551. 551: 551. 554

SOME EMPIRICAL STRESS-VALUES FOR THE LOWER TROPOSPHERE[1])

by

E. O. Holopainen

International Meteorological Institute, Stockholm

Abstract

An attempt has been made to determine shearing stresses in the lowest two kilometers of the atmosphere. Vertical integrations of the equations of motion have been made using data from aerological stations in southern U.S. during suitable types of weather situations. A mean value $1.5 \cdot 10^{-3}$ for the geostrophic drag coefficient was obtained from the computed surface stress values. The Austausch coefficient at one and a half kilometers height was found to be about 100 g cm^{-1} sec^{-1}.

1. *Introduction*

The purpose of this work is to clarify, in what amount daily wind observations can be used in a study of atmospheric friction. Turbulent friction is a factor, which according to our present knowledge had to be incorporated in the numerical prognoses, at least in those for longer time periods. Therefore it is of paramount importance to know eventual relations between friction and those parameters, which are used in prognostic models for the atmosphere.

The motion close to earth's surface is mainly determined by surface stress and geostrophic wind, and the latter is a parameter, which appears

---

[1]) The research reported here has been sponsored in part by the Geophysics Research Directorate, AFCRL, AFRD, of the Air Research and Development Command, USAF, through its European Office.

in every prognostic model. LETTAU [4] has introduced the concept of geostrophic drag coefficient, which relates the surface stress to the square of geostrophic wind at the surface. He has made a preliminary study of its dependence on Richardson-number, latitude and roughness of the surface. However, direct stress measurements exist only for relatively smooth surfaces, and it is an open question, how such a mean stress, which is needed in numerical prognoses, is dependent on the geostrophic wind.

Besides direct measurement of stress[1], another method of determining it is the use of equations of the mean motion. By this method some

Table 1. Distribution of Austausch-coefficient with height (after LETTAU [1] and [3]). Unit: g cm^{-1} sec^{-1}.

1. Leipzig wind profile
2. Scilly wind profile I (air warmer than sea)
3. Scilly wind profile II (air cooler than sea)

z in meters	1	2	3
950	58		
900	70		
950	81		
800	95		
750	108		
700	118		
650	128		
600	137		
550	142		
500	150	15	17
450	160	17	21
400	166	18	23
350	175	21	25
300	179	24	27
250	180	26	30
200	179	29	35
150	173	32	37
100	163	35	40
50	132	13	29
20		4	20

[1]) In this article »stress» means horizontal force per unit horizontal area due to turbulent eddies.

information has been obtained concerning the magnitude of internal stresses in the atmospheric first kilometer. Already classical is the work done by MILDNER [5] and his »Leipzig wind profile», which he in 1932 obtained through double-theodolite wind observations at the Leipzig airport. LETTAU [1] has reanalyzed his data and obtained the vertical distribution of the Austausch coefficient (usually denoted by A) up to 950 meters in height. From Scilly Island data (SHEPPARD *et al.* [7]) LETTAU [3] has worked out the corresponding distribution of A over the sea up to five hundred meters in height. These only available empirical A-profiles have been tabulated in Table 1. For the variation of the Austausch coefficient in the atmosphere above the first kilometer we have no empirical knowledge.

One of the restrictions in the use of the equations of mean motion in stress computations is that the magnitude of the stress must be known at some height. In the absence of strong thermal turbulence it can be assumed, that $\tau_{xz} = 0$, where $\frac{\partial u}{\partial z} = 0$ and $\tau_{yz} = 0$ where $\frac{\partial v}{\partial z} = 0$ (τ denotes stress. x, y, z, u, v have their usual meaning).

In the present work this assumption has been made for a special weather situation, which often prevails in the southern part of U.S. At the surface anticyclonic, easterly or southeasterly winds blow. Due to a strong northward directed temperaturegradient the flow turns with height into westerly or northwesterly winds, which increase rapidly with height. In this situation the zonal wind component reaches a minimum within the first kilometer, while the meridional wind component often has a maximum. The vertical shear of the zonal wind is already large at 1500-meters. The above minimum and maximum heights are suitable beginning-points in a vertical integration of the equations of motion, and the big shear value of zonal wind higher up makes it possible to compute the stress term for this component.

