqrt{A_\nu}}{\sqrt[3]{L_\nu}} = \frac{\sqrt{A_1}}{\sqrt[3]{L_1}}$$

which can also be written

$$\frac{A_\nu}{A_1} = \left(\frac{L_\nu}{L_1}\right)^{\frac{2}{3}} . \tag{25}$$

This is the second equation. We obtain the third equation needed by expressing the fact that at bursting the thickness of the balloon rubber is always the same. The weight of the rubber cover for balloons B_ν and B_1 is

$$\begin{aligned} B_\nu &= g d_{0\nu} F_{0\nu} = g d_\nu F_\nu \\ B_1 &= g d_{01} F_{01} = g d_1 F_1 \end{aligned} \tag{26}$$

Here g is the specific gravity of rubber (0.95), $d_{0\nu}$ and d_{01}, $F_{0\nu}$ and F_{01} the thicknesses and areas of the walls of balloons B_ν and B_1 at filling,

d_ν and d_1, F_ν and F_1 these quantities at the bursting of the balloons. The condition of bursting is

$$d_\nu = d_1 \tag{27}$$

When this is taken into account, we get from (26)

$$\frac{B_\nu}{B_1} = \frac{d_{0\nu}}{d_{01}} \frac{F_{0\nu}}{F_{01}} = \frac{F_\nu}{F_1} \tag{28}$$

When we now consider that the area of a sphere is proportional to the 2/3 power of the volume, we obtain instead of (28)

$$\frac{B_\nu}{B_1} = \frac{d_{0\nu}}{d_{01}} \left(\frac{V_{0\nu}}{V_{01}}\right)^{\frac{2}{3}} = \left(\frac{V_\nu}{V_1}\right)^{\frac{2}{3}}. \tag{29}$$

Further, the total load $L_\nu = K_\nu + B_\nu + A_\nu$ remains unchanged through the whole ascent. Therefore at filling and at bursting we have:

$$\begin{aligned} L_\nu &= \varrho_0(1-s) V_{0\nu} = \varrho(1-s) V_\nu \\ L_1 &= \varrho_0(1-s) V_{01} = \varrho(1-s) V_1 \end{aligned} \tag{30}$$

or

$$\frac{L_\nu}{L_1} = \frac{V_{0\nu}}{V_{01}} = \frac{V_\nu}{V_1} \tag{31}$$

When this relation is taken into account, it follows from (29) that

$$d_{0\nu} = d_{01} \tag{32}$$

and further from (29) and (31) that

$$\frac{B_\nu}{B_1} = \left(\frac{L_\nu}{L_1}\right)^{\frac{2}{3}} \tag{33}$$

which is the third equation sought.

We remember that L_1, B_1 and A_1 are given quantities:

$$B_1 = 350, \; A_1 = 1000 \; L_1 = 1650 \tag{34}$$

Therefore we can express A_ν and B_ν by means of L_ν due to (25) and (33):

$$A_\nu = \left(\frac{L_\nu}{L_1}\right)^{\frac{2}{3}} A_1 \text{ and } B_\nu = \left(\frac{L_\nu}{L_1}\right)^{\frac{2}{3}} B_1. \tag{35}$$

In (24) we replace A_ν and B_ν by these values and obtain

$$K_\nu = L_\nu - \frac{A_1 + B_1}{L_1^{2/3}} L_\nu^{2/3}. \tag{36}$$

Finally, replacing the given quantities (34) we have

$$K_\nu = L_\nu - 9.69\, L_\nu^{2/3}\,. \tag{37}$$

By solving this equation for the different instrument weights K_ν in regard to L_ν and using the relations (35) and (4) we obtain the numerical Table 3.

Table 3. $v_{0\nu} = v_{01} = 297$ m/min, $p_\nu = p_1 = 40$ mb

K_ν g	L_ν g	B_ν g	A_ν g	$V_{0\nu}$ m^3	$H_\nu - H_1$ m^3
240	1 520	332	948	1.38	−88
300	1 650	350	1 000	1.50	0
400	1 865	380	1 085	1.70	146
600	2 275	434	1 241	2.07	416
1 000	3 020	523	1 497	2.75	911
2 000	4 725	705	2 020	4.30	2 040

The examination of Tables 1, 2 and 3 illustrates us the questions arised at the beginning of the paper.

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