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ON THE INTERPRETATION OF BATHYTHERMOGRAMS

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Abstract

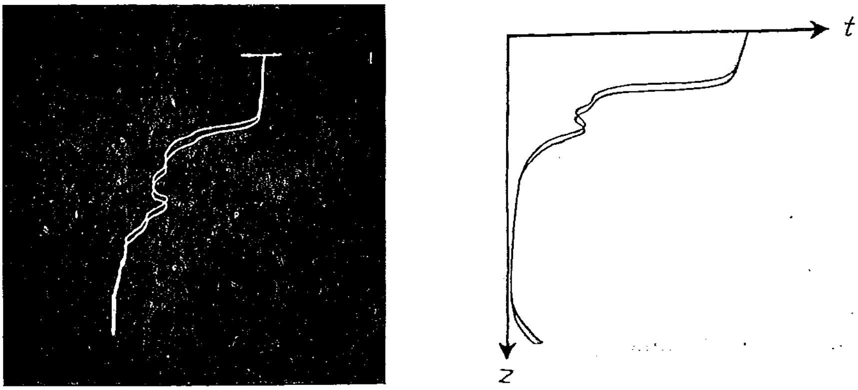
The bathythermographic temperature measurements from a stationary ship show to a certain extent erroneous values due to the slowness of the instrument. These may be corrected by means of the following method using the two curves produced when the instruments is lowered and hoisted respectively.

General. The Bathythermograph[1]) is designed to record the temperature of sea water as a function of depth. In deep water the instrument is usually handled from a ship going full speed, but in shallower water, as in the Baltic, where we have been using the instrument, safety reasons and horisontal temperature differences made it advisable to make the readings while the ship was at full stop. The inherent lag of the instrument and the restricted time available for the measurement under those circumstances will, however, cause the temperature curves registered when lowering and hoisting the bathythermograph through the same water layers to differ from one another (temperature hysteresis). The bathythermograms obtained in this way may thus show a pattern as *e.g.* that shown in figure 1.

Construction of the Bathythermograph. The temperature-sensitive element of the bathythermograph is a capillary containing a fluid. The changes in the volume of the fluid relative to the capillary cause a stylus at the end of a Bourdon tube to move transversely over a smoked glass slide. The depth

1) Spilhaus, A. F., 1938: A Bathythermograph. *Journ. Marine Research*, 1, 95—100.

is registered by a spring-loaded Tombak tube functioning like a simplified aneroid barometer; this system moves the slide longitudinally. The combination of the two movements produces a temperature-pressure (i.e., depth) curve. The curve may be read against a calibration grid appropriate to each individual instrument.



a b

Figure 1. a) A photographic copy of a real bathythermogram. b) A schematic representation of the same.

Errors and their Correction. The temperature-sensitive element does not operate instantaneously, but gradually approaches the temperature to be registered as time elapses. Since the measurement must be carried out in a limited time, the bathythermogram does not show the temperature with complete accuracy, the error being greater the more quickly temperature varies with depth. This error may be allowed for to a certain extent, as the bathythermogram shows two curves (figure 1), one produced when the instrument is lowered, and other when it is hoisted. For mathematical discussion of these curves, the following notation has been adopted:

$t \equiv t\ (z) \equiv$ true temperature at depth z

$t_1 \equiv t_1\ (z) \equiv$ bathythermogram reading at depth z when the instrument is lowered

$t_2 \equiv t_2\ (z) \equiv$ bathythermogram reading at depth z when the instrument is hoisted

$v_1 > 0 > v_2 \equiv$ the velocities of the instrument corresponding to the values t_1 and t_2

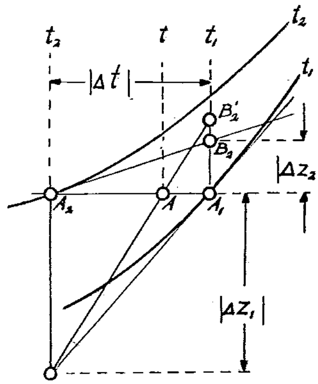


Figure 2. Correction of errors by the double curve method.