2. *Data. Computational procedure*

Stress computations have been made for the station Jackson [lat. 32° 20′ N long. 90° 13′ W]. Some aerological observations from years 1956—60 were selected, in which the wind profile has the character described above. The horizontal momentum equations

$$\frac{\partial \tau_{xz}}{\partial z} = \varrho \frac{\partial u}{\partial t} + \varrho \mathbf{v}_H \cdot \nabla u + \varrho w \frac{\partial u}{\partial z} - f\varrho (v - v_g)$$
$$\frac{\partial \tau_{yz}}{\partial z} = \varrho \frac{\partial v}{\partial t} + \varrho \mathbf{v}_H \cdot \nabla v + \varrho w \frac{\partial v}{\partial z} + f\varrho (u - u_g) \tag{1}$$

have been integrated from the surface to the height, where $\frac{\partial u}{\partial z} = 0$ and $\frac{\partial v}{\partial z} = 0$, respectively, and from these heights up to 1400 meters height (wind in Jackson is reported for the surface and for 150, 300, 400, 900, 1400, 1900 ... meters heights). Local time derivatives have been evaluated from sequent wind observation, which in best cases exist with six hours intervals. When computing horizontal advection terms, winds for five other stations (Montgomery, Burrwood, Lake Charles, Shreveport and Little Rock) were considered. Surface wind observations were not used for this purpose but the horizontal advection terms in the layer 0—150 m were assumed to have the same value as at the 150 meters level. Vertical velocities have been estimated from horizontal divergence using the continuity equation. For determination of the geostrophic wind, surface maps and 850 mb topography charts were used. In addition it was assumed, that the variation of geostrophic wind between 1000 mb and 850 mb is linear.

For surface stress the geostrophic departure term appeared to be the dominating one, while for the stress at 1400 meters height all terms in eq. (1) were important.

3. *Results*

a) S u r f a c e s t r e s s

From the τ_{x0}, τ_{y0} — values the magnitude of surface stress vector was computed. In Figure 1 this quantity τ_0 is plotted against magnitude of the geostropic wind at the surface (V_{g0}). Correlation between τ_0 and V_{g0}^2 is 0.9, while between τ_0 and V_{g0} it is 0.8. The results give thus some support to the use of the concept of geostrophic drag coefficient. From Fig. 1 a mean value $1.5 \cdot 10^{-3}$ for this coefficient

$$c_g = \frac{\tau_0}{\varrho_0 V_{g0}^2} \tag{2}$$

can be obtained (density ϱ_0 has been taken a constant $= 1.2 \cdot 10^{-3}$g cm^{-3}).

An attempt was made to explain the scattering of the τ_0, V_{g0} points

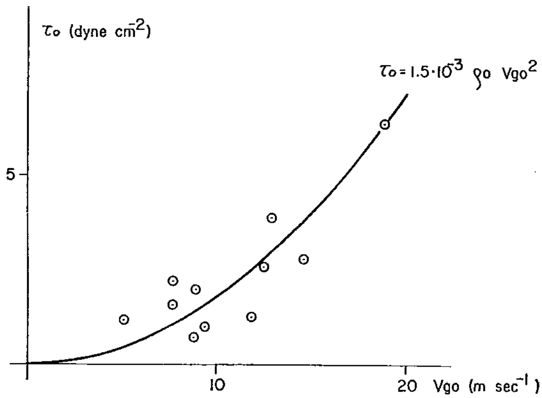

Figure 1. Variation of surface stress with the geostrophic wind at the surface.

with aid of static stability, as measured by the first temperature readings of a radiosonde observation. The eleven observations were divided into two groups: 1) six observations with $\frac{\partial T}{\partial z} > 0$ (inversion) and 2) five observation with $\frac{\partial T}{\partial z} < 0$. The group means of the geostropic drag coefficient in the first and second class were found to be $1.4 \cdot 10^{-3}$ and $1.6 \cdot 10^{-3}$, respectively. These numbers can, of course, only be considered as a qualitative indication of the dependence between surface stress and static stability at the surface.

From the »Leipzig wind profile» Lettau obtained drag coefficient of $1.4 \cdot 10^{-3}$ and the direct stress measurements on the prairie in Nebraska [2] gave a daily mean value of $1.1 \cdot 10^{-3}$. These values can be considered as representative for uniform fields with no bigger roughness elements, while in the value $1.5 \cdot 10^{-3}$ for Jackson such elements (trees, houses) are present. On the basis of these numbers one could think, that the geostrophic drag coefficient, and therefore also the surface stress, may not be very sensitive to the roughness of the surface. This question needs, however, much more investigations.

The mean value of the cross-isobaric angle of the surface wind was found to be 43° with a standard deviation of 17°. The ratio of the mean surface wind to the mean geostrophic wind at the surface was 0.37.

Direct measurements of stresses have shown, that over uniform, flat fields the surface stress is related to the wind speed through formula

$$\tau_0 = c\varrho_0 V_0^2 \tag{3}$$

where the drag coefficient c depends besides the stability of the air and the roughness of the surface, also upon the height at which v_0 is measured. This formula has in some investigations been assumed generally valid, V_0 being the wind at the »anemometer level». If the formula (3) is used in relating the stress-values of Fig. 1 to the wind at anemometer level in Jackson, a value of $c = \frac{\bar{\tau}_0}{\varrho_0 \overline{\overline{V_0^2}}} = 12 \cdot 10^{-3}$ can be deduced. This value is considerably larger than values used in studies involving determination of mean surface stresses in synoptic scale (*e.g.* [6]). However, the anemometer level is not uniqly determined and surface wind observations at any station are influenced by local effects. Considering general conditions, the formula (3) defines therefore a very complicated drag coefficient c, and the value $12 \cdot 10^{-3}$, deduced above, may not have any generality. In studies of large scale motion eventual empirical relationships of that type as in Figure 1 are really desirable.

b) Zonal stress at 1400 meters height

Figure 2 shows the computed $\tau_{x\,1400}$-values against the vertical shear of the zonal wind at this height. The value of the linear correlation is only 0.3 and depends highly upon three large stress values with relatively small shear. Because the distribution of the other points is more regular, these three values have been disregarded when finally computing mean stress and mean wind shear. The value of the ratio $A = \bar{\tau}_{x\,1400} \Big/ \left(\frac{\partial \bar{u}}{\partial z}\right)_{1400}$ was then obtained to be 84 g cm^{-1} sec^{-1}. An attempt was made to explain the scattering of the computed $\tau_{x\,1400}$-values with the aid of static stability at this height. The computational errors are, however, too big to allow any success in such attempts.

$\tau_{y\,1400}$ was also computed. The correlation between $\tau_{y\,1400}$ and $\left(\frac{\partial v}{\partial z}\right)_{1400}$ was found to be 0.3. Any functional dependence between these quantities could, however, not be observed. The main reason for this was probably the location of neighbouring aerological stations, which did not allow accurate enough determination of meridional derivative of geopotential, as needed in equation for v-momentum.

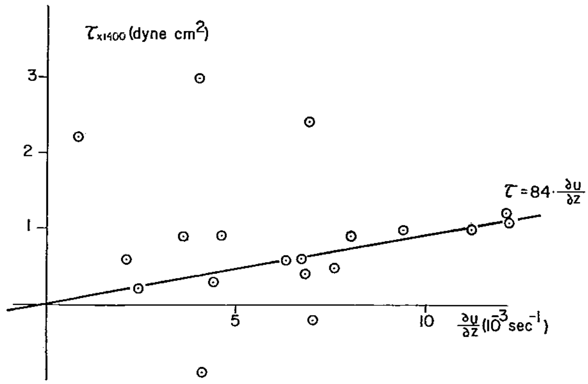

Figure 2. Variation of the zonal stress with vertical shear of the zonal wind at 1400 meters.

The only result of section b) is thus, that a value $A \simeq 100$ g cm^{-1} sec^{-1} $\left(K = \dfrac{A}{\varrho} \simeq 10^5 \text{ cm}^2 \text{ sec}^{-1}\right)$ seems, on the average, to describe the magnitude of turbulent transfer of momentum at one and a half kilometers height over relatively flat land surface.

The role of small scale turbulent friction in the middle and upper troposphere can hardly be investigated by direct use of the equations of motion because of quasi-balance between the mass and momentum fields in these layers and due to increase of observational errors with height. The problem can perhaps be approached by use of the kinetic energy equation.

REFERENCES

1. LETTAU, H., 1950: A re-examination of the »Leipzig Wind Profile» considering some relations between wind and turbulence in the frictional layer. *Tellus*, **2**, 125—129.
2. —»— and B. DAVIDSON 1957: *Exploring the atmospheric first mile*, Vol. I. Pergamon Press, New York, p. 337.
3. —»— 1957: Windprofil, Innere Reibung und Energie Umsatz in den unteren 500 m über dem Meer. *Beitr. Phys. Atmos.*, **30**, 78—96.
4. —»— 1959: Wind profile, surface stress and geostrophic drag coefficients in the atmospheric surface layer. *Advances in geophysics*, **6**, 241—257.
5. MILDNER, P., 1932: Über Reibung in einer speziellen Luftmasse in den untersten Schichten der Atmosphäre. *Beitr. Phys. fr. Atmos.*, **19**, 151—158.
6. PALMÉN, E., 1958: Vertical circulation and release of kinetic energy during the development of hurricane Hazel into an extratropical storm. *Tellus*, **10**, 1—23.
7. SHEPPARD, P. A., H. C. CHARNOCK and J. R. D. FRANCIS, 1952: Observations of the westerlies over the sea. *Quart. J. Roy. Met. Soc.*, **78**, 563—582.