The rate at which the temperature of the instrument changes is proportional to the difference between the temperatures of the instrument and of its environment. Hence

$$dt_1 = k(t_1 - t)\frac{dz}{v_1}, \text{ and} \tag{1}$$

$$dt_2 = k(t_2 - t)\frac{dz}{v_2}, \tag{2}$$

from which we obtain

$$t = t_2 + \frac{t_1 - t_2}{1 - \dfrac{v_1}{v_2}\dfrac{dt_1}{dt_2}}. \tag{3}$$

This may also be written

$$t = t_2 + \frac{\Delta z_1 (t_1 - t_2)}{\Delta z_1 - \dfrac{v_1}{v_2}\Delta z_2}, \tag{4}$$

where $\Delta z_1 = \frac{dz}{dt_1}\Delta t$ and $\Delta z_2 = \frac{dz}{dt_2}\Delta t$,

Δt being the difference between the temperatures read from the curves recorded on lowering and on hoisting at depth reading z.

Equation (4) enables us to obtain the true temperature t at depth z (Figure 2). Through the points A_1 and A_2 on the curves for lowering and hoisting respectively, corresponding to the depth reading z, the ordinates and the tangents are drawn. Their intersections are designated B_1 and B_2. Point B_2' is now determined in such a way that

$$\frac{B_2' A_1}{B_2 A_1} = \frac{v_1}{v_2} .$$

The point A at which A_1A_2 and B_1B_2' intersect corresponds to the temperature t.

Maximal & Minimal Values on the Bathythermogram. Equations (1) and (2) may be written in the general form

$$dt_n = k\,(t_n - t)\,\frac{dz}{v_n} .$$

The solution of this equation is

$$t_n = t - \int_{z'}^{z} e^{\frac{k}{v_n}(u-z)}\,\frac{dt}{du}\,du \tag{5}$$

where the value of z' is such that $t_n(z') = t(z')$. Where the bathythermogram shows an extreme temperature

$$\frac{dt_n}{dz} = 0;$$

and, therefore:

$$0 = \frac{dt_n}{dz} = \frac{dt}{dz} - e^{\frac{k}{v_n}(z-z')}\,\frac{dt}{dz} + \int_{z'}^{z} e^{\frac{k}{v_n}(u-z)}\,\frac{k}{v_n}\,\frac{dt}{du}\,du$$

or

$$\int_{z'}^{z} e^{\frac{k}{v_n}(u-z)}\,\frac{k}{v_n}\,\frac{dt}{du}\,du = 0$$

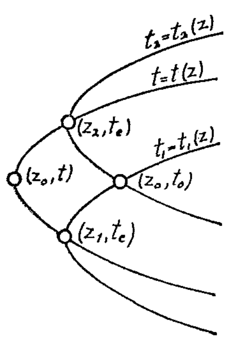


Figure 3. The method of deriving the extreme temperature from the bathyhermogram extremes.

and according to equation (5)

$$t_n = t.$$

The maximal & minimal points on the bathythermogram thus show the true temperature of the environment.

Maximal & Minimal Temperatures. To determine the true temperature extremes an approximation method is used. To this end, two assumption are made:

Firstly it is assumed that both curves of the bathythermogram show the same extreme temperature reading t_e at depths z_1 and z_2 respectively; and

secondly, that near the extreme values the curves closely resemble the parabolas

$$t_1 = t_e + a\,(z - z_1)^2 \qquad (6)$$
$$t_2 = t_e + a\,(z - z_2)^2$$

These parabolas intersect at (z_0, t_0), and according to equations (6)

$$2\,z_0 = z_1 + z_2. \qquad (7)$$

The true extreme temperature may now be obtained from equation (3) by putting $z = z_0$. But since

$$v_1 \frac{dt_1}{dz} = v_2 \frac{dt_2}{dz} \qquad (8)$$

we have to adopt the limiting value method, and write

$$t = \frac{v_2 t_1 \dfrac{d^2 t_2}{dz^2} + v_2 \dfrac{dt_1}{dz}\dfrac{dt_2}{dz} - v_1 t_2 \dfrac{d^2 t_1}{dz^2} - v_1 \dfrac{dt_1}{dz}\dfrac{dt_2}{dz}}{v_2 \dfrac{d^2 t_2}{dz^2} - v_1 \dfrac{d^2 t_1}{dz^2}}$$

which, on simplification, becomes:

$$t + t_0 = 2\,t_e.$$

The true temperature extreme and the intersection of the bathythermogram curves are approximately symmetrical to the value $t = t_e$. This approximation is admittedly rather rough, but as equations (3) and (4) cannot be directly applied in this case, the method offers certain advantages.